Rigid body setup correction in three-dimensional external beam radiotherapy represents a homogeneous transformation. The reference point set P from treatment planning CT and the transformed point set P' from the patient treatment setup are related by the equation P'=RP+T, where R and T are the rotation and the translation components. The translation is commonly calculated as the difference in centers of mass (CM), T_{cm}=P_{cm}'-P_{cm}. This study proposes a revised formalism derived from the unified transformation equation P'=MP. The matrix inversion solution $M=P'P^{-1}$ is not in agreement with T_{cm} if the isocenter is away from the center of mass. The isocenters for the reference set (I) and the transformed set (I') are expressed as an invariant linear combination of a body bases vectors u_{123} , $I=au_1+bu_2+cu_3$ and $I'=au_1'+bu_2'+cu_3'$. The translation between these two isocenters $T_{iso}=I'-I$ is consistent with the result from the matrix inversion. This technique then enables us to accurately calculate the rotation R. A congruent set of four corresponding noncoplanar points are required for P and P'. Subsequent clinical setup corrections are the couch translation T_{iso} and the gantry, collimator, and couch angles calculated from the rotation R. Data from a prostate patient with two sequential CTs and virtual simulations corroborate the theory. Unlike search methods with many approximate solutions, this formalism yields a single exact solution in closed form. With the translation calculated by the improved algorithm it will now be possible to implement full 3D rigid body transformations for robust on-line setup correction.