Positron Emission Tomography II: Data Corrections and Calibrations

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Outline

I. Brief Introduction to PET
II. Organization of PET data
III. Data Correction Methods for PET

NOTE: TOPICS DISCUSSED ARE SUBJECTS OF ACTIVE RESEARCH - HERE WE DESCRIBE SOME OF THE ALGORITHMS CURRENTLY IMPLEMENTED IN COMMERCIAL CLINICAL SYSTEMS.

What is Positron Emission Tomography? (PET)

PET is a Nuclear Medicine tomographic imaging technique that uses a tracer compound labeled with a radionuclide that is a positron emitter. The resulting radio-emissions are imaged.

RESULT
- Cross-sectional image slices representing regional uptake of the radio-chemical
- Quantitative information in absolute units of µCi/cm³ or in terms of actual rates of biological processes that utilize or incorporate that chemical
**What Does a PET Scanner Look Like?**

- **Full Ring**
- **Discrete Crystal**
- **Partial Ring**
- **Discrete Crystal**
- **Full Ring**
- **Curved Plate**

**“Dedicated”**

**“Scintillation Cameras”**

**PET Annihilation Photon Detectors**

**“Block Detector”**

Example

- **Scintillator:** Sodium Iodide (NaI(Tl))

**PET Detectors - ctd.**

**“Curved-Plate Scintillation Detector Head”**

Example

- **Scintillator:** Sodium Iodide (NaI(Tl))

From Siemens/CTI “ECAT 931” (BGO)

From Philips-ADAC “C-PET” (NaI(Tl)), Courtesy of J. Karp, UPENN
PET Data Collection Modes

Two-dimensional (SEPTA IN)
- N direct + N(N-1) oblique = 2N-1 sinograms
- Axial sensitivity profile

Three-dimensional (SEPTA OUT)
- N direct + N(N-1) oblique = 2N-1 sinograms
- Axial sensitivity profile

Types of Coincident Photon Events

Organization of PET Data
Organization of Tomographic Data

For reconstruction the tomographic projection data is often organized into a “sinogram”

Line of Response (LOR): Number of coincident photon pairs detected along a given line between two detector elements.

Sinogram = Stacked profiles or “projections” from all angles.

PET Data Corrections for Quantitative Accuracy

- Photon attenuation (largest correction factor)
- Detector efficiency non-uniformity (Normalization)
- Detector saturation (Dead-time)
- Random coincidences
- Scattered coincidences
- Isotope decay
- Blurring (Partial volume)
- Calibration factor
Photon Attenuation in PET

Reconstructed Images

Example:
Uniform activity cylinder

\[ \text{Photon attenuation produces non-uniform activity artifacts} \]

Photon Attenuation Correction (AC)

Attenuation Correction—the largest correction factor

\[ A_{\text{t}} = N_0 e^{-\mu D} \]

\[ \text{Attenuation Correction Factor} = \frac{N_i}{N} = e^{-\mu D} \]

How do you correct for photon attenuation?

- Can be measured directly with transmission source.
- Can be calculated by contour finding algorithm (brain, phantoms only)
- Correction factors calculated for each detector line-pair

For measured transmission data can use:
- Rotating rod source \(^{137}\text{Cs} or \(^{68}\text{Ge}\) (traditional PET)
- X-ray source (PET/CT)

Measured correction factors calculated from:
- Measured transmission data
- Segmented transmission data

Example: Rotating Rod Source
**Measured Attenuation Correction**

Attenuation correction factors calculated for each detector pair

Blank-represents non-attenuated flux, $N_o$

Transmission-represents attenuated flux, $N$

Attenuation correction factors $= N_o/N = (\text{blank data/ transmission data})$

$= e^{\mu D}$ for every detector line-pair

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**Measured Attenuation Correction (AC)**

<table>
<thead>
<tr>
<th>Before Attenuation Correction</th>
<th>After Attenuation Correction</th>
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</table>

(human thorax)

Traditional measured attenuation correction introduces additional noise

Attenuation Correction accounts for absorption of photons in body

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**Segmented Attenuation Correction**

- Drawbacks of measured attenuation correction are that takes a relatively long time to acquire transmission data and noisy transmission data propagate errors into corrected result.

- Segmented attenuation correction utilizes a short transmission study to simply define boundaries of lungs and soft tissue for segmentation.

- The soft tissue partition is assigned one fixed, non-fluctuating, noiseless $\mu$-value. Typically the lung partition is allowed to float as measured but could be assigned a distinct fixed $\mu$-value as well.

- The result is that less noise is propagated into the attenuation correction process using the segmented attenuation map to generate attenuation correction factors; That is the S/N is improved. Since the transmission study is shorter, the overall study time is also reduced.
Segmented Attenuation Correction

Breast Imaging

Emission - no correction  Segmented Transmission  Emission with SAC

Attenuation Correction using CT

“Biograph” PET/CT System (Siemens Medical)

Combining PET and CT into one system offers:

• Anatomic map to fuse onto functional map
• Improved fusing accuracy
• Photon attenuation coefficients from CT
• Faster photon attenuation correction
• Less noise generated in attenuation correction
Detector Normalization

- There are inherent PET detector variations in parameters such as light output from the individual scintillation crystals, reflectors, coupling to the PMTs, PMT gain variations, etc.
- Different detector pairs register different count rates when viewing the same activity.
- Normalization factors are measured by irradiating each detector pair with the same amount of activity and recording the coincidence count variations.
- Activity can be configured as a thin plane source or a line source that rotate within the field-of-view or a precisely centered uniform cylinder.
- Normalization takes care of both geometric and intrinsic sources of non-uniformity.
- High counts must be acquired so that the normalization factors do not introduce noise into the corrected data.

Normalization: Correction for non-uniform detector response

Example: Using a rotating plane source

Normalization corrects for variations in crystal geometric and intrinsic detection efficiencies throughout the detector gantry.
Random Coincidences

- Random coincidence rate along any LOR may be directly measured using the Delayed Coincidence Method.
- May be calculated from single photon measured rate.

Correction for Random Coincidences

**Delayed Coincidence Method**
- Delayed Coincidence Method uses two coincidence circuits
- The first circuit is used to measure the true coincidences + randoms along all lines-of-response
- The second has a delay of several hundred microseconds inserted so all true coincidences are thrown out of coincidence.
- The average detected single photon rate is the same for both circuits.
- Along each line-of-response, the counts measured in the delayed circuit are subtracted, on-line, from those of the prompt circuit.
- Note: Due to statistical fluctuations, the random events included in the prompt (trues $T$) + randoms $R$) circuit are not equal to those of the delayed circuit, so subtraction of measured random events increases the statistical noise. Thus, if $N = \text{the number of prompt + delayed events} = (T+R) - R$, assuming Poisson statistics the error or noise in $N$ propagates as: $\Delta N = \sqrt{T + 2R} = \sqrt{T + 2R}$

**Calculated Method**
The random coincidence rate for annihilation photons detected along a given LOR is given by:

$$R_{ij} = 2\Delta \tau \cdot S_i \cdot S_j$$

where $\Delta \tau$ is the coincidence time resolution, which accounts for difference in arrival times of the two photons, the scintillation light decay time, variations in photoelectron transit times in the PMT, delay variations in electronic processing circuits, etc.; $S_i$ is the single photon detection rate in detector crystal $i$.

Note: Increasing the activity by a factor $f$ increases the trues and singles rates $T$ and $S$ by the same factor but the randoms rate $R$ increases by $f^2$.

Note: In this case since a separate measurement was not required, $\Delta N = \sqrt{T+R}$
Scatter Coincidences

Techniques for Scatter Removal:
- Partially rejected by energy discrimination
- Deconvolution method (2D PET)
- Interpolation Fitting (3D PET - brain only)
- "Model-Based Scatter Estimation" (a.k.a. Activity and attenuating media dependent scatter estimation) (3D PET).
- Scatter corrections for 3-D whole-body PET that have been implemented in commercial clinical systems are just estimates and not highly accurate.

\[ \mu_{\text{water}} \approx 0.095 \text{ cm}^{-1} \]
\[ \mu_{\text{bone}} \approx 0.15 \text{ cm}^{-1} \]
\[ \mu_{\text{lung}} \approx 0.03 \text{ cm}^{-1} \]

>99% of 511 keV photon attenuation in body tissues is due to Compton scatter.

Removal of Effects of Compton Scatter

Energy Discrimination

\[ E_s = \left( \frac{511}{2 - \cos \theta_s} \right) \]

where \( E_s \) is the detected scatter photon energy when the 511 keV photon scatters by an angle \( \theta_s \).

For example, if the photon scatters 57° before entering a detector in the ring, \( E_s \approx 350 \text{ keV} \). So setting the LLD at 350 keV would mean the system accepts all incoming photons that have scattered through angles ranging 0-57° (assuming perfect energy resolution). This is rather poor scatter rejection capability. A 475 keV LLD would mean only scatter photons with \( \theta_s < 22^\circ \) contaminate data, which would reject significantly more scatter photons, but this could significantly reduce photon count sensitivity.

Removal of Effects of Scatter - ctd.

Deconvolution (2-D PET only)

- Scatter distribution is variable across the field of view.
- The measured data (true+scatter) is assumed to be a convolution of a position-sensitive scatter kernel with the true data.
- The position-dependent scatter kernel is determined by measurements of the scatter distribution across the FOV. This is typically done with a line source positioned at many precise locations on a grid within a tissue-equivalent phantom.
- The scatter kernel is estimated by fitting a Gaussian function to the tails of the measured line source distribution (see figure).
- These measured scatter kernels are deconvolved from measured projection data (true+scatter) to result in an estimate for the true.
- Assumes the shape of the line-spread-function and scatter is the same for all activity and attenuation media distributions in a PET study, and so the resulting scatter estimate serves as only a rough approximation when the scatter fraction is low (2D PET).
Removal of Effects of Scatter in 3D PET

Interpolation-fitting (3-D PET, brain studies only)

- Raw 3-D PET data contains a high fraction of counts from scattered photon events (40-70%).
- The tails of the measured activity distributions outside the boundaries of the head seen from any projection angle are a result of only scatter coincidences.
- If one can accurately fit a Gaussian (or Cosine) function to the tails of the distributions one can interpolate an estimated shape of the scatter distributions inside the head.
- To correct for scatter, for every LOS in every projection throughout the 3-D data set, fit the tails of the measured distribution to a Gaussian, and subtract that function from the measured data.
- This approach assumes the object being imaged is far from the FOV edge so that tails are present, the tails must contain adequate statistics for an accurate fit, and the object should contain homogeneous scatter media that always produces a Gaussian shaped scatter distribution. Thus, this approach is most useful only for scatter correction in brain studies.

Scatter Correction in 3D PET - ctd.

Model-Based Scatter Estimation (3-D PET, entire-body)

- Assume for scatter events that only one photon scatters per annihilation pair and that photon scatters only once before detection.
- Designate the object by preliminary 2-D estimates of the scatter and attenuation distributions from the direct photon data reconstructed without scatter correction.
- Calculate an estimation for single-scatter contamination into every LOS. Square scatter arising from sources outside of solid FOV. Subtract the estimate from each of the measured LOS in the 3-D data set.
- The single scatter contribution to a given LOS within a projection view can be expressed as the volume integral of a scattering kernel over the scatter position. It is the scattering medium (see figure):

\[
R_{\text{ss}} = \iint_{V} S_{\text{ss}}(x, y, z) \frac{dV}{d\Omega} e^{-\mu d} e^{-\sigma d} dV
\]

where \( R_{\text{ss}} \) is the mean total coincidence rate in an LOS due to single-scattered events, \( S_{\text{ss}} \) is a single-scatter sample point (see figure), \( \mu \) is the attenuation coefficient, \( \sigma \) is the photon original energy, \( d \) is scattered energy, \( \Omega \) is the scattering angle, \( q_b \) and \( q_o \) are the detector cross sections at points A and B, respectively, \( R \) and \( L \) are the distance from the scatter point to these detectors, and \( e_{\text{in}} \) and \( e_{\text{out}} \) are the two detector efficiencies. To reduce computational burden, discrete sampling of image volume and interpolation for points in between are used.

**IS A HIGHLY ACCURATE SCATTER CORRECTION FOR 3-D PET REQUIRED?**

**2-D PET**:

- Scatter fraction is only 10 - 20% effect. A 30% error in scatter estimate is ~3 - 6% error in result. (And systematic error in PET is ~10%)

**3-D PET**:

- Scatter fraction is 80 - 100% of the trues. A 30% error in scatter estimate is ~20 - 40% error in result. This is significantly larger than systematic errors in PET.

SO, YES!
Dead time Correction

• Data loss mechanisms in a positron camera are a result of two separate system deadtimes, one from the detector processing system, the other from the data processing system.

• For the activity concentrations typically used in PET (~1 µCi/ml) the live time fraction is described by the paralyzable model of system deadtime.

• Live time fraction for each detector block ($BL_i$) is given by:

$$BL_i = \exp(-N_S i \tau_{block})$$

where $S_i$ = average single count rate for detector $i$, $N$ is the number of crystals per detector block, and $\tau_{block}$ is the time constant of the front end block detector signal processing, including scintillation decay time, pulse height discrimination, and crystal identification. Typically $\tau_{block}$ ~2-3µs.

• Live time fraction of back end data handling acquisition system ($AL_i$) is given by:

$$AL_i = \exp(-C_L i \tau_{system})$$

where $CL_i$ = ideal coincidence count rate load for block $i$. $CL_i = true + scatter + multiples + randoms / BL_i^2$, and $\tau_{system}$, the data processing deadtime is typically ~200 ns.

Dead time Correction - ctd.

Thus, the overall system live time fraction for a line-of-response within detector $i$ is ($SL_i$) is given by:

$$SL_i = AL_i \cdot BL_i^2 = \exp(-C_L i \tau_{system}) \cdot \exp(-2N_S i \tau_{block})$$

Thus, the dead time correction ($DC_i$) for line-of-response $i$ is given by:

$$DC_i = CR_i / SL_i$$

where $CR_i$ is the measured coincidence rate.

Dead time correction is typically done on-line, during data collection.

Typical $SL_i$ values in 2-D PET are ~90% and typical $DC_i$ values are ~1.1.

Decay Correction

The activity strength $A$ of a radioactive isotope after a given time $t$ is given by:

$$A = A_0 \cdot e^{-0.693 t / \tau_{1/2}}$$

where $A_0$ is the initial activity of the radionuclide and $\tau_{1/2}$ is its half-life.

• PET studies may involve a short-lived isotope, multiple time-frame dynamic studies, multiple bed position whole-body studies, or a relatively long study duration.

• For qualitatively accurate images and quantitatively accurate data, the data measured in each time frame must be corrected for the decay of the isotope with time.

• The decay correction factor for each projection plane or sinogram is given by $e^{+0.693 t / \tau_{1/2}}$. 
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<thead>
<tr>
<th>Image Activity Calibration</th>
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<tr>
<td>• To reconstruct PET images in absolute units of µCi, it is necessary to calibrate the system with a standard source of known activity.</td>
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<td>• A cylinder filled uniformly with activity is imaged with all corrections applied.</td>
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<tr>
<td>• A small sample is taken from that cylinder and well-counted for the absolute activity concentration in the cylinder (µCi/ml).</td>
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<tr>
<td>• The counts in a selected region-of-interest (ROI) (area known) from an image slice (known thickness) from the uniform cylinder image volume are recorded to obtain counts/ml or counts per second (cps) per ml in the images.</td>
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<tr>
<td>• Dividing the two factors gives the calibration factor of image counts (or cps) into µCi.</td>
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<th>Quantitative PET</th>
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<tr>
<td>Before image data can be reconstructed, the absolute activity concentration (µCi/cm³) (X) of a projection data set must be determined by applying a series of correction factors to each LOR in the raw projection data:</td>
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<tr>
<td>[ X = (\text{RAW DATA} - \text{RANDOMS} - \text{SCATTER}) \times AC \times N \times DTC \times DC \times CF, ]</td>
</tr>
<tr>
<td>where (AC) and (N) are the attenuation correction and normalization factors, (DTC) and (DC) the dead time and decay correction factors, and (CF) the calibration factor.</td>
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<tr>
<td>• Corrections for physical effects inherent in PET data collection must be applied to acquired PET data to provide quantitative accuracy in order for there to be a direct correspondence between counts seen in the image and the true activity distribution of the tracer uptake.</td>
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<tr>
<td>• Measured corrections add noise into the data set. Calculated corrections may not be accurate.</td>
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