Cone Beam Reconstruction

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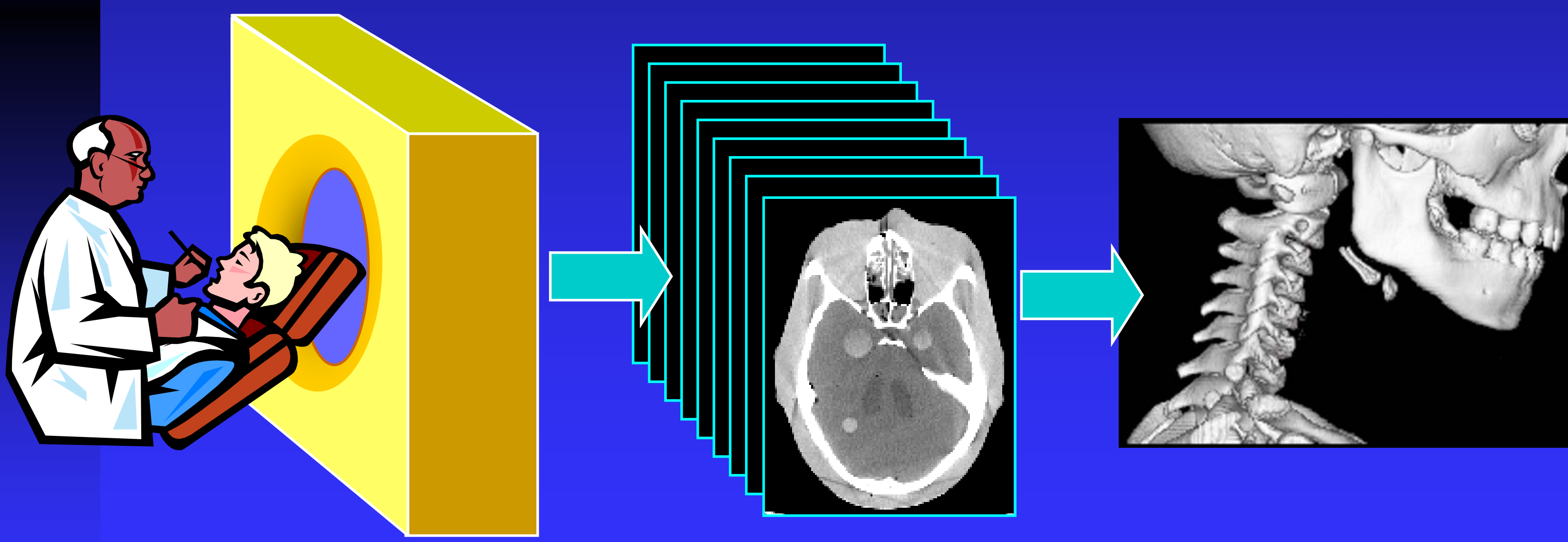
Image Generation

- Reconstruction of images from projections.
  - “textbook” reconstruction
  - advanced acquisition (helical, multi-slice)
  - advanced application (cardiac, perfusion)
- Formulation of 2D images to 3D volume.



reconstruction **Presentation**

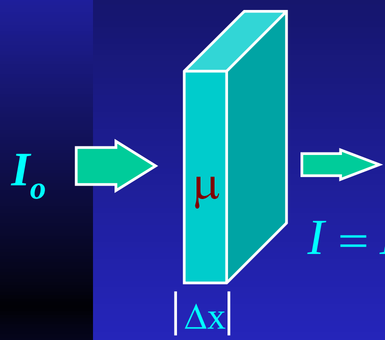
“Textbook” Reconstruction

- The mathematical foundation of CT can be traced back to 1917 to Radon.
- The algorithms can be classified into two classes: analytical and iterative.
- Some of the commonly used reconstruction formula was developed in the late 70s and early 80s.
- With the introduction of multi-slice helical CT, new cone beam reconstruction algorithms are developed.

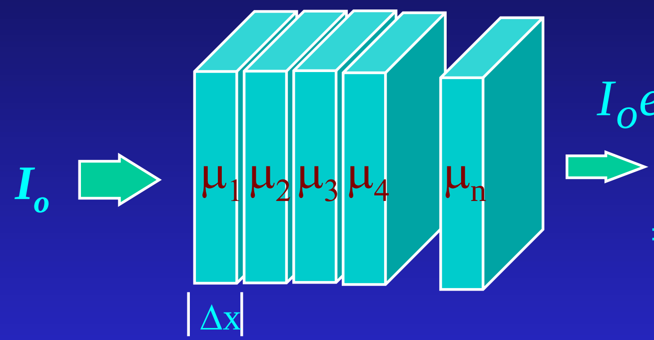
CT Data Measurement

-Under Ideal Conditions

- x-ray attenuation follows Beer's law.

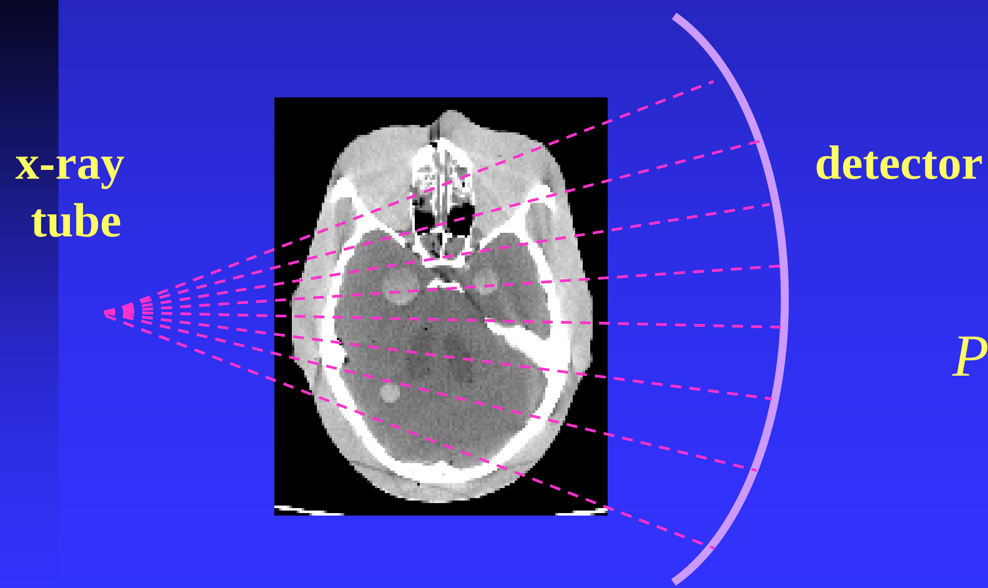


$$I = I_o e^{-\mu \Delta x}$$



$$I_o e^{-\mu_1 \Delta x} e^{-\mu_2 \Delta x} \dots e^{-\mu_n \Delta x}$$

$$= I_o e^{-(\mu_1 + \mu_2 + \dots + \mu_n)\Delta x}$$



$$\Delta x \to 0,$$

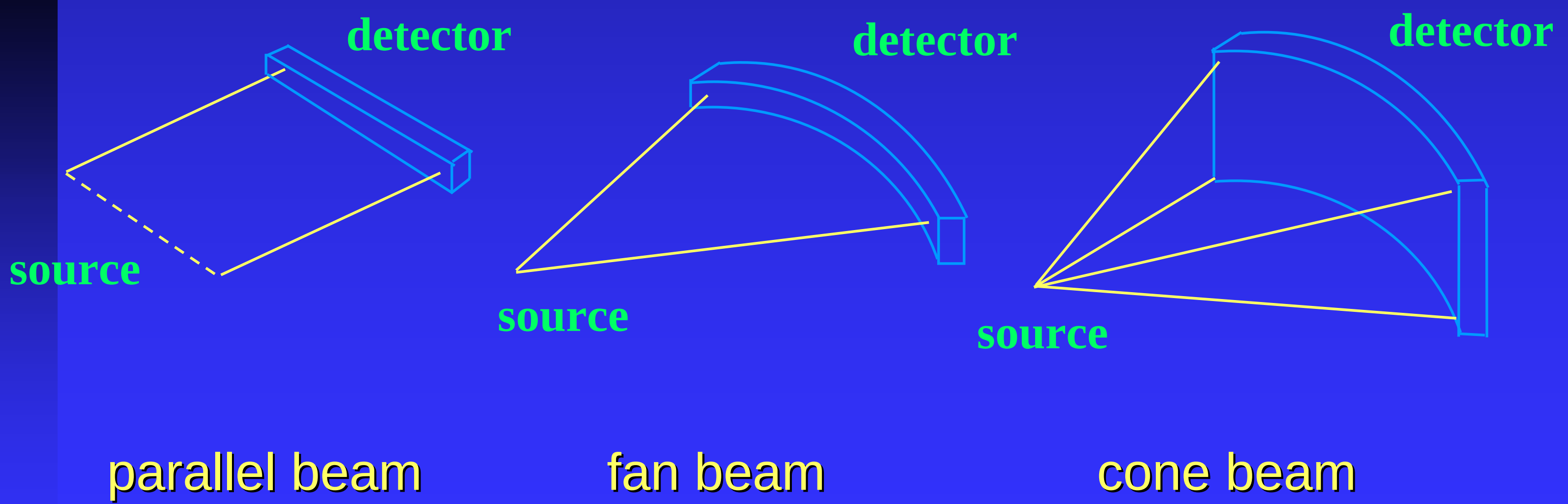
$$P = -\ln\left(\frac{I}{I_o}\right) = \int_{-\infty}^{\infty} \mu(x)\,dx$$

Ideal Projections

- The measured data are not line integrals of attenuation coefficients of the object.
  - beam hardening
  - scattered radiation
  - detector and data acquisition non-linearity
  - patient motion
  - others
- The data need to be calibrated prior to the tomographic reconstruction to obtain artifact-free images.

Sampling Geometries

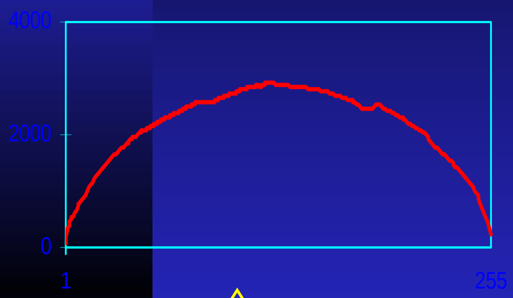
- The sampling geometry of CT scanners can be described three configurations.
- Due to time constraints, we will not conduct in-depth discussions on each geometry.



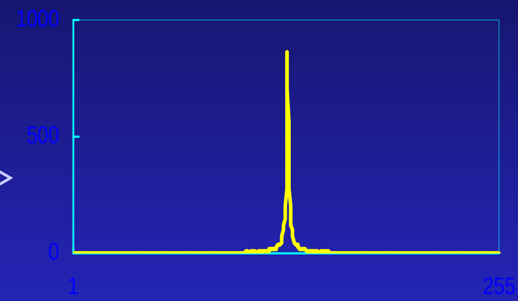
Fourier Slice Theorem

(Central Slice Theorem)

$$p(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad P(u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-i2\pi ux}\,dxdy \qquad P(u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-i2\pi ux}\,dxdy$$



FT



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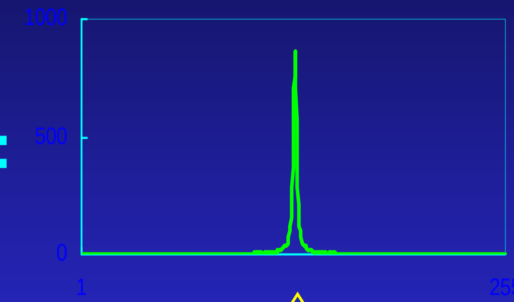
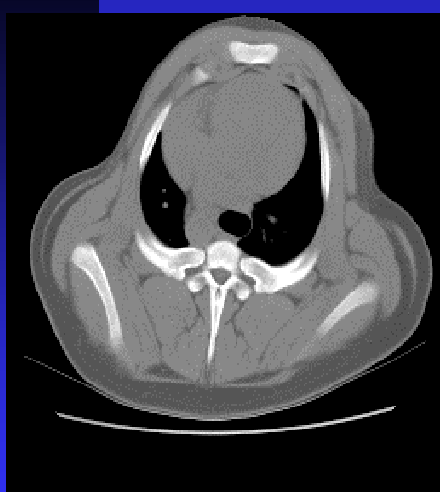
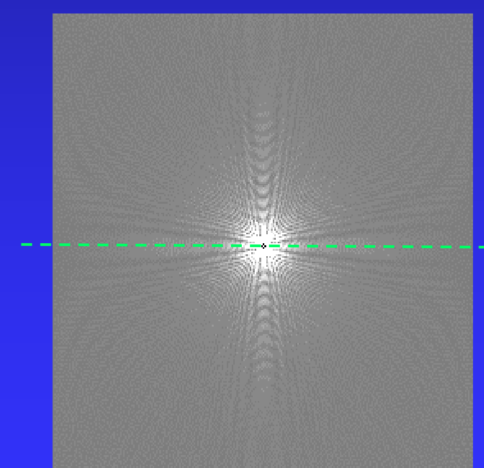


PROJECTION

$v=0$



2D FT



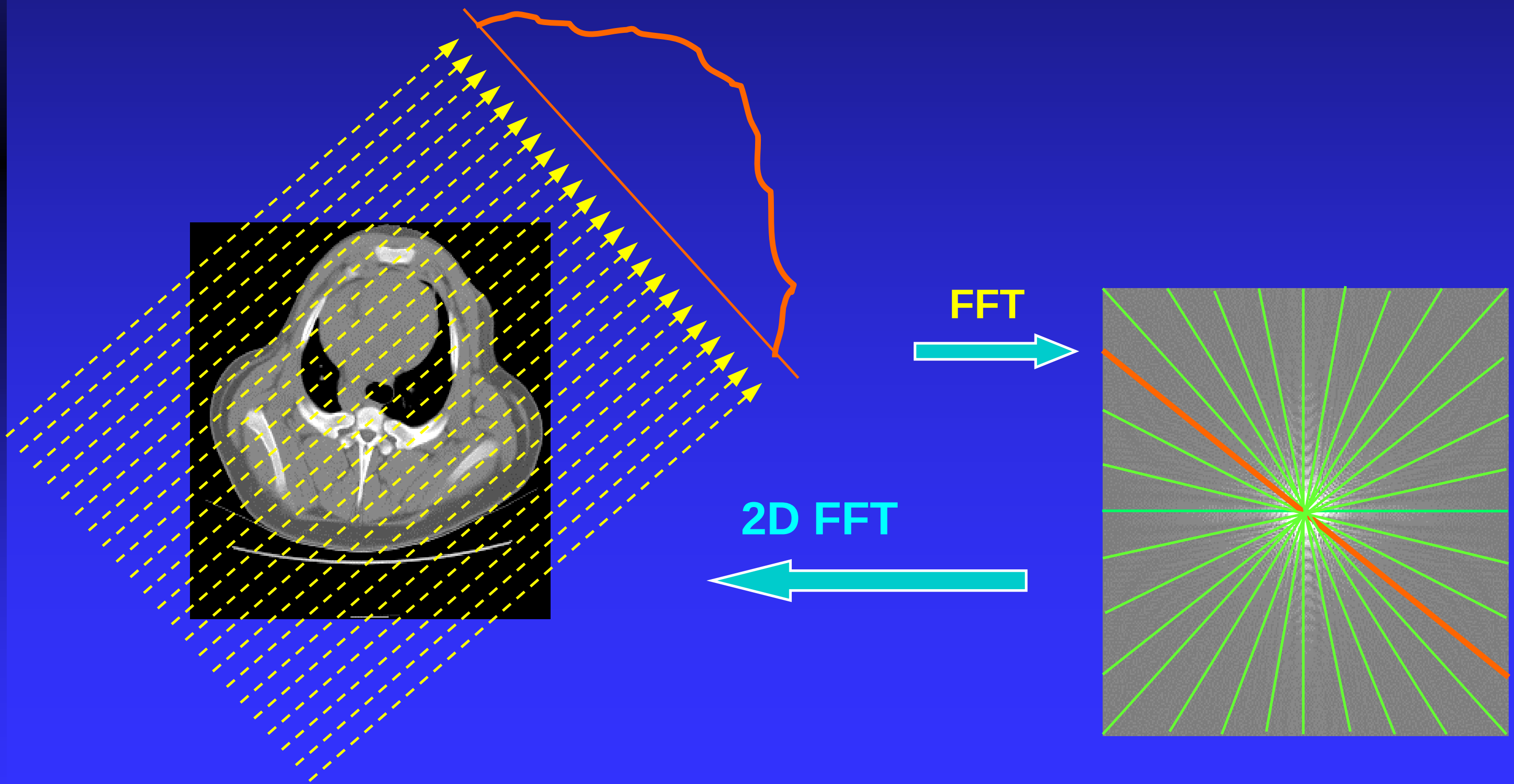
$f(x,y)$

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-i2\pi(ux+vy)}\,dxdy$$

Fourier Slice Theorem

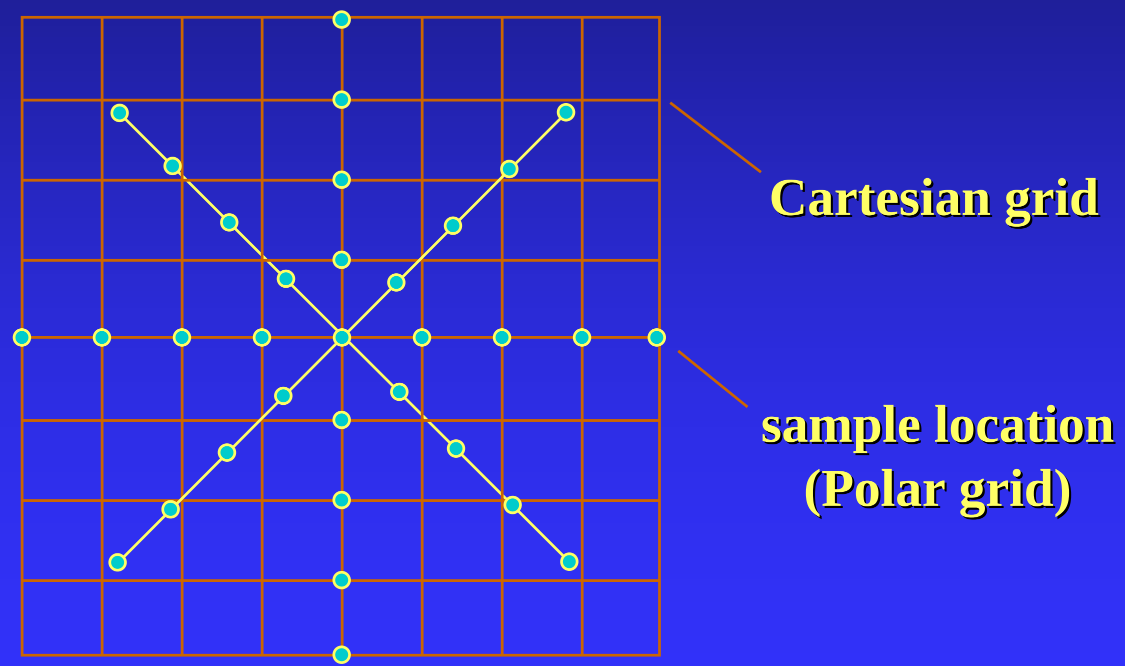
(central slice theorem)

- Fourier transform of projections at different angles fill up the Fourier space.
- Inverse Fourier transform recovers the original object.



Implementation Difficulty

- Due to sampling pattern, direct implementation of the Fourier slice theorem is difficult.



Filtered Backprojection

- The filtered backprojection formula can be derived from the Fourier transform pair, coordinate transformation, and the Fourier slice theory:

backprojection

$$f(x,y) = \int_0^{\pi} \boxed{\int_{-\infty}^{\infty} P_\theta(u)|\omega| e^{j2\pi\omega t} d\omega} d\theta$$

filtering

pre-processed data

⇓

filter the data

⇓

backprojection

parallel beam reconstruction

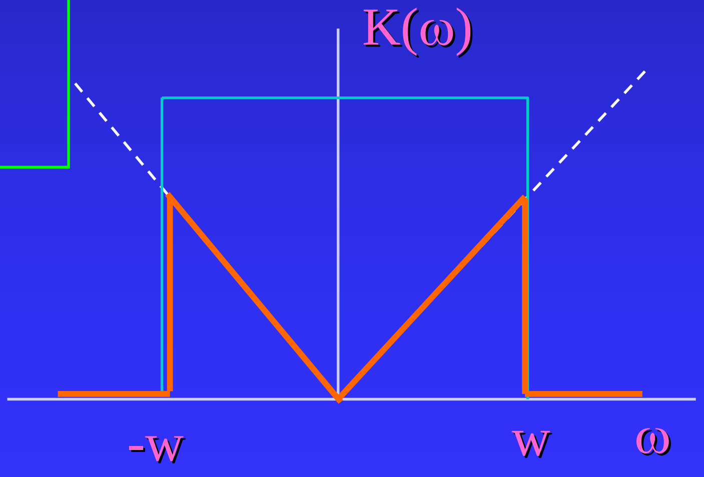
Filter Implementation

- The filter as specified does not exist.

$$k(t) = \int_{-\infty}^{\infty} |\omega| e^{j2\pi\omega t} d\omega$$

- The filter needs to be band-limited:

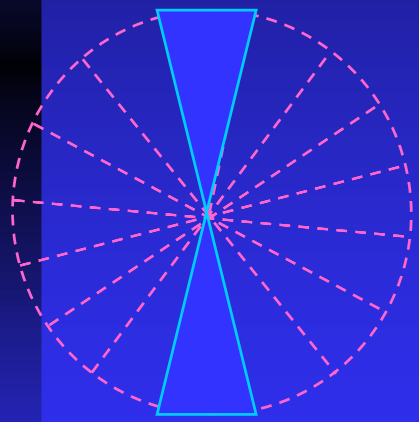
$$\boxed{k(t) = \int_{-W}^{W} |\omega| e^{j2\pi\omega t} d\omega}$$

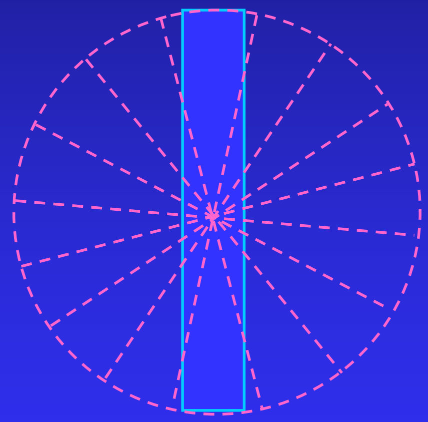


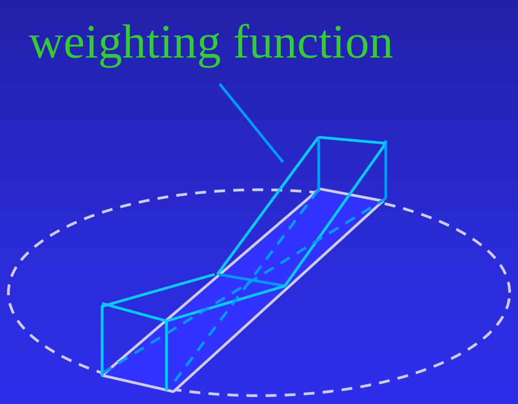
Filtered Backprojection

-an intuitive explanation

- Filtered backprojection uses weighting function to approximate ideal condition.







ideal frequency data from one projection

actual frequency data from one projection

weighting function for approximation

Filtering

- Consider an example of reconstructing a phantom object of two rods.

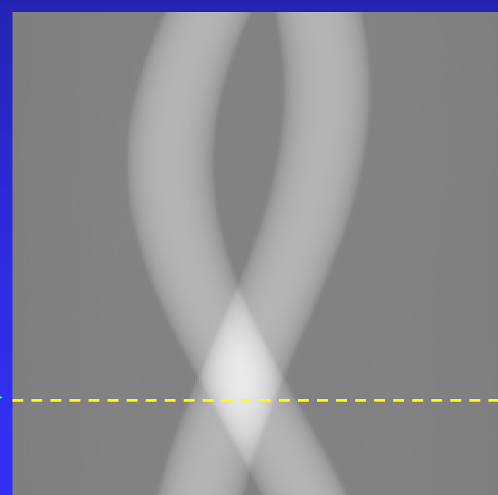
Object

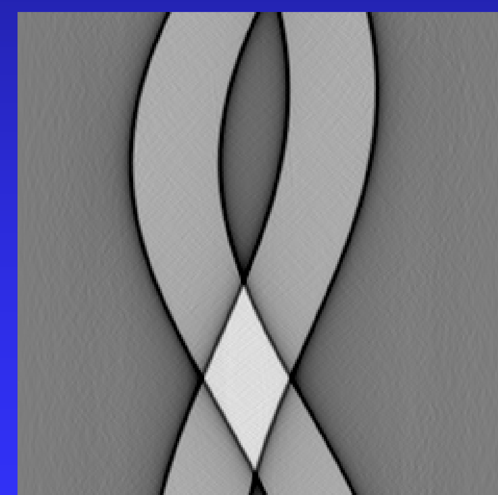
Original Sinogram

Filtered Sinogram

views

single projection

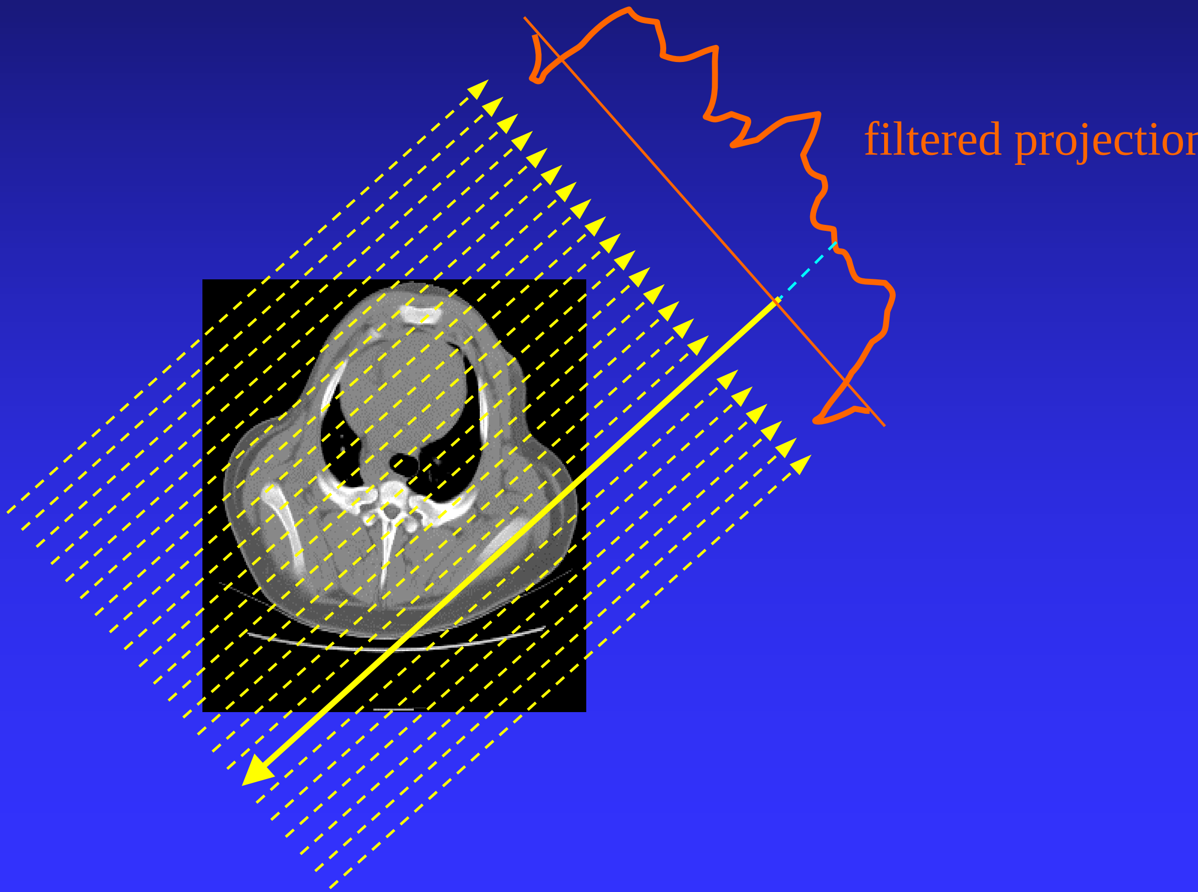




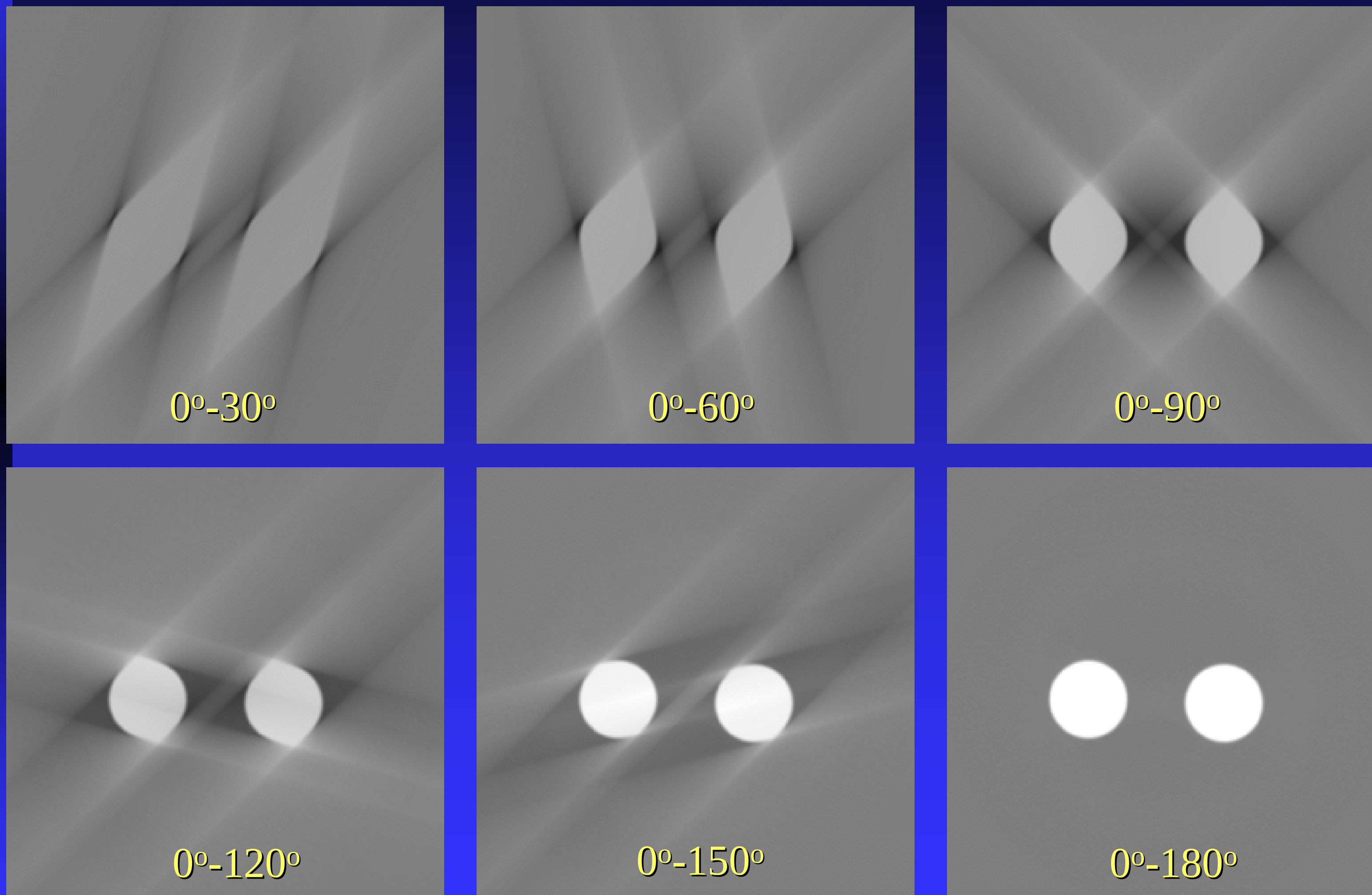
detector sample

Backprojection

- Backprojection is performed by painting the intensity of the entire ray path with the filtered sample.

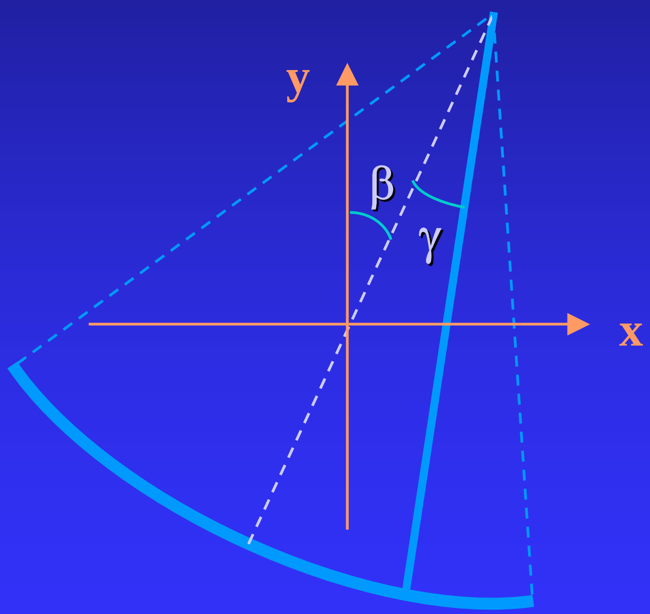


Backprojection

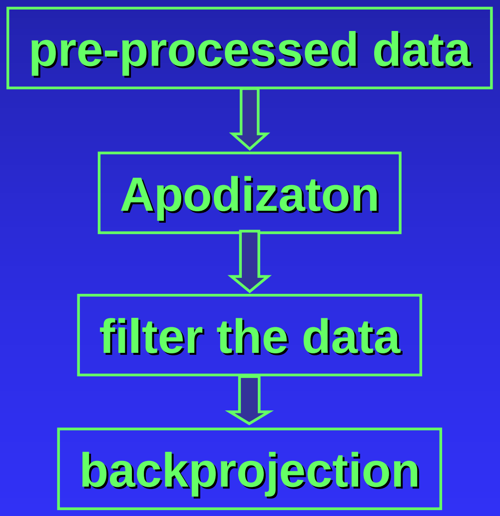


Fan Beam Reconstruction

- Each ray in a fan beam can be specified by β and γ.
- Reconstruction process is similar to parallel reconstruction except additional “apodization” step and weighting in the backprojection.



fan beam geometry



fan beam reconstruction

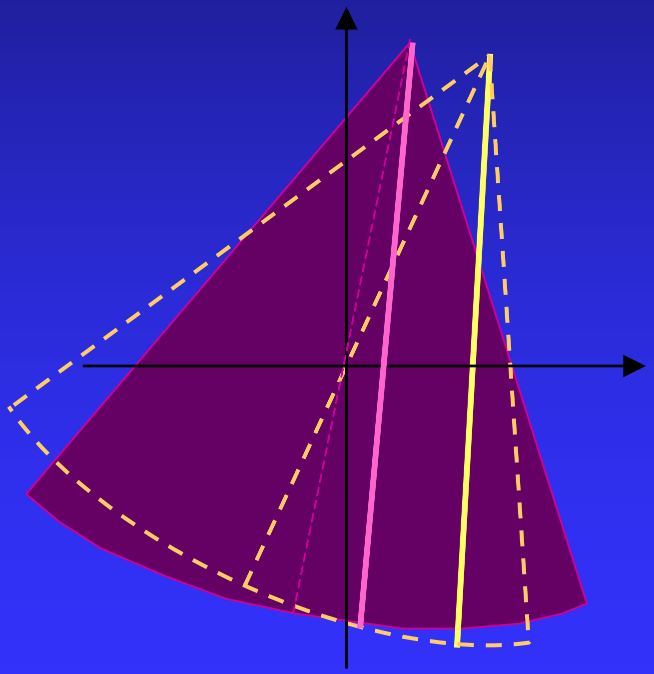
Equiangular Fan Beam Reconstruction

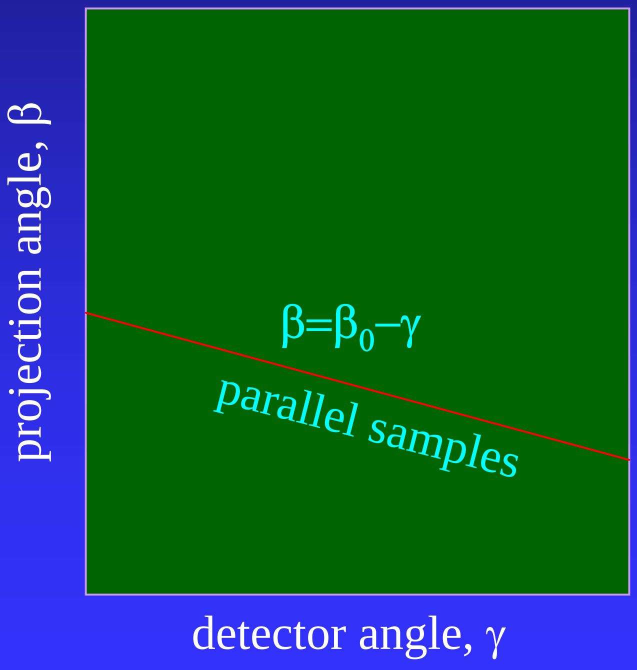
$$f(x,y)=\int_0^{2\pi} L^{-2}\,d\beta \int_{-\gamma_m}^{\gamma_m} p(\gamma,\beta)\,h(\gamma'-\gamma)\,D\cos\gamma\,d\gamma$$

- The projection is first multiplied by the cosine of the detector angle.
- In the backprojection process, the filtered sample is scaled by the distance to the source.

Fan Beam Reconstruction

- Alternatively, the fan beam data can be converted to a set of parallel samples. Parallel reconstruction algorithms can be used for image formation.





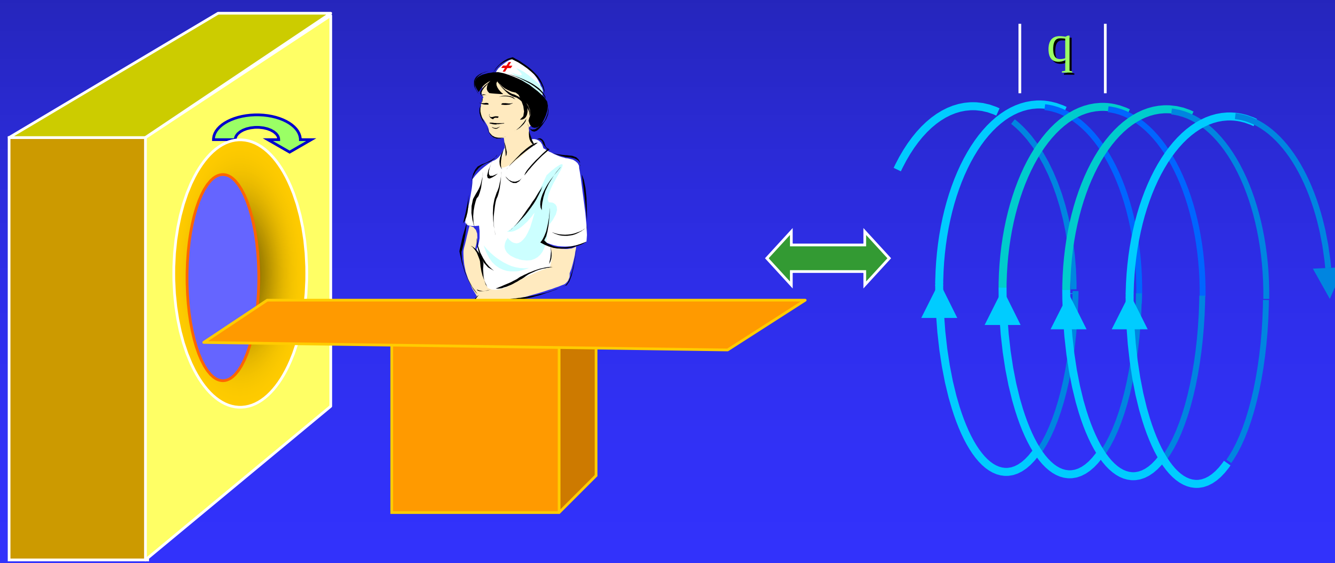
Helical Scanning

- In helical scanning, the patient is translated at a constant speed while the gantry rotates.
- Helical pitch:

$$h = \frac{q}{d}$$

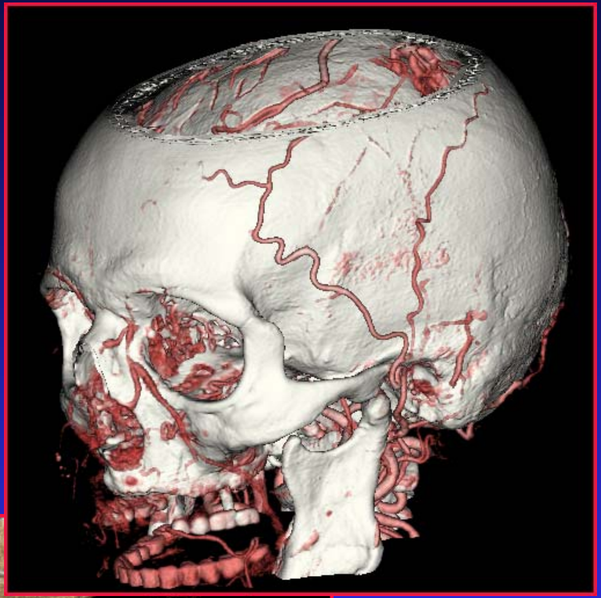
q — distance gantry travel in one rotation

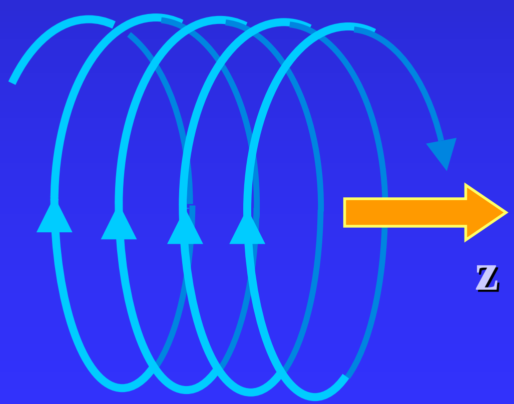
d — collimator aperture

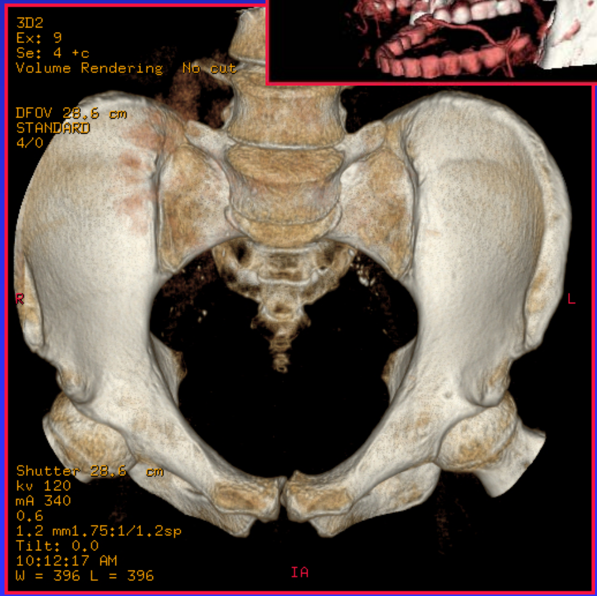


Helical Scanning

- Advantages of helical scanning
  - nearly 100% duty cycle (no inter-scan delay)
  - improved contrast on small object (reconstruction at any z location)
  - improved 3D images (overlapped reconstruction)







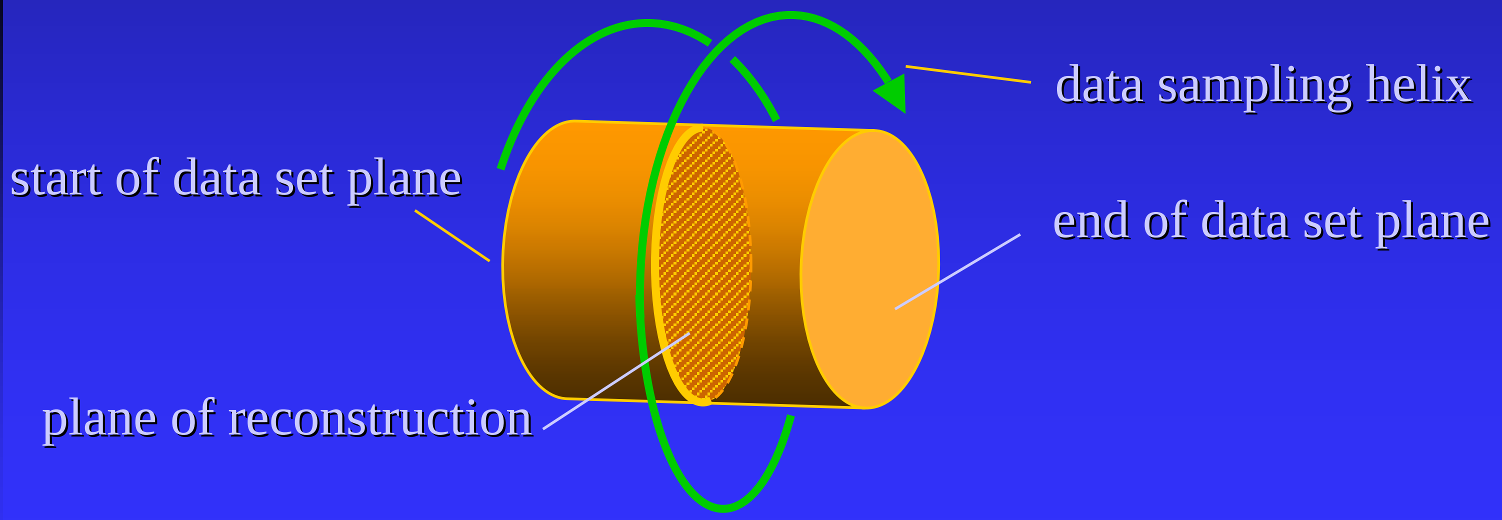
Helical Scanning

- The helical data collection is inherently inconsistent. If proper correction is not rendered, image artifact will result.

reconstructed helical scan without correction

Helical Reconstruction

- The plane of reconstruction is typically at the mid-point between the start and end planes.
- Interpolation is performed to estimate a set of projections at the plane of reconstruction.



Helical Reconstruction

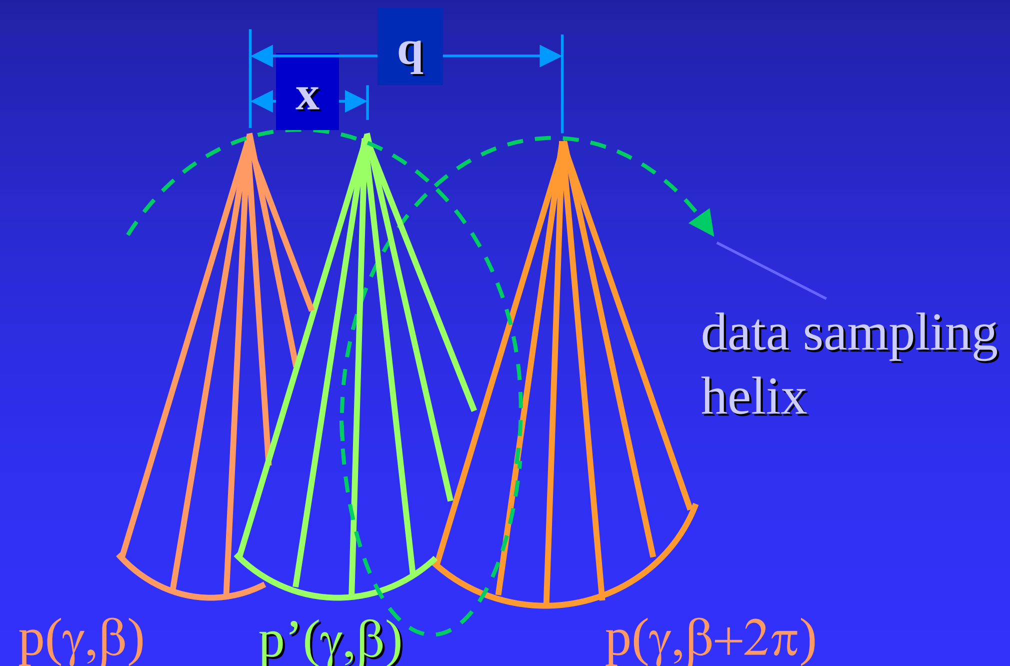
-360º interpolation

- Samples at the plane-of-reconstruction is estimated using two projections that are 360º apart.

$$p'(\gamma,\beta) = wp(\gamma,\beta) + (1-w)\,p(\gamma,\beta+2\pi)$$

where

$$w = \frac{q-x}{q}$$



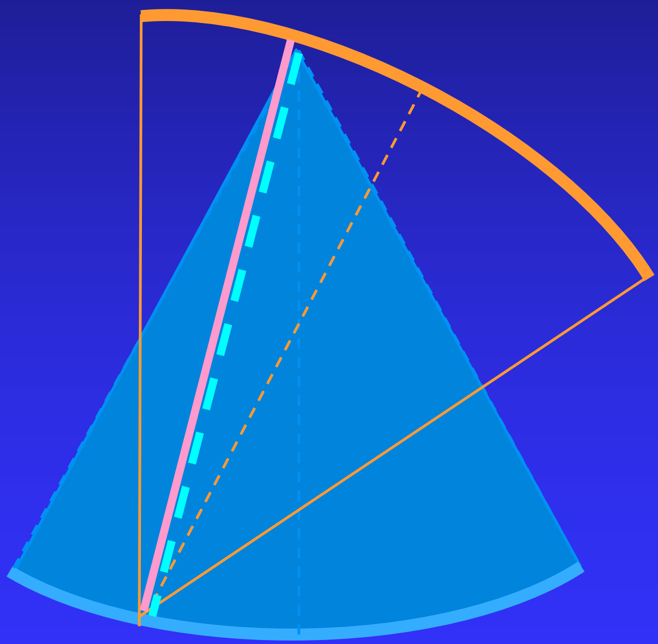
Helical Reconstruction

-180° interpolation

- In fan beam, each ray path is sampled by two conjugate samples that are related by:

$$\begin{cases} \gamma' = -\gamma \\ \beta' = \beta + \pi + 2\gamma \end{cases}$$

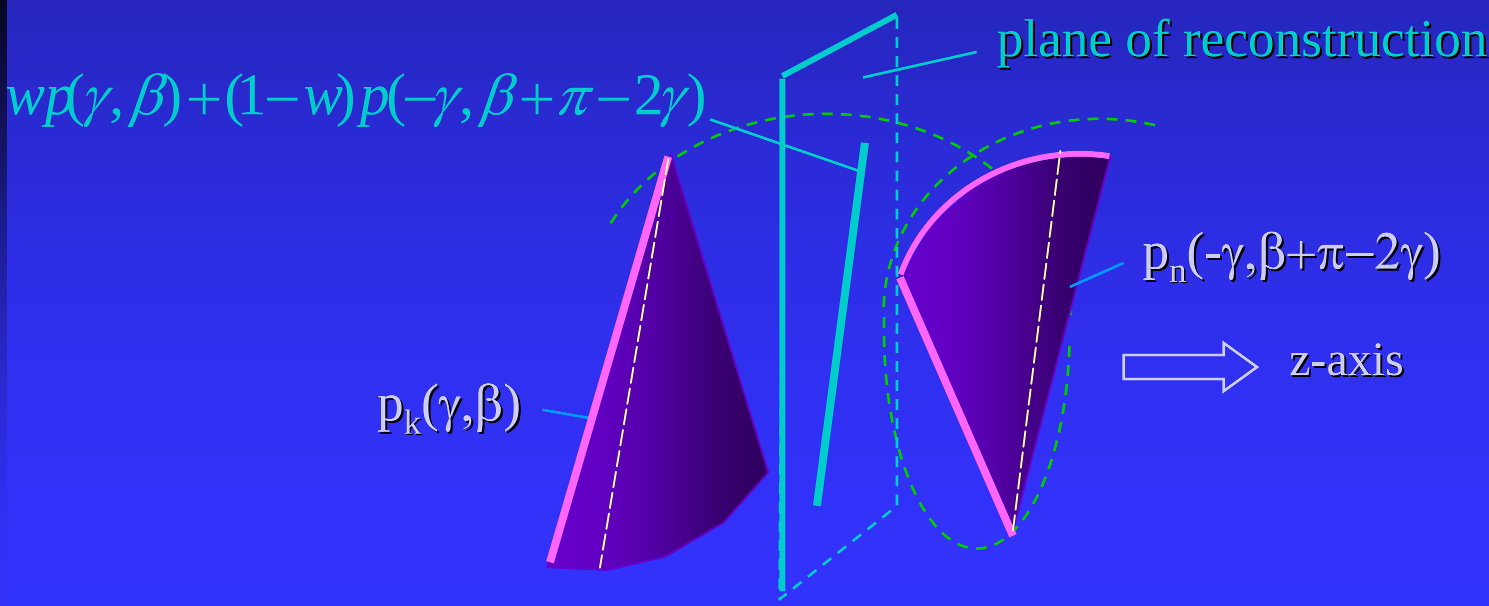
For helical scan, these two samples are taken at different z location because of the table motion.



Helical Reconstruction

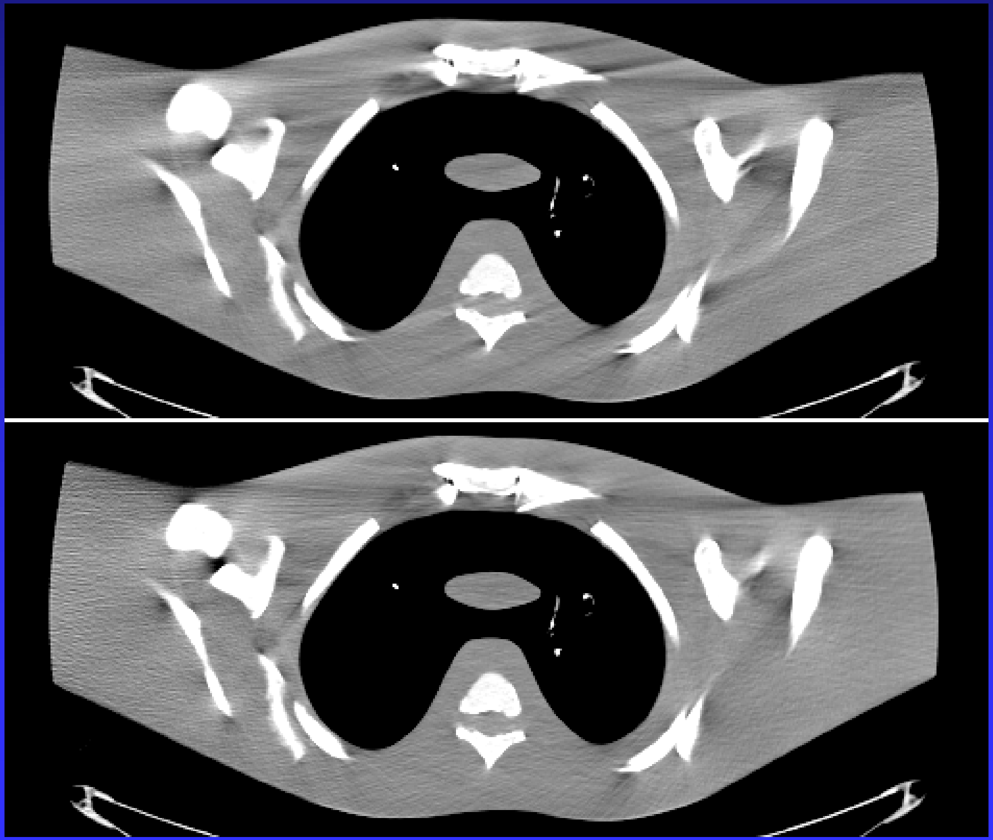
-180º interpolation

- Linear interpolation is used to estimate the projection samples at the plane of reconstruction.
- Because samples are taken at different view angles, the weights are γ– and β–dependent.



Artifact Suppression

- Helical reconstruction algorithm effectively suppresses helical artifacts.

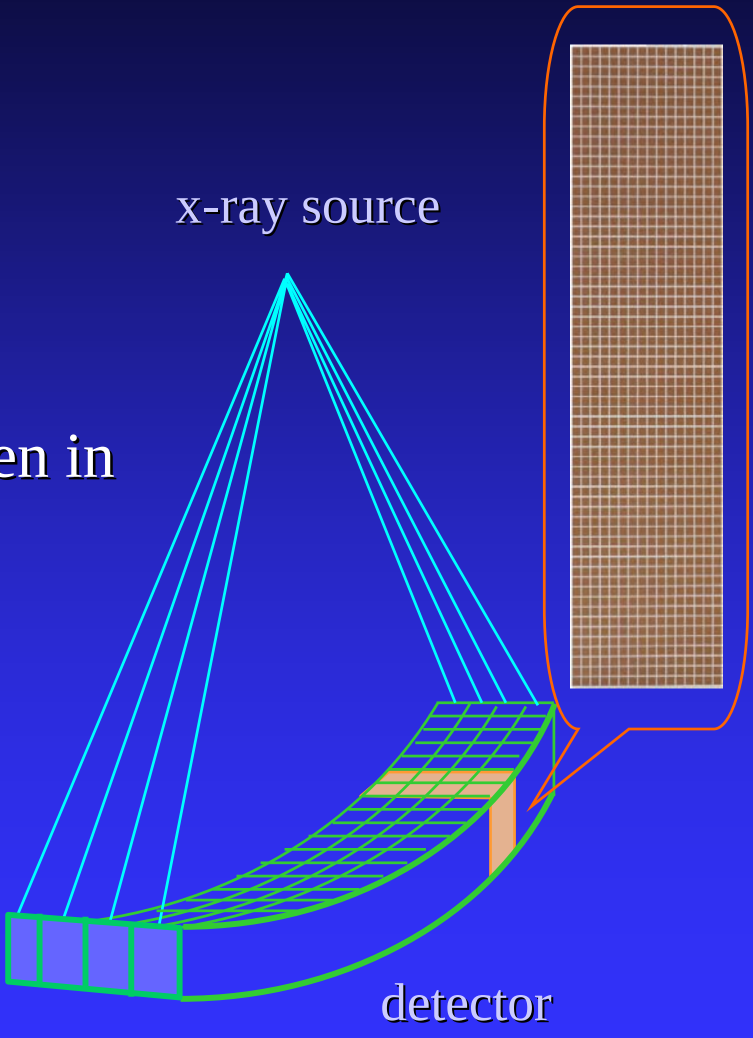


without helical correction

with helical correction

Multi-slice CT

- Multi-slice CT contains multiple detector rows.
- For each gantry rotation, multiple slices of projections are acquired.
- Similar to the single slice configuration, the scan can be taken in either the step-and-shoot mode or helical mode.

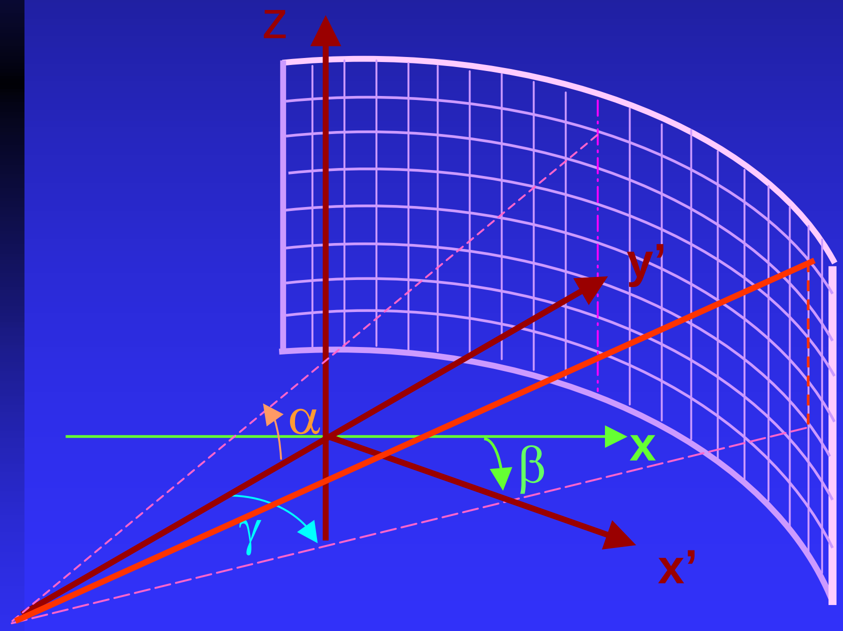


Advantages of Multi-slice

- Large coverage and faster scan speed
- Better contrast utilization
- Less patient motion artifacts
- Isotropic spatial resolution

Cone Beam Reconstruction
FDK Algorithm

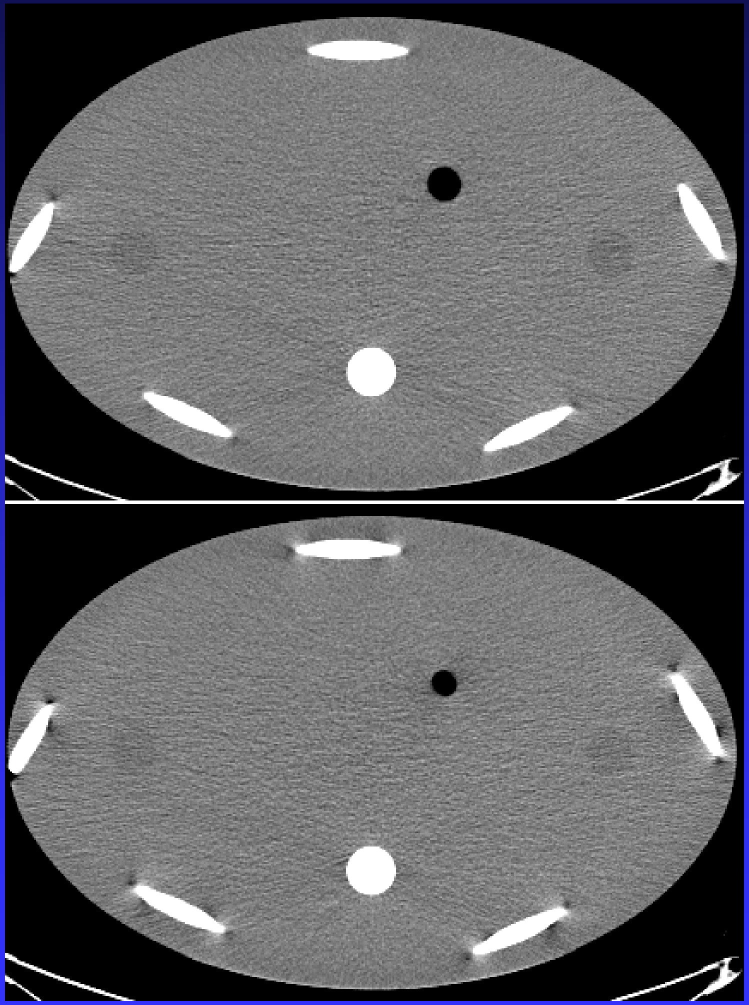
- Each ray in a cone beam can be specified by β, γ, and α.
- FDK algorithm was derived from fan-beam algorithm by studying the impact of cone angle to the rotation angle.

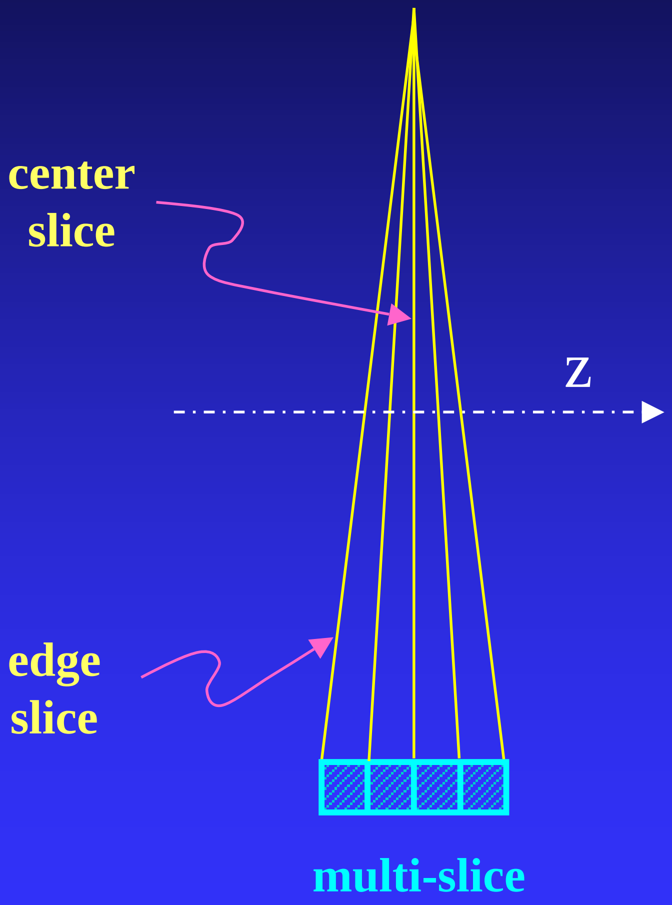


pre-processed data → weighting → filter the data along row → 3D backprojection

fan beam reconstruction

Cone Beam Artifact





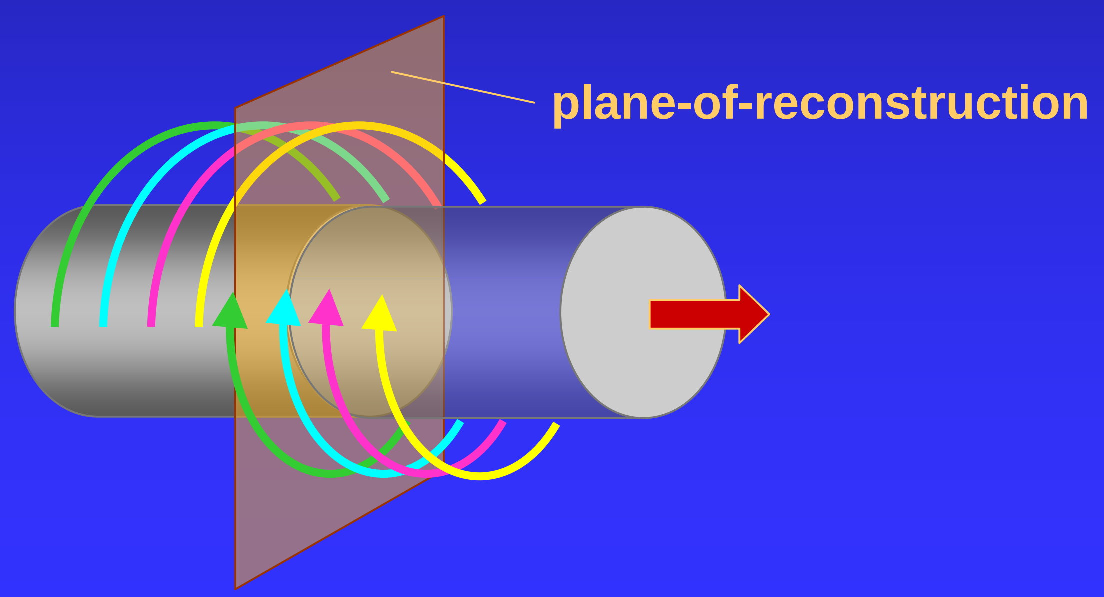
Multi-slice Helical

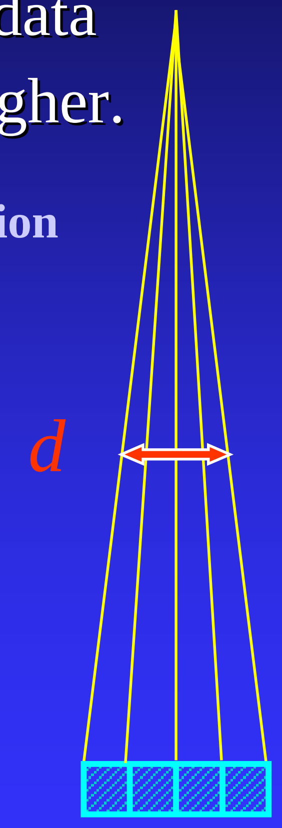
- When acquiring data in a helical mode, the N detector rows form N interweaving helixes.
- Because multiple detector rows are used in the data acquisition, the acquisition speed is typically higher.

$$h = \frac{q}{d}$$

q — distance gantry travel in one rotation

d — collimator aperture





multi-slice

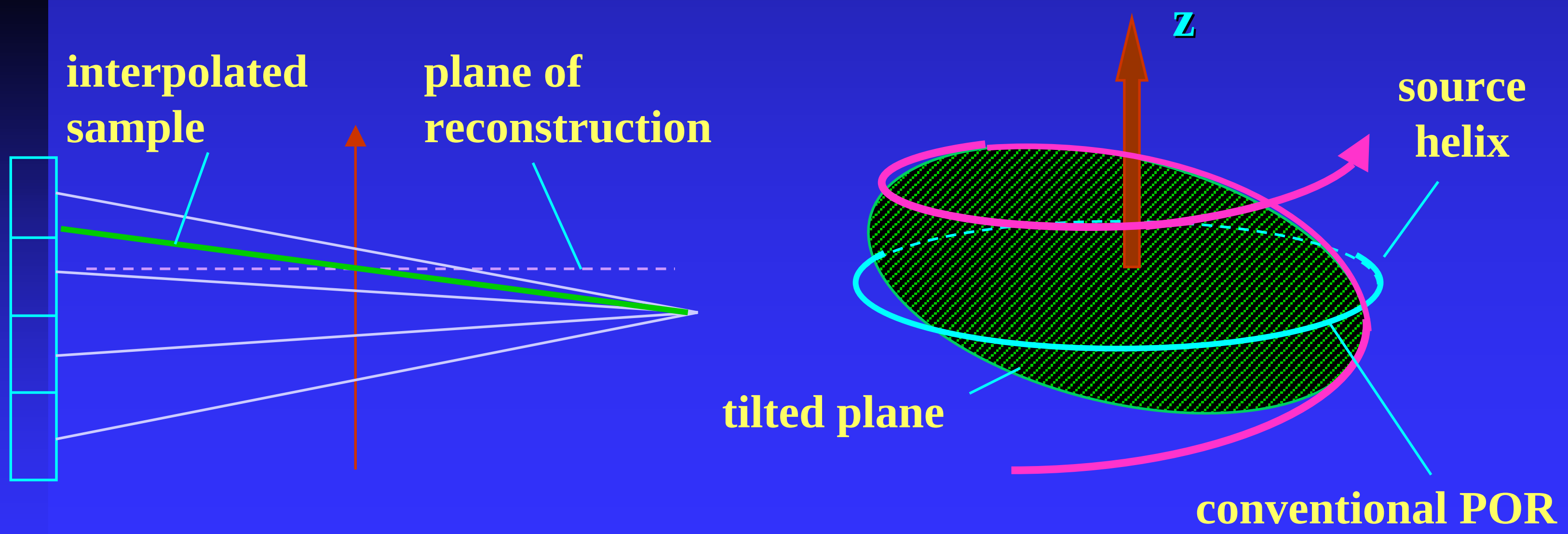
Cone Beam Helical Reconstruction

- Exact algorithms produce mathematically exact solutions when input projections are perfect.
  - Katsevich
  - Grangeat
  - Rebin PHI
  - FBP PHI
- Approximate algorithms, although non-exact, generate clinically accurate images.
  - FDK-type
  - N-PI
  - CB-virtual circle
  - Tilted Plane
  - ZB

Cone Beam Algorithm

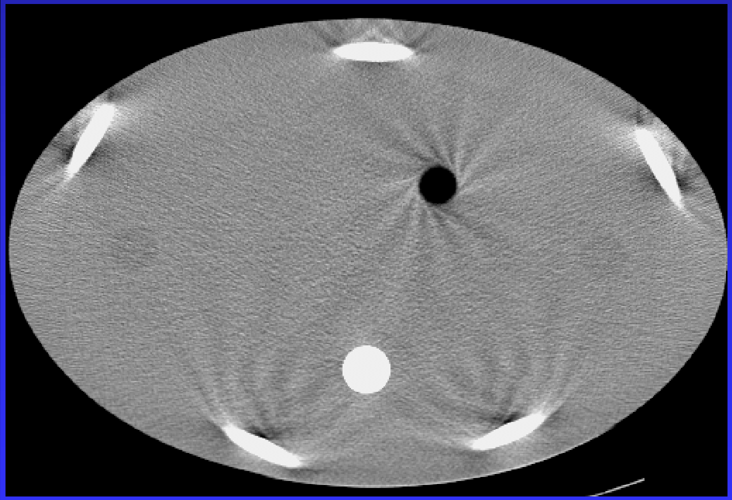
small cone angle

- From a computational point of view, 3D backprojection is more expensive than 2D backprojection.
- To overcome the discrepancy, tilted planes are defined as the plane of reconstruction so that 2D reconstruction algorithm can still be used.

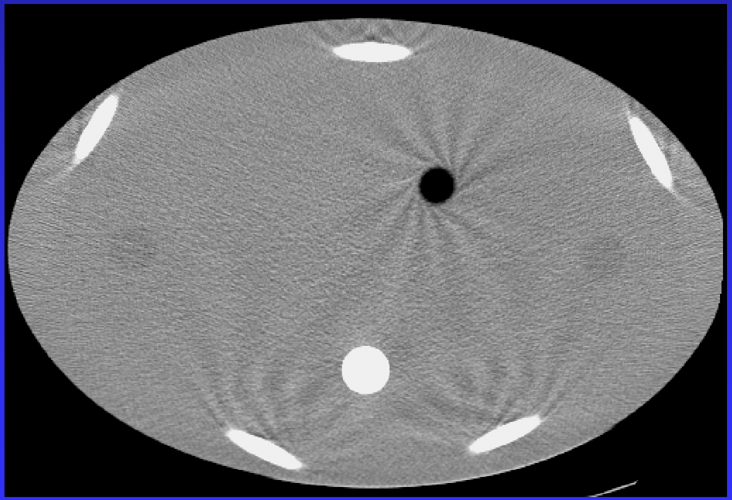


Tilted Plane Reconstruction

- For small cone angles, the flat plane and source helix match quite well.
- When the same weighting function is used, reconstructions with the tilted plane produces better image quality than the conventional reconstruction plane with 2D backprojection.



conventional plane

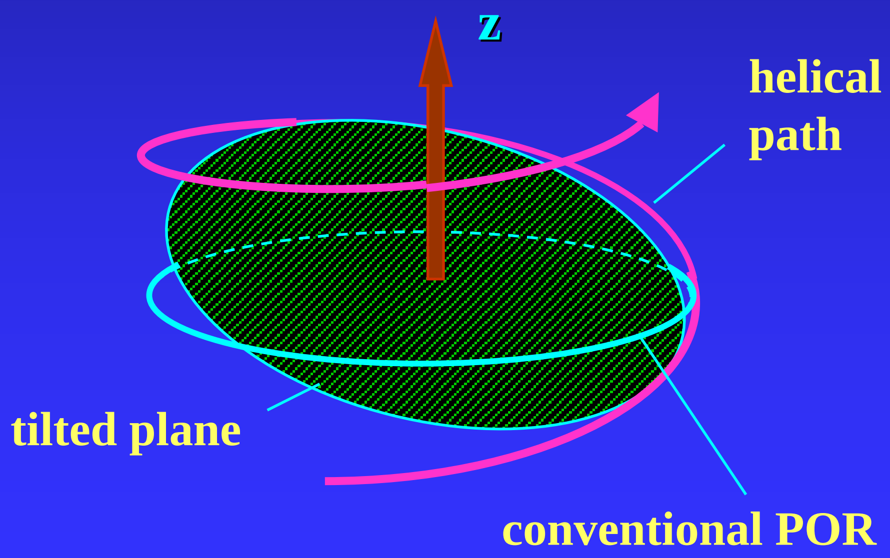


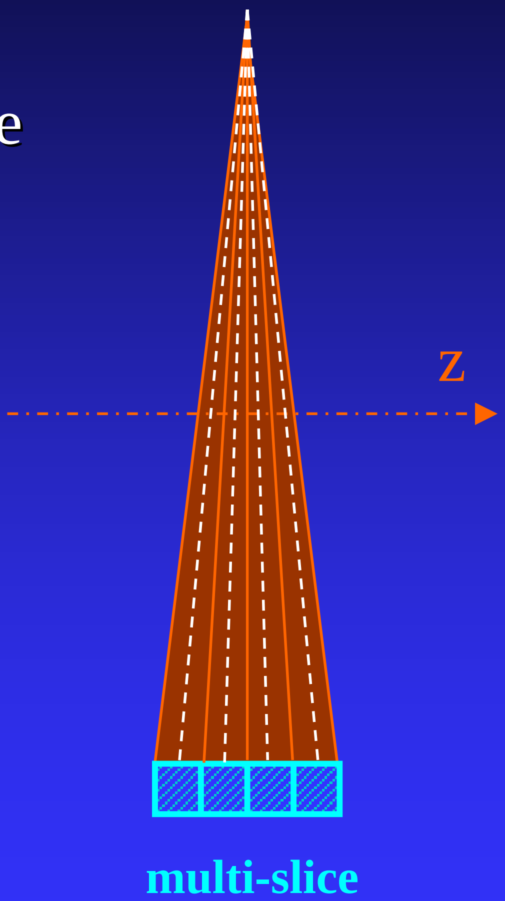
tilted plane

Cone Beam Reconstruction

moderate cone angle

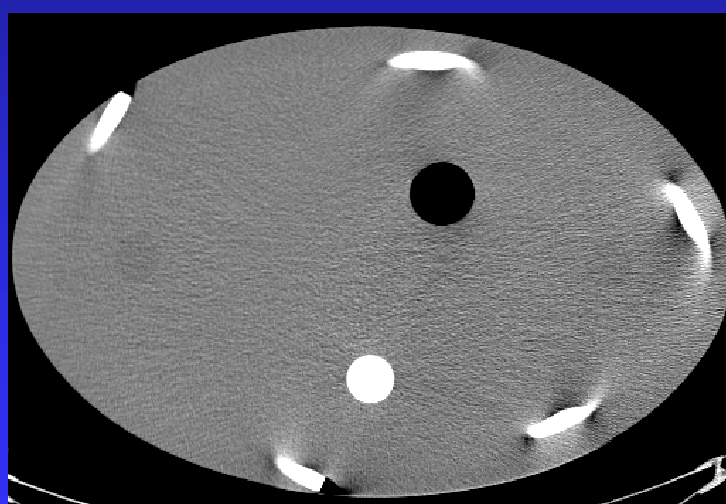
- For larger cone angles, tilted plane reconstruction is no longer sufficient, due to the larger difference between the flat plane and the curved helix.
- FDK-type algorithm with appropriate weighting is often used.



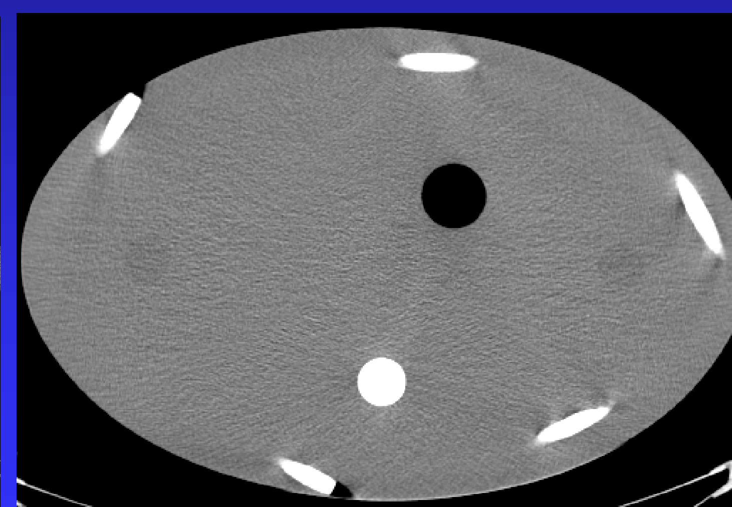


FDK-type Algorithm

- FDK-type algorithm can be combined with different weighting functions to optimize its performance in different performance parameters.
- Cone beam artifacts are suppressed but not eliminated.



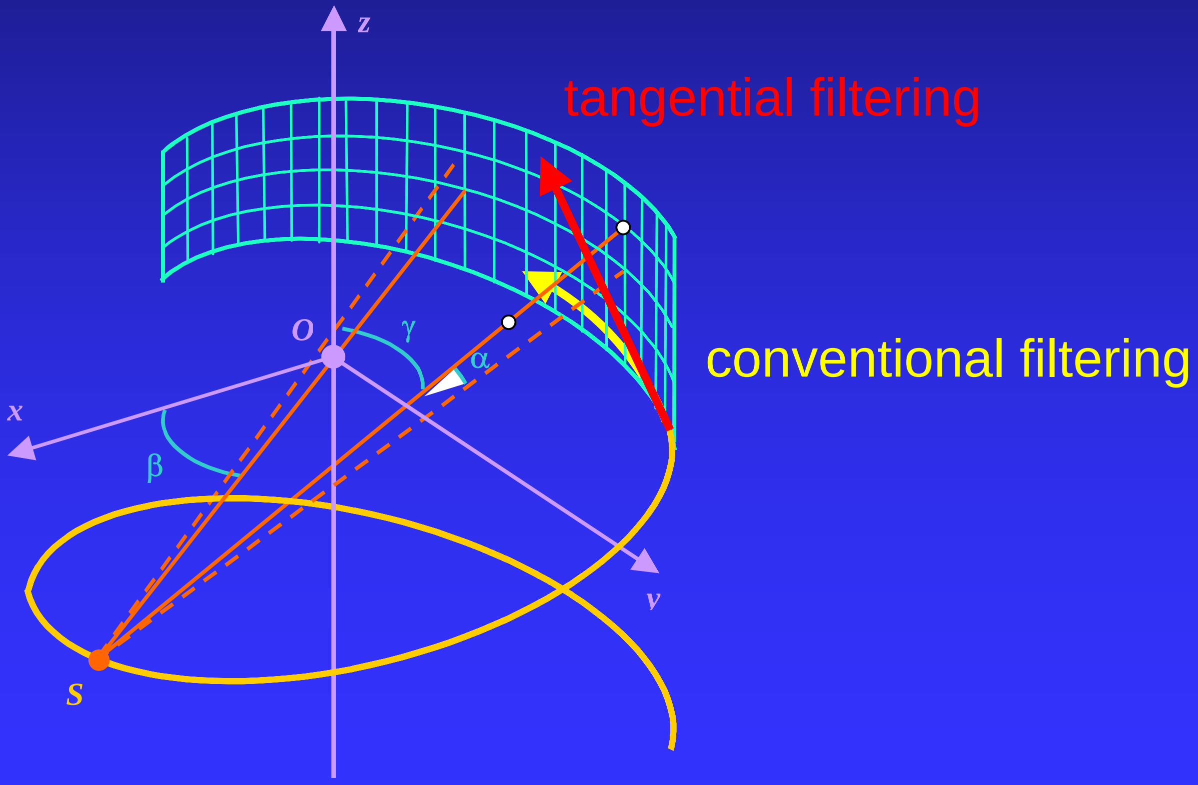
original



FDK-based

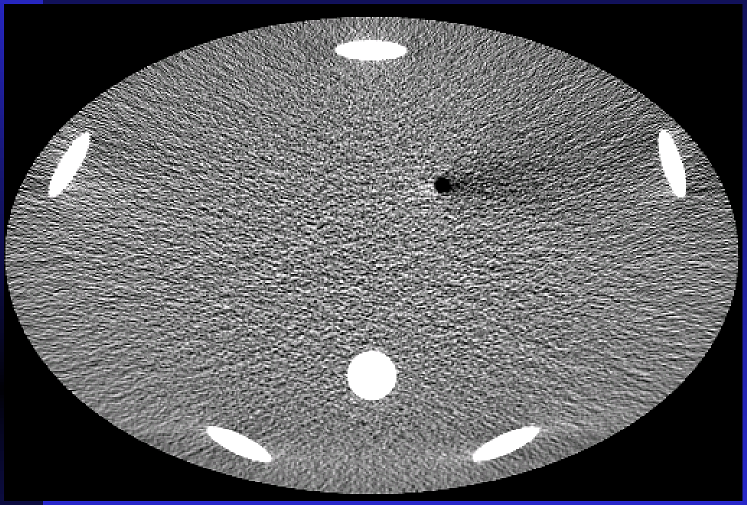
Tangential Filtering

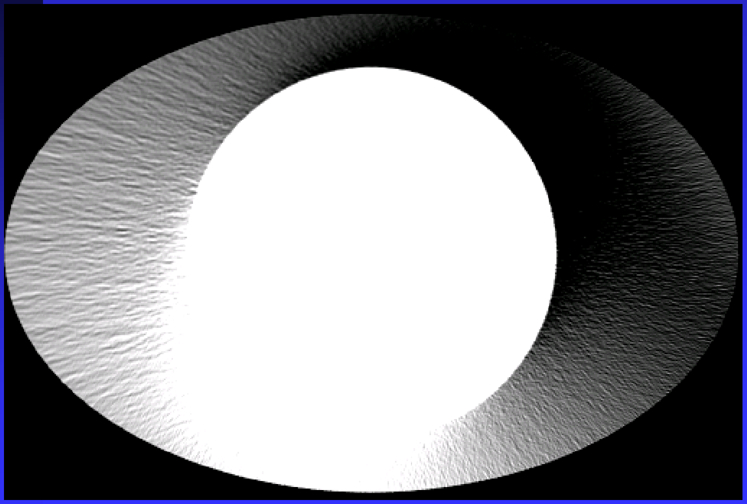
- Conventional filtering process is carried out along detector rows.
- Tangential filtering is carried out along the tangential direction of the source trajectory.



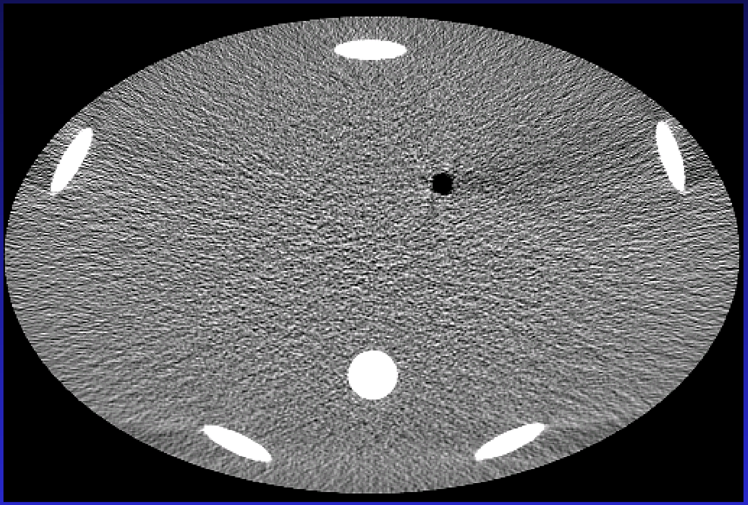
Tangential Filtering

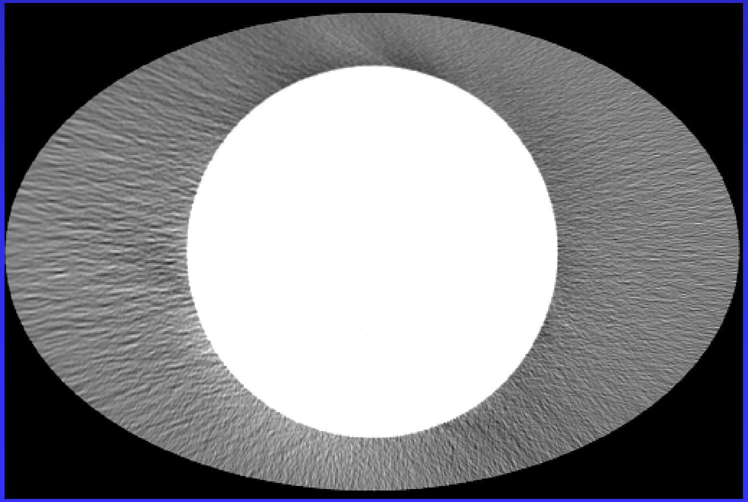
conventional filtering





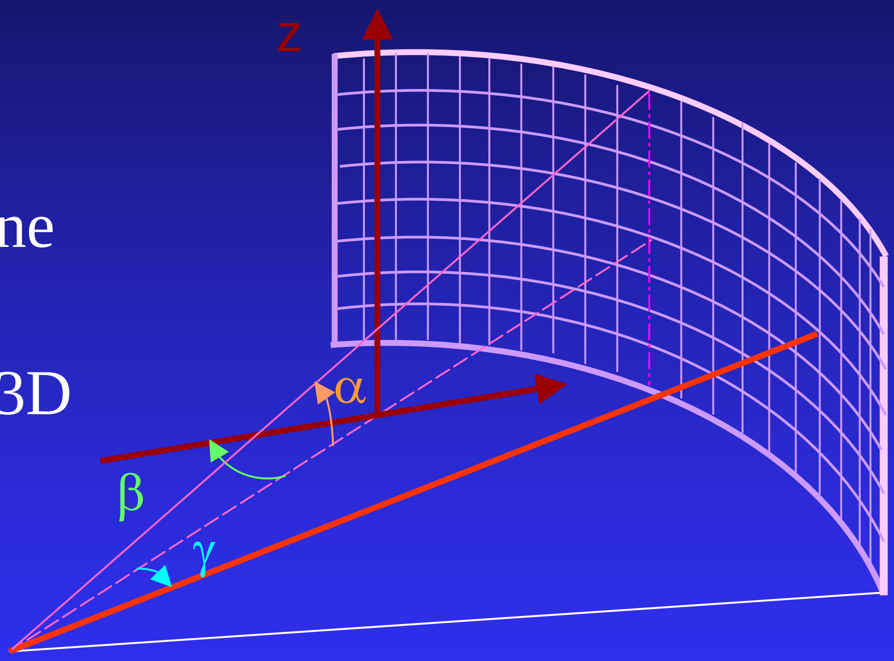
tangential filtering



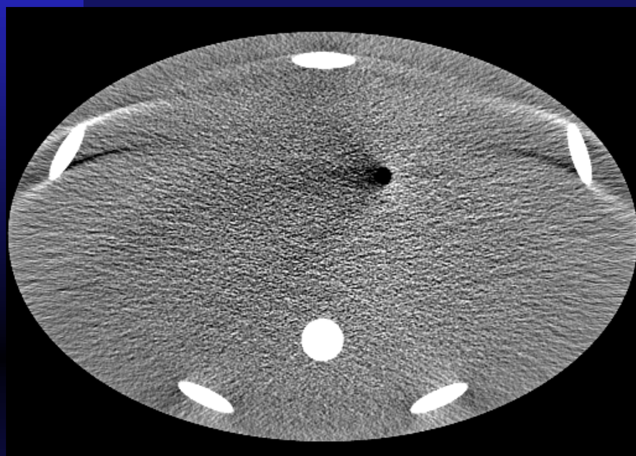


3D Helical Weighting

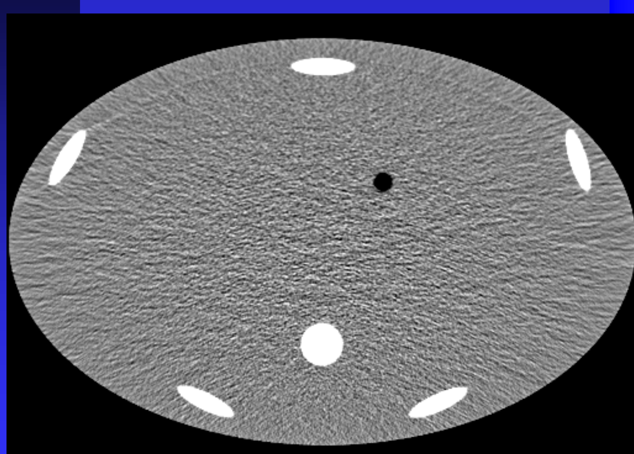
- The helical weighting function changes with projection angle β, detector angle γ, and cone angle α.
- Experiments show that 3D weighting function provides significant improvement in image quality.



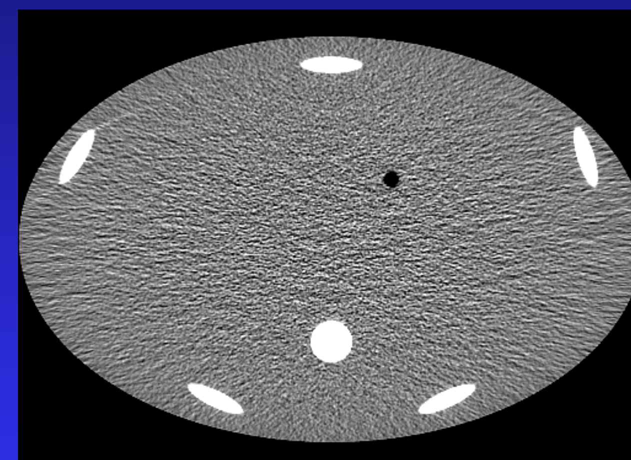
3D Helical Weighting



"off the shelf" recon



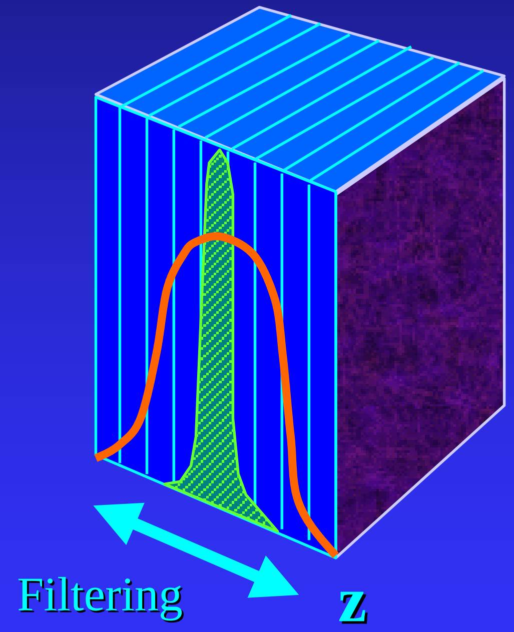
more expensive exact recon



3D weighting

Slice Thickness Change With Algorithm

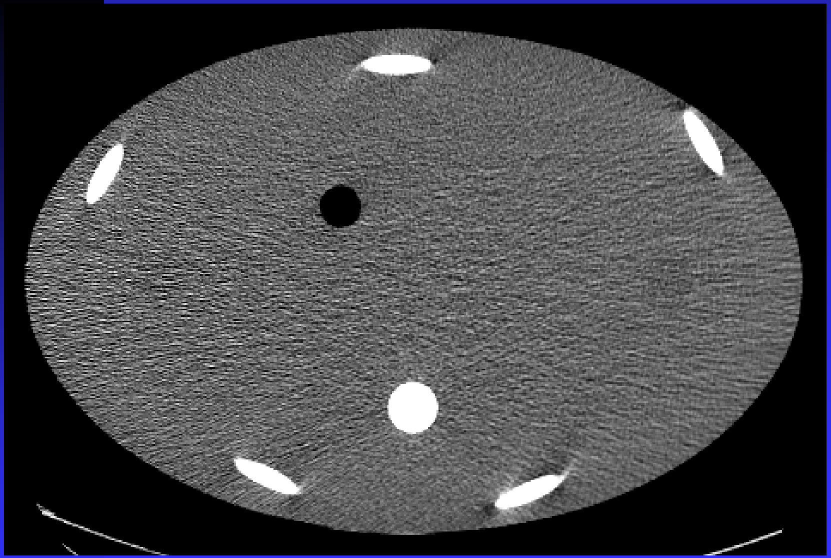
- Slice thickness can be selected by modifying the reconstruction process.
- By low-pass filtering in the z-direction, the slice sensitivity profile can be broadened to any desired shape and thickness.
- From an image artifact point of view, images generated with the thinner slice aperture is better.



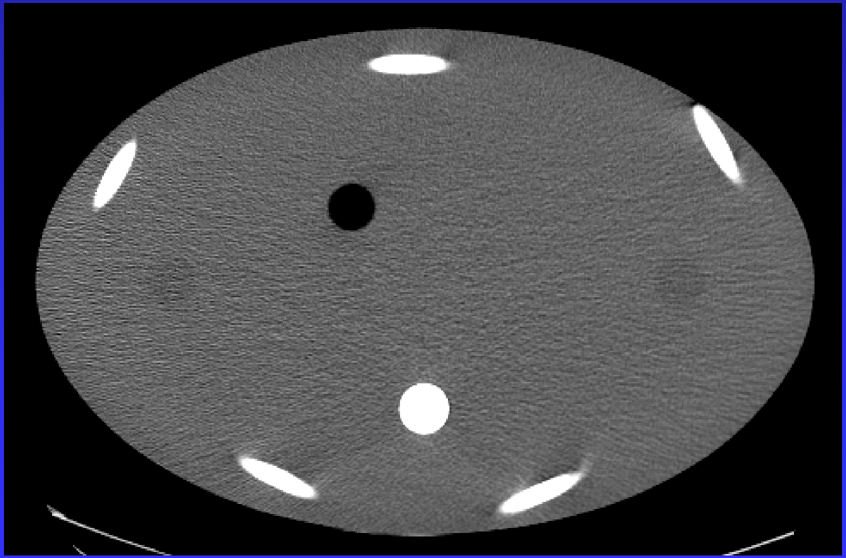
Example

- Z filtering can be applied in either the projection domain or the image domain.
- In general, z-smoothing provides artifact suppression capability.

16x0.625mm detector aperture at 1.75:1 helical pitch



FWHM=0.625mm



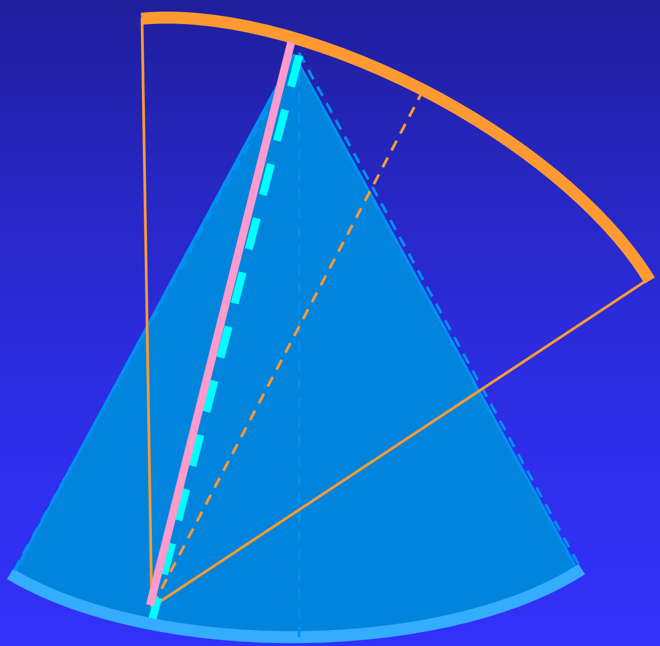
FWHM=2.5mm

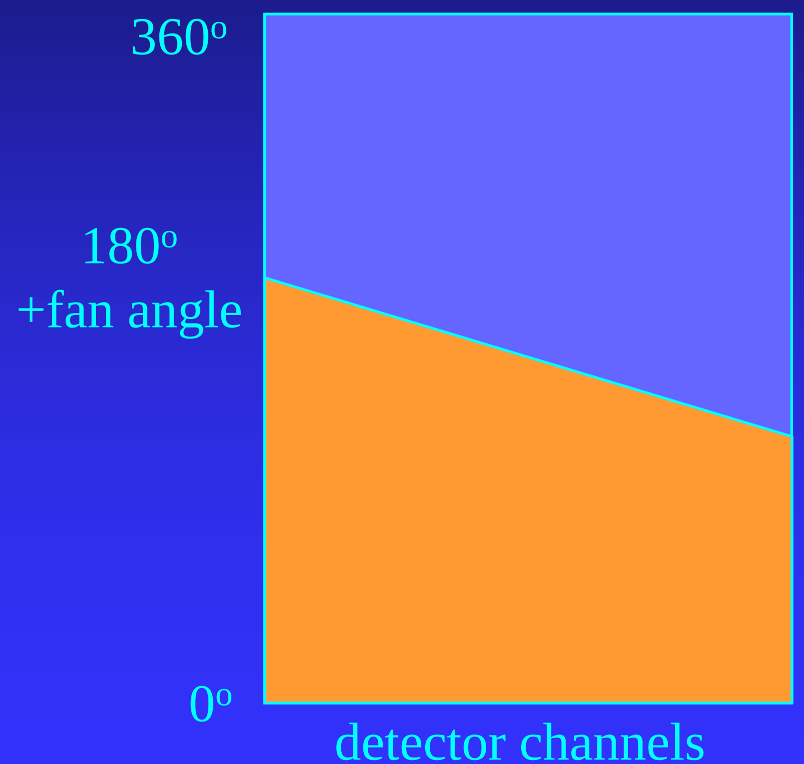
Cardiac Scans

- The most challenging problem in cardiac scanning is motion.
- Unlike respiratory motion, cardiac motion cannot be voluntarily controlled.
- For motion suppression, we could either reduce the acquisition time and/or acquire the data during the minimum cardiac motion.
- In cardiac motion, there are relative quiescent period: diastolic phase of the heart motion.

Halfscan

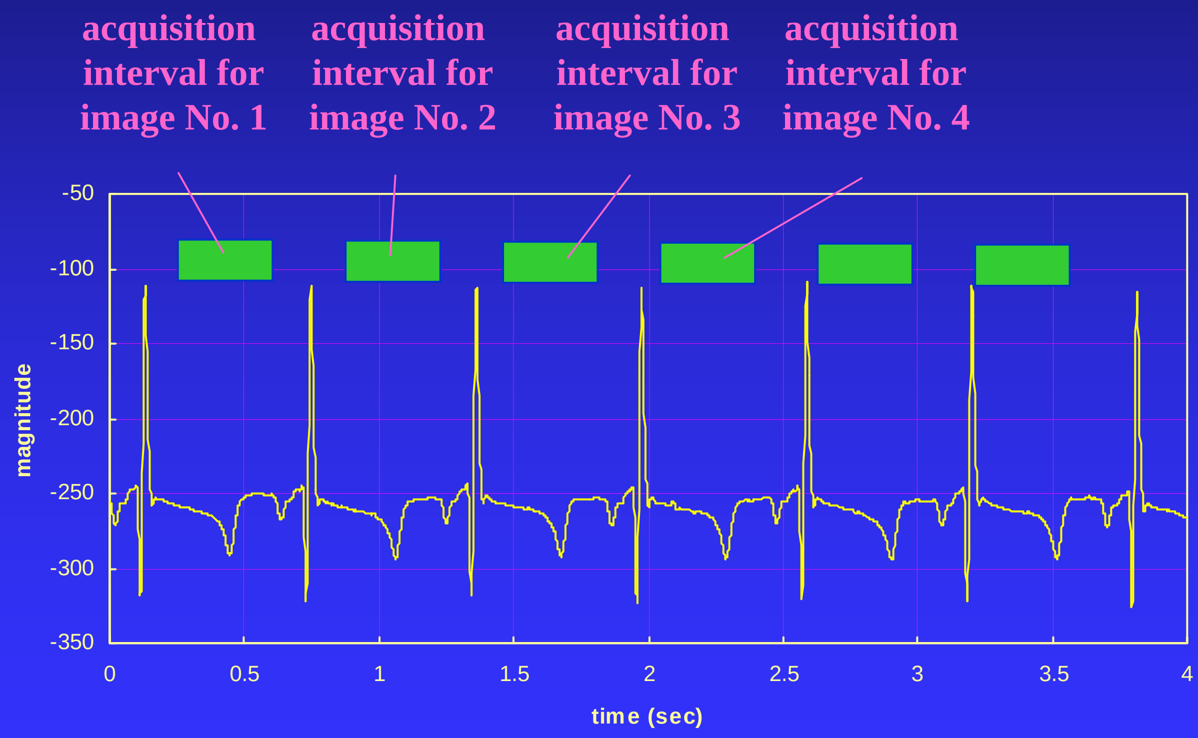
- In fan beam, each ray path is sampled by two conjugate samples.
- We need only 180 + fan angle data for complete reconstruction.



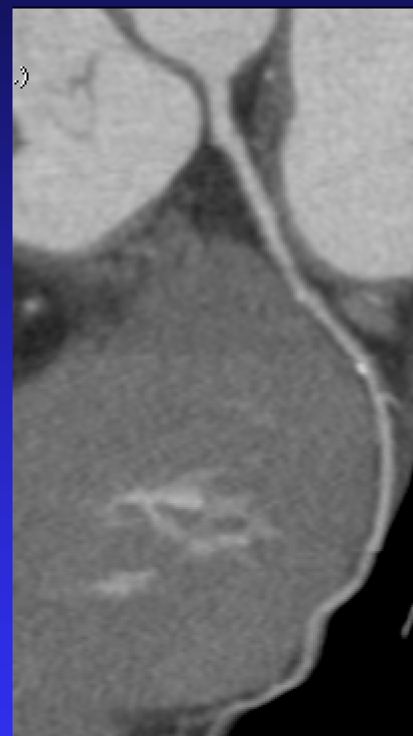


Single-cycle Cardiac Reconstruction

- Projection data used in the reconstruction is selected based on the EKG signal to minimize motion artifacts.



Cardiac Imaging

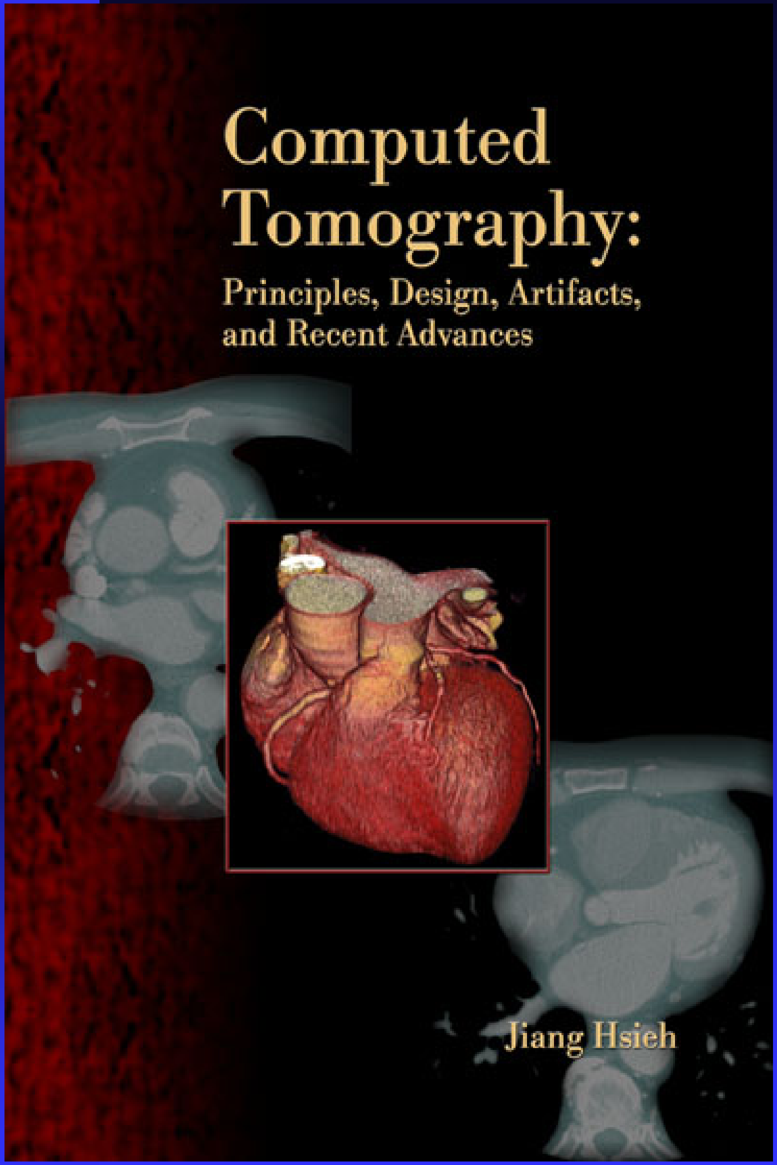


curved reformation

Bypass Graft Follow-Up
Gated Cardiac
20cm in 11s @ 0.625mm

Summary

- CT Image reconstruction techniques have been continuously developed over the years to match the advancement in new acquisition hardware and new acquisition techniques.
- With image explosion from the new CT scanners, advanced visualization tools are needed to improve the productivity of radiologists. Faster and better tools are constantly developed.



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