Proton Transport Based on a Generalization of Fermi-Eyges Theory and Energy Spectrum Evolution

Fermi-Eyges multiple scattering theory yields a simple Gaussian solution to the charged particle transport equation under pencil beam boundary conditions. There is an increasing number of applications of Fermi-Eyges theory to the proton beam therapy. One of the main difficulties encountered in these applications is that the protons' range straggling is primarily due to the energy straggling. Therefore, multiple scattering theory alone can not be used to predict the depth dose distribution.

Overcoming this difficulty was the motivation for developing of a new proton transport algorithm based on a generalization of Fermi-Eyges theory, known as the Proton Loss Model, and an energy spectrum evolution model. In this algorithm, we use N characteristics in the depth-energy phase space which are calculated using the continuous slowing down approximation (CSDA). For every characteristic, we solve analytically the Proton Loss transport equation which takes into account inelastic nuclear collisions and the range straggling due to multiple scattering. Then, we calculate the depth dependent energy spectra using the energy spectrum evolution model. The evolution of the proton spectrum describes the diffusion of protons between characteristics due to energy straggling. A final solution at every depth can be found as a linear combination of characteristic solutions with the weights which are defined by the energy spectrum at this depth. The effectiveness of the method is demonstrated by comparisons with the PTRAN Monte Carlo code and measurements.