## **Purpose:**

To analytically investigate a property of the integral dose-volume histogram (DVH) space.

## Method and Materials:

A curve called an  $\alpha$ -iso-NTCP envelope is constructed by connecting points belonging to step-like integral DVHs each corresponding to partial organ homogeneous irradiation of relative volumes  $v_{\alpha}$  to dose levels  $D_{\alpha}$  such that the resulting NTCP is in all cases  $\alpha \otimes$ . The two subspaces into which the envelope divides the DVH space are analytically explored through comparing the values of the equivalent uniform doses (EUDs) corresponding to the different DVHs and using the fact that NTCP is a monotonic function of EUD as well as the monotonic nature of the integral DVH itself.

## **Results:**

It is theoretically proven that any DVH passing through a point  $(D_a, v_a)$  from the  $\alpha$ -iso-NTCP envelope, i.e. any DVH that crosses or is tangential to the envelope, will result in an  $NTCP \ge \alpha\%$ , the equality being valid only for the step-like DVH corresponding to the partial organ homogeneous irradiation of  $v_a$  to  $D_a$ . Thus, it is proven that any DVH that at least partially lies above the envelope result in  $NTCP \ge \alpha\%$ . For some of the DVHs lying under the envelope, e.g. those that are tangential to the envelope, it is also true that the resulting  $NTCP \ge \alpha\%$ . However, it was numerically demonstrated elsewhere<sup>1</sup> that there exist DVHs lying entirely in the lower subspace that result in  $NTCP \le \alpha\%$ .

## **Conclusion:**

Since there is a chance that a DVH lying under the  $\alpha$ -iso-NTCP envelope will result in *NTCP*<a%, it would therefore be preferable in the treatment optimization process to seek solutions for DVHs lying entirely under an iso-NTCP envelope.

1. C. Schinkel-Ranger, P. Stavrev, N. Stavreva, and B. G. Fallone. "A theoretical approach to the problem of dose-volume constraint estimation and their impact on the dose-volume histogram selection," submitted to Medical Physics, 2006.