AbstractID: 5555 Title: A simple iterative method to invert a deformation field

Purpose

Inversion of a deformation field is applied frequently to map dose and regions of interest to a reference frame. A prevailing naïve approach that takes the opposite displacement of the forward deformation as the displacement of the inverse is mathematically wrong and can cause large errors for large or accumulative deformation. Inversion by "scattered data interpolation" has $O(N^2)$ complexity and is difficult to implement. We propose a simple iterative approach with $O(N)$ complexity.

Method

Instead of calculating the inverse, we calculate the displacement of the inverse. The displacement of the inverse is iteratively refined through the displacement of the forward map. We prove that such iterative scheme converges exponentially to the true solution when the deformation field is subject to a condition of the Lipschitz type. The Lipschitz type condition essentially states that the difference of the deformation of two points can not be too far. This is a mild restriction on the deformation field and is usually a valid assumption for any deformable registration method with regularization.

Results

We tested the proposed method on both simulated 2D data and real 4D CT data of lung patient. The simulations showed that the proposed method has exponential convergences to the true inverse. For real 4D CT data, the forward deformation field constructed by deformable registration mapped the test phase to the reference phase and the inverse of that deformation field accurately map the reference phase to the test phase.

Conclusions

A simple, accurate and fast method for inverting a deformation field is presented. Both the mathematical proof and the simulations showed its exponential convergence. Simulations and real data tests demonstrated its efficacy in medical image analysis and radiotherapy applications. Typically less than 10 iterations are needed to get an inverse deformation field with clinically relavent accuracy.