AbstractID: 7239 Title: Theoretical investigation of the pencil beam solution from the Fermi-Eyges partial differential equation

Purpose: To demonstrate the rigorous mathematical derivation of the Fermi-Eyges partial differential equation for the pencil beam solution and to compare with the original Eyges solution.

Method and Materials: In the radiation therapy community, there is a debate about the solution of the Fermi-Eyges equation presented by the original Eyges paper as to whether the equations are correct or not. But nobody has presented the detailed solution of this equation and compared with the original Eyges solution yet. In this manuscript, we present the rigorous mathematical derivation of the Fermi-Eyges equation for the distribution function. Several standard mathematical techniques such as two dimensional Fourier transform, change of variables by introducing the transformation function, different boundary conditions and use of the Jacobean to separate the coupled variables have been implemented to calculate the closed-form solution of the distribution function for the electron transport of the pencil beam algorithm. Also the derivation and the solution of the Fermi-Eyges equation have been compared with the original Eyges solution at the different check points.

Results: Examination of the detailed derivation of the Fermi-Eyges partial differential equation indicates that there is an error in the equation (14) of the original Eyges solution.

Conclusion: We have demonstrated the rigorous derivation for the pencil beam solution of the Fermi-Eyges equation. In radiation oncology physics, it's always important to fully understand the theoretical basis and the limitations of the solution of the Fermi-Eyges equation because this is the only deterministic solution for the pencil beam algorithm to calculate electron dose distribution. And, it's also important in order to develop the theory further and to apply it successfully in the clinical practice.