

Purpose: Repair of sublethal damage plays an important role in radiation therapy. Conventional radiobiology concepts are based on the assumption that the repair rate remains constant during the entire radiation course. However, increasing evidence from animal experiments report that the repair process may slow down with time and the data does not fit an exponential pattern. To address this dilemma, we developed a new closed form of Linear-Quadratic (LQ) model based on the repair pattern with a reciprocal time. The new formulas were tested with published experimental data.

Methods and Materials: The LQ model has been widely used in radiation therapy, $S = \exp(-\alpha D - \beta G D^2)$. The parameter G represents the repair process of sublethal damage with T_r as the repair half-time: $G(t) = \frac{2}{D^2} \int_0^t dw I(w) \int_0^w dv I(v) \frac{1}{1 + (w-v)/T_r}$ in the reciprocal pattern. A closed form of G was derived analytically for arbitrary radiation schemes: dose rate $I(t) = I_i$, $t_{i-1} < t < t_i$, $i = 1, \dots, N$. A set of published animal data was adopted to test the reciprocal formulas, in which rat foot skin was irradiated in split-doses with increasing time intervals (0-22h). The complication data of moist desquamation were analyzed using the generalized LQ model.

Results: A closed form of the LQ model to describe the repair process in a reciprocal pattern was obtained: $G = \frac{2T_r^2}{D^2} \sum_{i=1}^N \left\{ I_i \left[Y(t_i - t_{i-1}) + 1 \right] + \sum_{j=1}^{i-1} I_j \left[Y(t_i - t_{j-1}) - Y(t_i - t_j) + Y(t_{i-1} - t_j) - Y(t_{i-1} - t_{j-1}) \right] \right\}$, where function $Y(t) = \left(1 + \frac{t}{T_r} \right) \left[\ln \left(1 + \frac{t}{T_r} \right) - 1 \right]$.

Formulas for special cases were derived from this general form. The reciprocal model showed a better fit to the animal data than the exponential model, especially for the ED₅₀ data (reduced χ^2_{\min} of 2.0 vs. 4.3, $p = 0.11$ vs. 0.006), with the following LQ parameters: $\alpha/\beta = 2.6-4.8\text{Gy}$, $T_r = 3.2-3.9\text{h}$.

Conclusions: The repair process following a reciprocal-time has been investigated in this study and a closed form of the generalized LQ model was presented and validated. These formulas can be used to analyze the experimental and clinical data, where a slowing-down repair process appears during the radiation course.