A review of the Monte Carlo method in external photon beam treatment planning

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Outline

A. Definition of Efficiency - Factors that render current MC codes fast enough for clinical treatment planning - Variance reduction and efficiency enhancing techniques

B. Beam modeling: A review of available methods for beam characterization

C. Experimental Verification of MC-based systems
Outline

0. Statistical Uncertainties in MC-based dose

E. CT-number to material conversions

F. Dose-to-water and dose-to-medium

H. Summary of the recommendations of AAPM Task Group Report No. 105
AAPM TG 105 report


IJ Chetty, B Curran, J Cygler, J DeMarco, G Ezzell, B Faddegon, I Kawrakow, P Keall, H Liu, C-M Ma, DWO Rogers, D Sheikh-Bagheri, J Seuntjens, JV Siebers
A. Definition of Efficiency

The efficiency, $\varepsilon$, of a Monte Carlo calculation is defined as:

$$\varepsilon = \frac{1}{(\sigma^2 T)}$$

$\sigma^2$ is the variance on the quantity of interest (e.g. dose) and $T$ is the CPU time required to achieve the variance.

Since $\sigma^2 \propto 1/N$ and $T \propto N$, the quantity $\sigma^2 T$ is $\sim$ constant and the efficiency expresses how fast a given algorithm can calculate dose at a desired level of statistical accuracy.
Definition of Efficiency

\[ \varepsilon = 1/(\sigma^2 T) \]

Two approaches to increase \( \varepsilon \):
(a) either reduce \( \sigma^2 \) for a given \( T \)
(b) reduce \( T \) for a given \( N \) (no. of histories)
VRT’s and EET’s

AAPM TG-105 classification:

(a) methods reducing $\sigma^2$ while producing a mathematically unbiased result are called variance reduction techniques (VRT’s): ex. photon forcing, bremsstrahlung splitting, Russian roulette

(b) methods which reduce $T$ for a given $N$ but are approximate (introduce bias) are termed efficiency enhancing techniques (EET’s): ex. Range rejection and electron/photon energy cutoffs, Condensed History Technique (CHT)

Both of the above methods are vital in MC-based clinical calculations
The Condensed History Technique (CHT) is the single most important method enabling the use of MC calculations in radiotherapy. Berger (1963), developed the CHT for transporting electrons.

A typical electron in the MeV range, undergoes ~10 collisions

Interaction-by-interaction (analog) transport of electrons would therefore be prohibitively long.

The CHT method groups $e'$ interactions into single "steps" that account for aggregate effects of scattering along the path.
The Condensed History Technique (CHT)

The CHT introduces an artificial parameter, the “step size”; the electron step algorithm (transport mechanics) can strongly influence speed and accuracy.

The significant improvements in efficiency with “second generation” codes (e.g. VMC++, XVMC, EGSnrc, DPM, MMC, etc.) are mainly a result of differences in the transport mechanics and boundary crossing implementations, relative to “first generation codes” (EGS4/Presta, MCNP, GEANT4, Penelope, etc.).

In general, “second generation” codes employ e-step algorithms that converge faster, i.e. fewer CH steps are required for the same precision.
Other efficiency enhancing techniques

Range rejection: Electron’s history is terminated when it’s residual range is so low that it cannot escape from the current region; note that this method ignores the possible creation of brems. photons while slowing down and is therefore biased

Energy cutoffs: Photons and electrons are terminated if their energies fall below a low energy cutoff. This method is biased as it ignores photon and electron interactions that could take place below the energy threshold
Variance reduction techniques

Bremsstralung splitting: Electron is forced to produce many brems. photons; these secondary photons have appropriately adjusted weights to conserve particle weights.

Russian roulette: Typically played with events of little interest; particles are terminated with a given probability but surviving particle weights are increased with inverse probability to render the game unbiased.
Improper use of VRTs

Reynaert et al. MC tx planning for photons and e’s (Rad Phys Chem 2007)
B. Treatment head simulations and virtual source modeling

Phase space: $x, y, u, v, E, q$

Patient-independent structures

Phase space plane

Courtesy: Siebers (VCU)
The possible options for specifying a beam model

1. Linac simulation
2. Direct phase space
3. BEAM MODEL delivers PS particles
4. Measurement-driven models
5. Measured data
1. Direct Phase Space Simulation

A phase space file is generated at a plane above the patient-dependent components (jaws and MLC) and is used as input for the patient-dependent simulation.

Methods for simulation through the patient-dependent components include: direct simulation with or without approximations.
Direct simulation with approximations

First order Compton scattered photons are transported
Electrons are ignored (deposit energy locally)
Tongue and groove effect, and other details of the MLC design are incorporated

From Siebers et al.
Med Phys 2002
Direct Simulation of the linac (Siemens Primus) and phantom (40x40)

x-profile at 10.0 cm depth calculated with vmc++

VMC++ direct simulation
2.6 GHz, CPU
2. Virtual Source Modeling

Motivation: Virtual source models provide a more concise characterization of the PS file – they do not require GB of disk space, and are possibly more efficient.

Fluence distributions for individual treatment head components (sub-sources) are reconstructed from the phase space file acquired in a plane above the patient-dependent components.

Distributions for particle fluence, mean energy and angle for sub-sources are correlated.
Example Energy Fluence Distributions
(Varian, 6 MV)

Schach von Wittenau et al.: Med Phys 1999
3. Measurement Driven Models

Analytical representations or parameterized forms describing the fluence distributions and returning the phase space for calculations within the patient.

Optimal model parameters are derived from fitting procedures comparing calculations and measurements.

Beam modifiers may also be modeled using analytical approaches and parameters to account for primary and scattered photons.
FWHMs and relative weights of the 3 sources are adjusted iteratively to produce the best agreement between analytical calculations of the energy fluence and measured profiles in air.

Energy spectrum is derived by calculating mono-energetic depth doses water and minimizing the differences between measurements and the superposition of the calculated doses - includes an off-axis softening term.

Includes a geometry package for beam modifiers, such as MLCs.
What methods are used in the major MC vendor systems?

The majority of vendors are using measurement-driven models. Measurement-driven models do not require detailed knowledge of the treatment head and are very similar to the analytical models used over the years with conventional algorithms.

Using these models one may not be utilizing the full potential of the MC technique in simulating complicated delivery techniques, such as IMRT.

AAPM TG-157: Commissioning of beam models in Monte Carlo-based clinical treatment planning, C-M Ma, et al
C. Experimental Verification

How should one commission and verify a MC-based dose algorithm?

One strategy is to design measurements to test the algorithm performance with emphasis on:

(a) the beam model: to verify characterization of $e'$ parameters (e.g. energy), Tx head components (e.g. FF) and beam-modifiers (e.g. MLC) - this is done against depth dose and profile measurements in water phantoms for square and shaped fields.

(b) radiation transport accuracy in the patient: done against measurements in heterogeneous phantoms: small field sizes, low-density media, high beam energies, non-equilibrium conditions are useful.
Issues with measurements – buildup region

6x, 10x10

Courtesy of P. Roberson, S. Yokoyama (UM)
Issues with measurements – small field sizes

Measurements with small field sizes are complicated!

Measurements with small field sizes in inhomogeneous media are even more complicated!

W. Laub and T. Wong,
Accurate measurements are a requirement for accurate simulations!

Measurements at the 2% level of accuracy in clinically realistic geometries are difficult.

D. Statistical Uncertainties
Sources of uncertainty

Two sources of uncertainty: treatment head simulation (latent uncertainty – term coined by Sempau) and the patient simulation

The statistical uncertainty in calculated dose will approach (as a function of $1/\sqrt{N}$, where $N$ is the number of simulated particles), the finite, latent uncertainty associated with the phase space, regardless of the number of times the phase space is sampled.
Latent variance and beam models

Beam models consisting of full PS simulation of the treatment head are subject to latent variance.

Virtual source models reconstructed from the PS will also be subject to latent variance; fluctuations may be somewhat smoothed out.

Measurement-driven models will not be subject to latent variance although other, systematic uncertainties may exist in the generation of these models.
Statistical uncertainties: Dose prescriptions

MC-based dose prescriptions should be volume-based (e.g. to the PTV); doses should not be prescribed to the max. or min. dose points

In a region of uniform dose (e.g. the PTV), the MC calculated dose distribution will fluctuate about the mean dose; the statistical outliers (max. or min. dose points) can deviate from the mean dose by many standard deviations
Statistical uncertainties: Recommendations

Probability that the max. dose differs from the uniform dose by $y$ std. devs. in a region with $N$ voxels.

Prescribing doses to the max. pt. will underdose the target and vice versa for the min. pt.

Statistical uncertainties: Recommendations

DVHs and dose indices, such as TCP and NTCP are not highly sensitive to statistical noise; calculations with statistical precision of <2% are sufficient to accurately predict these values.

Dose volume indices for parallel organs like the lung (e.g. the mean lung dose) are minimally impacted by statistical noise.

For serial organs, where point doses are important, (e.g. the max. cord dose) higher statistical precision may be necessary; volume-based uncertainties will be more reliable.
E. MC-based treatment planning: CT number to material conversions
Methods for CT-to-material conversions

Patient tissues (via imaging data) need to be converted into cross sections required for MC simulation.

CT image (HU) → Convert to densities → HU vs. density conversion ramp

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Relative Electron Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>air</td>
<td>0.0</td>
</tr>
<tr>
<td>lung</td>
<td>0.2 (0.1-0.5)</td>
</tr>
<tr>
<td>Soft tissue, water</td>
<td>1.0</td>
</tr>
<tr>
<td>spongy bone</td>
<td>1.2</td>
</tr>
<tr>
<td>skull</td>
<td>1.65</td>
</tr>
<tr>
<td>compact bone</td>
<td>1.85</td>
</tr>
</tbody>
</table>
Methods for CT-to-material conversions

Directly convert CT HU to material cross sections; simple relationships between mass density and mass scattering and stopping powers have been derived by Kawrakow et al. (Med Phys, 23: 445 ('96)

CT-to-material conversions: Recommendations

Both mass density and material compositions (atomic no.) are needed for accurate MC calculation. Failure to incorporate atomic no. compositions can result in notable errors at higher tissue densities (Verhaegen and Devic, PMB, 50:937, '05).

From Siebers et al PMB: 45: 983 (2000)
F. Dose-to-water and dose-to-medium
Converting $D_m$ to $D_w$

The conversion can be accomplished using the Bragg-Gray formalism:

$$D_w = D_m \left( \frac{S}{\rho} \right)_m^w$$

Unrestricted wat-to-med mass collision stopping power averaged over the energy spectrum of electrons at the pt. of interest.

This can be applied either as a post-processing step or as a multiplication factor to the energy loss step.
Clinical Examples: $D_w$ and $D_m$

$D_m$

$D_w$

$D_w$ increases dose to mandible

Clinical Examples: $D_w$ and $D_m$

Clinical Examples: $D_w$ and $D_m$

Dose-to-medium and dose-to-water: Recommendations

The report recommends that vendors report both $D_m$ and $D_w$ as part of their dose calculation output.

The method of conversion from $D_m$ to $D_w$ should be clearly documented.
G. AAPM TG-105: Summary of Recommendations

Treatment Head Simulation:
(a) Vendors should provide the necessary support and assistance with the beam modeling and benchmarking process, e.g. fine-tuning of the models
(b) If the model is based on direct PS simulation, the latent variance in the model should be estimated by the vendor and be made available to users

Patient Simulation:
Statistical Uncertainties: Should be specified to doses within volumes consisting of many voxels; single-voxel dose uncertainty estimates should be avoided as should be specification to the maximum or minimum dose voxels
Patient Simulation:
VRTs and EETs: Users should understand the influence on the dose accuracy of VRTs and EETs. Vendors should provide documentation on these methods and on their influence, as well as flexibility to adjust these parameters where possible.

Dose Prescriptions: Vendors are strongly discouraged from prescribing doses to single voxels (point doses). Doses should be prescribed to volumes consisting of more than a single voxel; e.g. an isodose volume.

CT to material conversions: Should be based on both mass density and atomic no. compositions of materials.
Patient Simulation:
Dose to water and dose to medium: Vendors should: (a) state explicitly to which material dose is reported; (b) allow for conversion between $D_w$ and $D_m$

Experimental Verification: (a) The MC method should be subjected to the same level of testing as reported in articles on commissioning of dose algorithms, such as AAPM TG-53. In addition to standard commissioning tests, verification should include testing in complex situations to verify the expected improved accuracy with the MC method; (b) Detector perturbations need to be carefully assessed particularly under conditions of electronic disequilibrium; (c) Measurement uncertainties should be understood and estimated, where possible, in the verification process.
Conclusion

Clinical implementation of MC-based systems must be performed thoughtfully and physicists must understand the differences between MC-based and conventional dose algorithms.

Successful implementation of clinical MC algorithms will require strong support from the clinical team and an understanding of the paradigm shift with MC algorithms.

A properly commissioned MC-based dose algorithm will improve dose calculation accuracy in 3D-CRT and IMRT treatment planning and may improve dose-effect correlations.
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Which of the following regarding the condensed history method is/are \textit{false}?

1. It is an algorithm for the transport of photons
2. It is an algorithm for the single-scatter transport of electrons
3. It is an algorithm for the transport of photons in an analog manner
4. Second generation codes require more CH steps to converge than first generation codes
5. All of the above
Which of the following is an example of a variance reduction technique according to the TG-105 report classification?

0%  1. Range rejection
0%  2. Energy cutoffs
0%  3. The condensed history technique
0%  ★4. Bremsstrahlung splitting
0%  5. None of the above
Which of the following statements regarding the statistical uncertainty in dose is/are true?

1. It is proportional to the number of histories simulated
2. It is proportional to the square root of the number of histories simulated
3. It is inversely proportional to the number of histories simulated
4. It is inversely proportional to the square root of the number of histories simulated
5. All of the above
Thank You!