Dual Energy CT Image Reconstruction Algorithms and Performance

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Special thanks to Bruce R. Whiting and N. Agbeko

Dual Energy CT Image Reconstruction

SOMATOM Definition CT Scanner
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- Data Models ➔ Reconstruction Algorithms
- Image Reconstruction Approaches
  - "Linear" Approaches
  - Statistical Iterative Reconstruction
- Simulation Study of the Dual Energy Alternating Minimization Algorithm
- Performance Quantification: the Cramer-Rao Lower Bound
- Conclusions

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Motivation

- Dual Source, Fast kVp Switching, Phton Counting, and Energy Selective Detectors Are Available
  - Dual energy image reconstruction algorithms are needed now
  - Imaging III session yesterday; this session
- System Selection and Algorithm Design
  - Basis for comparing systems based on performance
  - Fundamental approach that extends to new systems (multiple energies, photon counting, etc.)
  - Quantifying the impact of modeling errors

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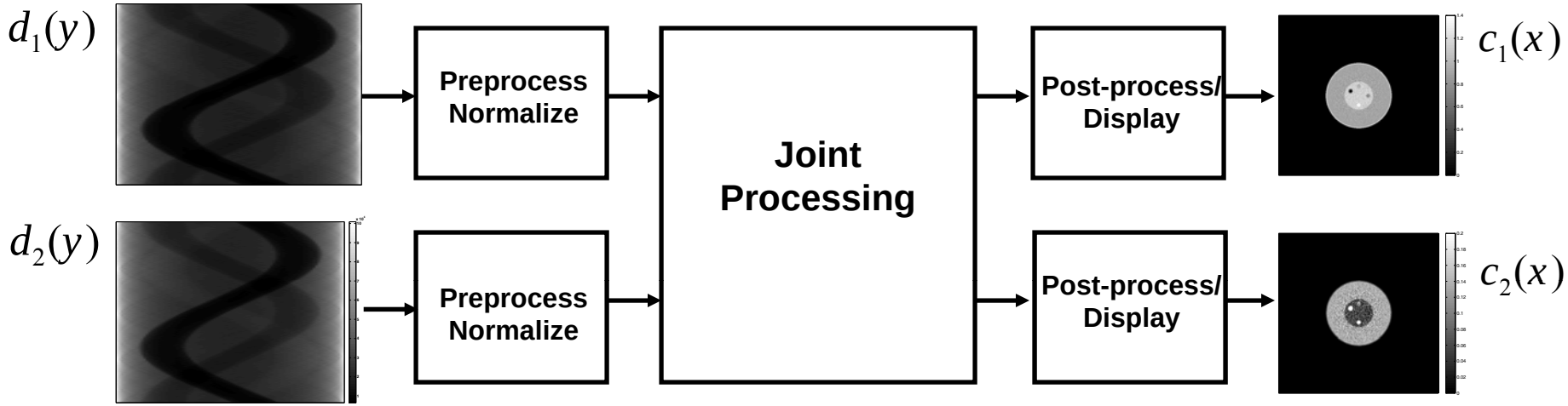
Dual Energy Image Reconstruction



- Multiple inputs with different spectral sensitivities
- Some standard normalizations (e.g., relative to air scans)
- Joint processing combines the data sets
- Post-processing can extract the desired image(s)
  - Components
  - Estimated images

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X-Ray Transmission Tomography—Back to Basics

- Source Spectrum, Energy E
- Beer's Law and Attenuation

$$S(E)e^{-\int_l \mu(x,E)dx}$$

- Mean Photons/Detector I_0

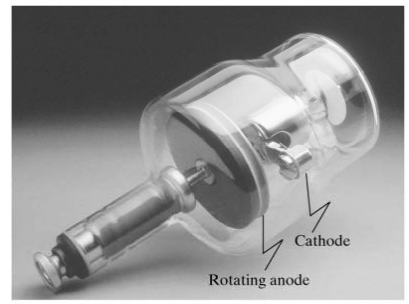
$$I_0 S(E)e^{-\int_l \mu(x,E)dx}$$

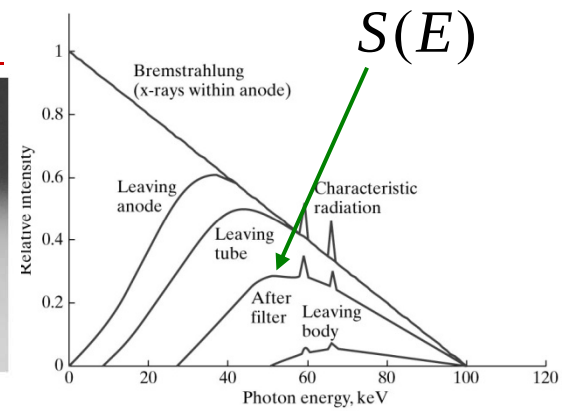
- Beam-hardening
- Detector Sensitivity Spectrum
- Mean (Photon) Flux

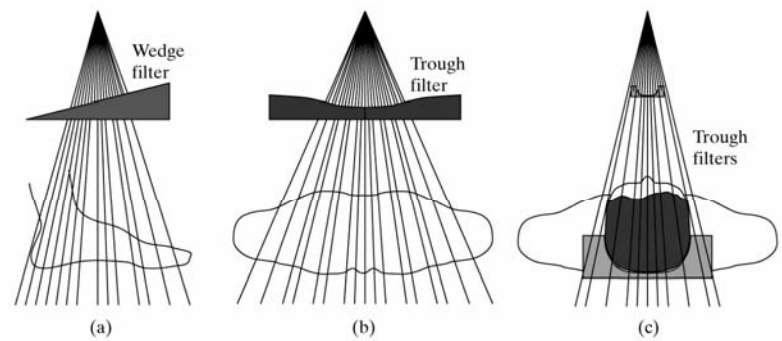
$$D(E), \quad \Phi(E) = S(E)D(E)$$

$$I_0 \Phi(E)e^{-\int_l \mu(x,E)dx}$$

$$\int_0^{kVp} I_0 \Phi(E)e^{-\int_l \mu(x,E)dx} dE$$







J. L. Prince and J. Links, *Medical Imaging Signals and Systems*. Pearson Education: Upper Saddle River, NJ, 2006.

"Linear" Image Reconstruction

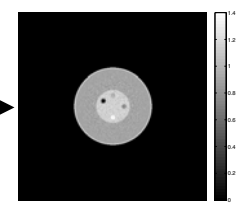
- Normalize relative to an air scan
- Beam-hardening correction to target energy
- Negative log to estimate line integrals at target energy
- Linear inversion, normalize (e.g. water-equivalent)

$$\int_{l(y)} \mu(x,\bar{E})dx \approx -\log\left[BH^{-1}\left(\frac{\int_0^{kVp} I_0 \Phi(E) e^{-\int_{l(y)} \mu(x,E)dx} dE}{\int_0^{kVp} I_0 \Phi(E) dE} \right)\right]$$

$$\hat{\mu}(x,\bar{E}) = FBP\left(\int_{l(y)} \mu(x,\bar{E})dx\right), \quad \hat{c}(x) = \frac{\hat{\mu}(x,\bar{E})}{\mu_{water}(\bar{E})}$$

$d(y)$

Normalize & correct → **Negative logarithm** → **FBP normalize** →



$c(x)$

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X-Ray Transmission Tomography—Back to Basics

- Detector Sensitivity $D(E)$
  - Probability that a photon of energy E is detected
  - Mean response to a photon of energy E:
    - photon counting ➔ response = 1
    - energy integrating ➔ response = E
    - other
- Detector Statistics
  - Photon counting, Beer's Law as survival probability ➔ Poisson distribution
  - Energy integrating or other ➔ compound Poisson

Poisson mean λ: $$P(N=k)=\frac{\lambda^k}{k!}e^{-\lambda},\quad k\geq 0$$

Log-likelihood function: $$l(\lambda)=k\ln\lambda-\lambda$$

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X-Ray Transmission Tomography—Reconstruction Principles

- Deterministic Model
  - Data equal a function of the desired image
  - Approximately invert that function to reconstruct image (minimize a measure of error between the data and the model)

$$\min_{\mu} \left\| d - g(\cdot : \mu) \right\|^2$$

- Random Model
  - Find the log-likelihood function for the data
  - Maximize the (possibly penalized) log-likelihood function over possible images

$$\max_{\mu} l(d \mid g(\cdot : \mu))$$

Statistical Image Reconstruction

- Source-detector pairs indexed by y; voxels indexed by x
- Data $d_j(y)$ Poisson, means $g_j(y{:}\mu)$, log-likelihood function

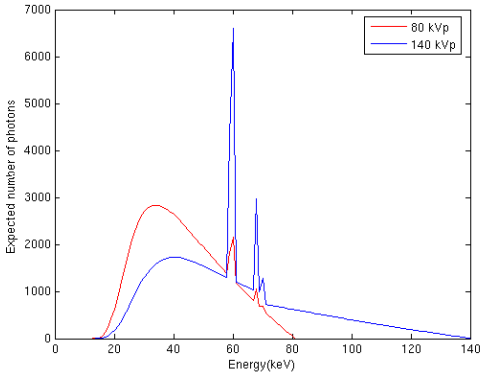
$$l(d_j \mid g_j(\cdot : \mu)) = \sum_{y \in \mathcal{Y}} d_j(y) \ln g_j(y:\mu) - g_j(y:\mu)$$

$$g_j(y:\mu) = \sum_E I_j \Phi_j(y,E) \exp\left(-\sum_{x \in \mathcal{X}} h(y,x)\mu(x,E) \right) + \beta_j(y)$$

- Mean unattenuated counts I_j, mean background β_j
- Attenuation function $\mu(x,E)$, E energies

$$\mu(x,E) = \sum_{i=1}^{I} c_i(x)\mu_i(E)$$

- Maximize over μ or c_i



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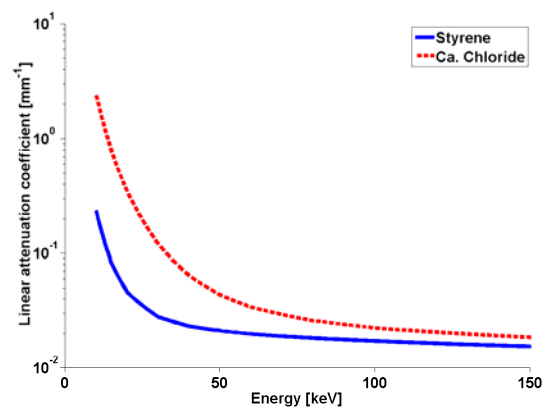
Attenuation Function Approximation

- Voxels as function approximation
  - Constant attenuation over a small volume, or
  - Linear combination of basis functions
- Energy dependence
  - Water equivalent
  - Linear combination of basis functions
- Basis functions
  - Physics (photoelectric and Compton scatter)
  - Physiological (e.g., fat and bone)
  - Signal processing (e.g., SVD)
  - Hand selected (e.g., CaCl and styrene)
- Constrained system
  - Dual Energy, 3 images

$$\mu(x,E) = \sum_{i=1}^{I} c_i(x)\mu_i(E)$$



$$c_i(x) \geq 0, \quad c_1(x) + c_2(x) + c_3(x) = 1$$

$$\mu(x,E) = \sum_{i=1}^{3} c_i(x)\mu_i(E)$$

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Statistical Image Reconstruction—AM Algorithm

- Alternating Minimization Algorithm
  - Several papers from our group (O'S & Benac, TMI 2007)
  - Monotonically increasing log-likelihood

$$\boxed{g_j(y) = \sum_E q_j(y,E)}$$

$$g_j(y:\mu) = \sum_E I_j \Phi_j(y,E) \exp\left(-\sum_{x\in\mathcal{X}} h(y,x) \sum_{i=1}^{I} \mu_i(E) c_i(x) \right) + \beta_j(y),$$

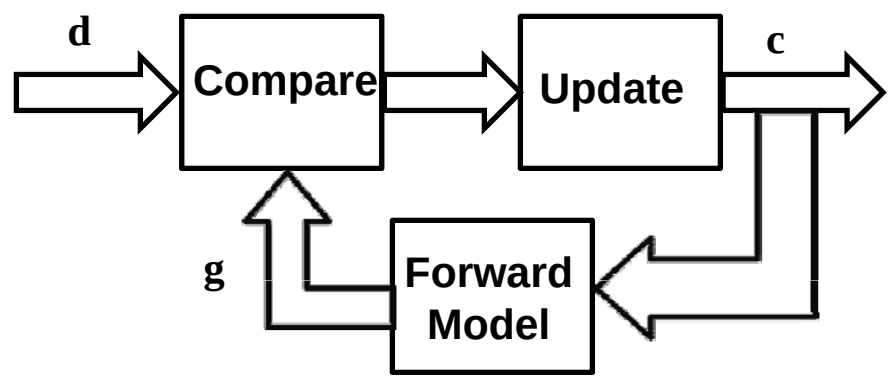
$$\max_{\{c_i(x)\}} \sum_{j=1}^{2} l(d_j \mid g_j(\cdot:\mu)) = \sum_{j=1}^{2} \left[\sum_{y\in\mathcal{Y}} d_j(y) \ln g_j(y:\mu) - g_j(y:\mu) \right]$$

- Compare using backprojection of data, estimated means

$$\hat{c}_i^{(k+1)} = \hat{c}_i^{(k+1)}(x) - \frac{1}{Z_i(x)} \ln\left(\frac{\sum_j \tilde{b}_{i,j}^{(k)}(x)}{\sum_j \hat{b}_{i,j}^{(k)}(x)} \right)$$

$$\hat{b}_{i,j}^{(k)}(x) = \sum_{y,E} h(y,x) \mu_i(E) q_j^{(k)}(y,E)$$

$$\tilde{b}_{i,j}^{(k)}(x) = \sum_{y,E} h(y,x) \mu_i(E) \frac{d_j(y) q_j^{(k)}(y,E)}{\sum_{E'} q_j^{(k)}(y,E')}$$



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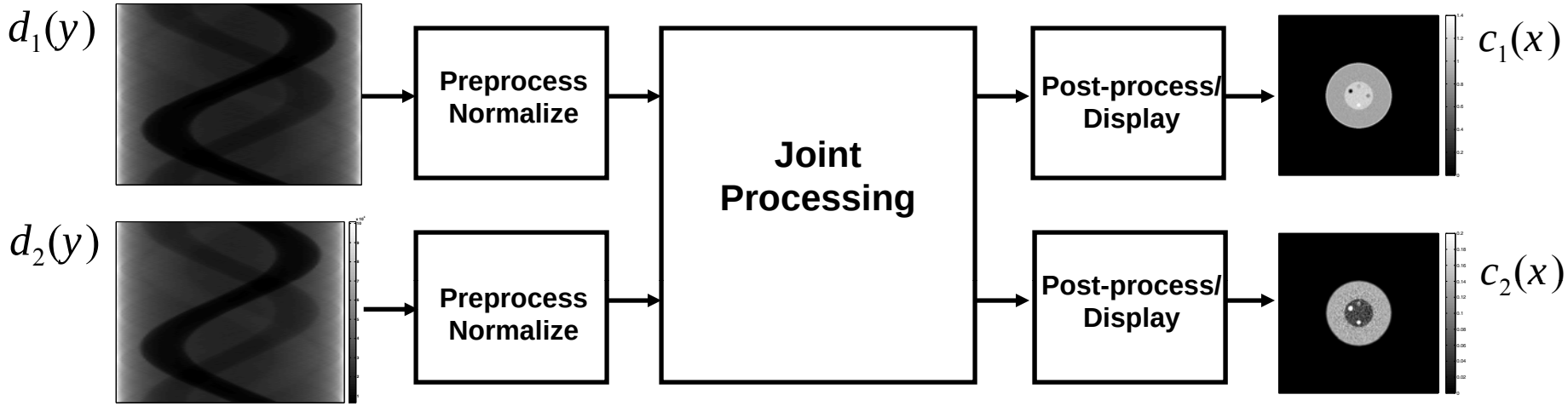

- Multiple inputs with different spectral sensitivities
- Some standard normalizations (e.g., relative to air scans)
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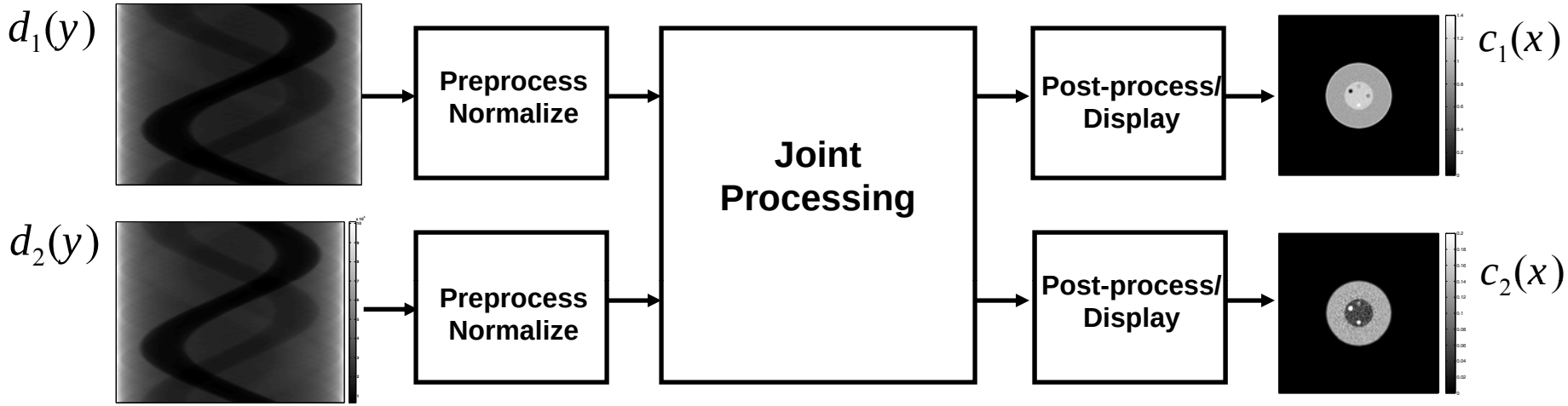
Dual Energy Image Reconstruction



- Multiple inputs with different spectral sensitivities

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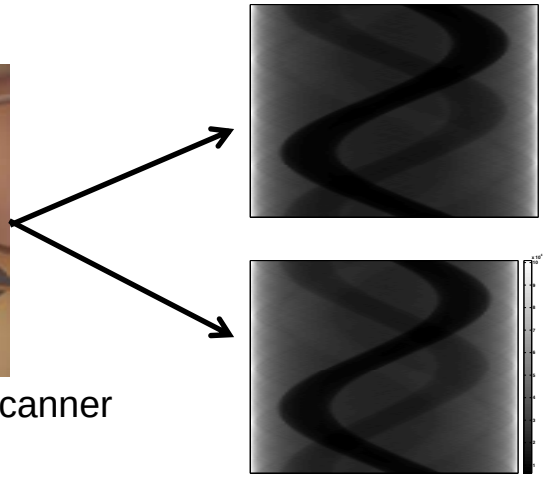
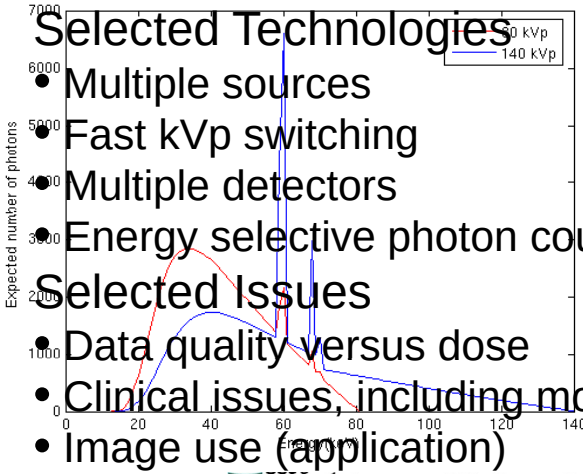
Dual Source





Selected Technologies
- Multiple sources
- Fast kVp switching
- Multiple detectors
- Energy selective photon counting

Selected Issues
- Data quality versus dose
- Clinical issues, including motion
- Image use (application)

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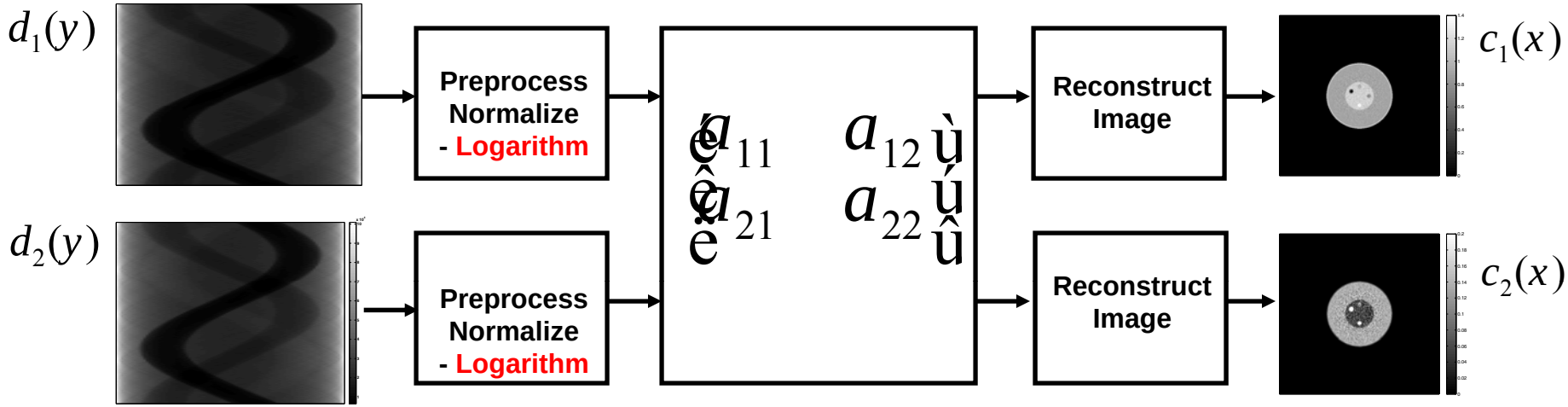
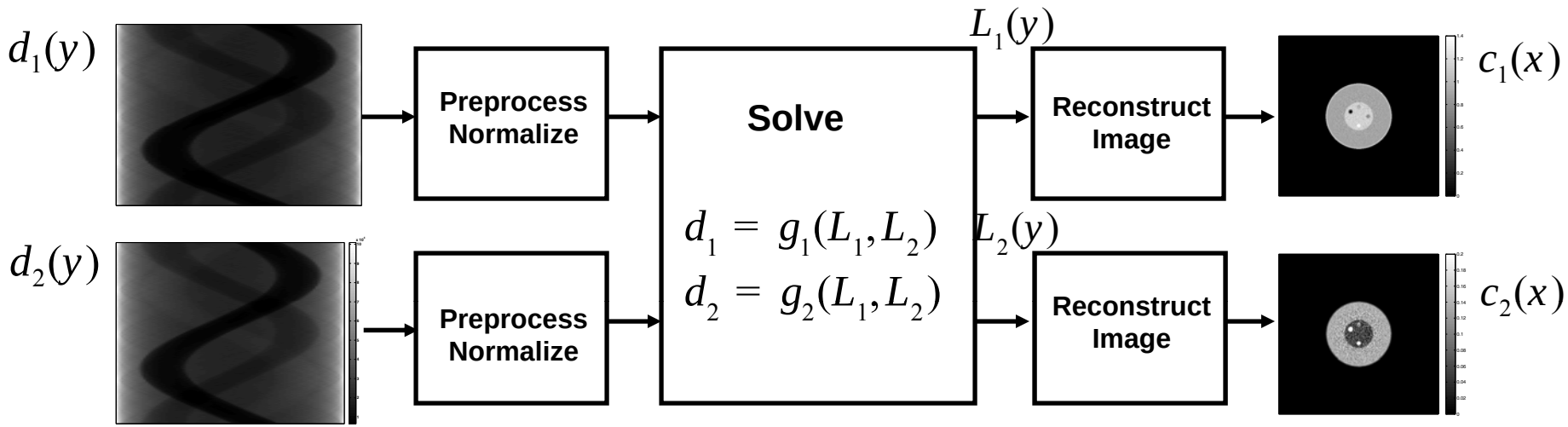
Dual Energy Image Reconstruction



- Joint processing combines the data sets
  - Linear combination of attenuations, then reconstruct individual images independently

Dual Energy Image Reconstruction



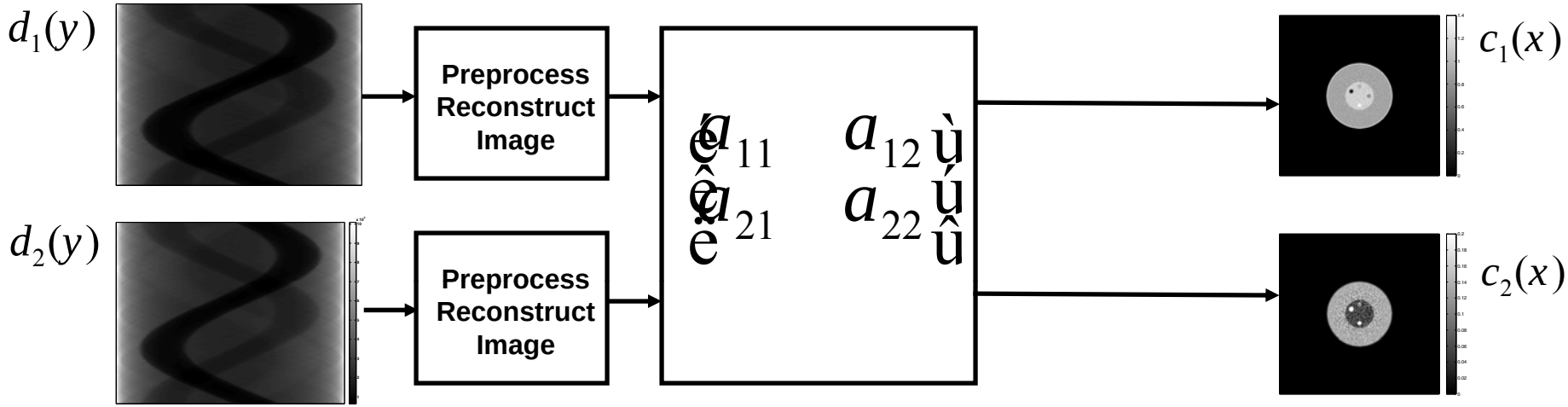
- Joint processing combines the data sets
  - Nonlinear inversion of raw data, then reconstruct individually

$$g_1(L_1, L_2) = \sum_E I_1 F_1(E) \exp\left(-L_1 m_1(E) - L_2 m_2(E)\right)$$

$$g_2(L_1, L_2) = \sum_E I_2 F_2(E) \exp\left(-L_1 m_1(E) - L_2 m_2(E)\right)$$

Kalendar, Klotz, Kostaridou, TMI 1988.

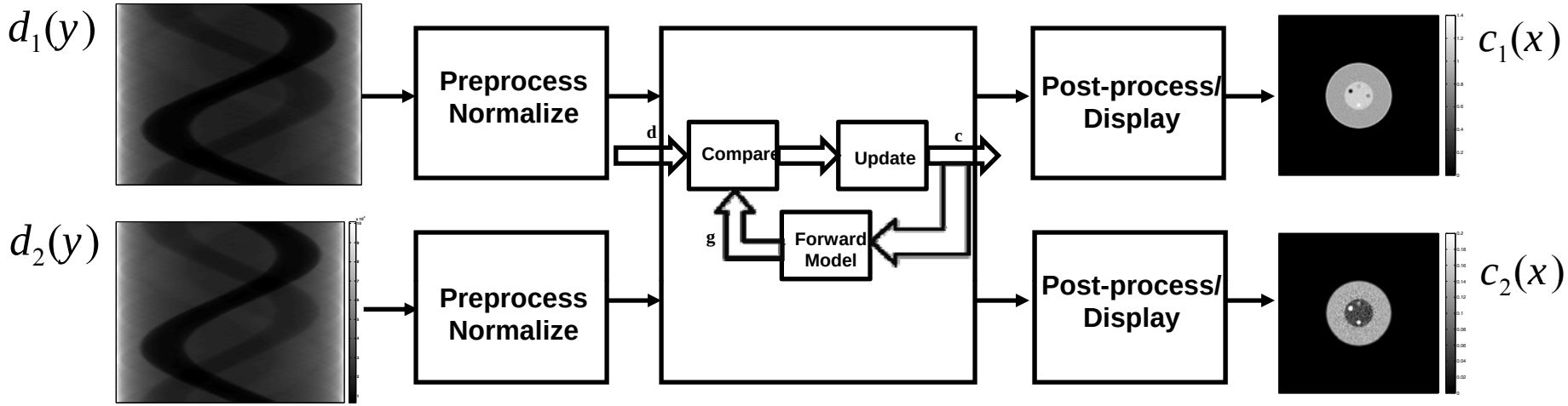
Dual Energy Image Reconstruction



- Joint processing combines the data sets
  - Reconstruct individual images from each data set, then (linearly) combine the resulting attenuation images to estimate desired outputs

Williamson, Li, Whiting, Lerma, Med. Phys. 2006.

Dual Energy Image Reconstruction



- Joint processing combines the data sets
  - Use knowledge of source spectrum and detector sensitivity spectrum
  - Use a model of joint data dependence on unknown underlying attenuation maps
  - Jointly estimate component images based on that model (e.g., statistical iterative reconstruction)

See also: Sukovic and Clinthorne, TMI 2000.

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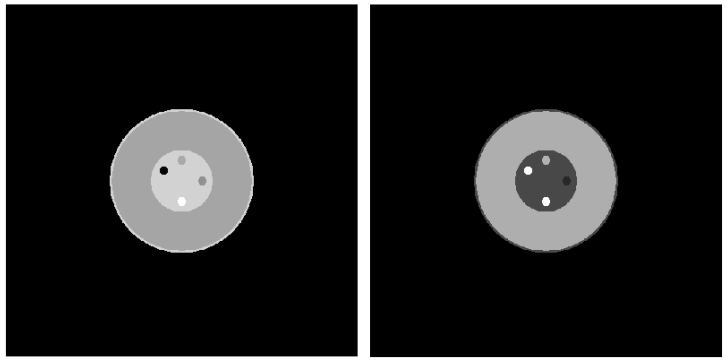
Simulations: Post-reconstruction vs. Joint Statistical Image Reconstruction

- **Large Phantom**: 20cm in diameter; thin outer lucite shell; water; four rods in inner 60mm lucite cylinder.
  - Calibration rods: calcium chloride, ethanol, teflon and polystyrene in the 12, 3, 6, and 10 o'clock positions, resp.
  - Test phantom rods: muscle, ethanol, teflon and substance X (a bonelike material) in the 12, 3, 6, and 10 o'clock positions, resp.
- **Small Phantom**: 60mm diameter lucite cylinder with rods
- **FBP-BVM** (Basis Vector Method; JF Williamson, et al. 2006)
  - Water-equivalent beam hardening correction
  - Requires calibration data to estimate linear transformation that generates the component images
  - FBP uses a ramp filter
- **AM-DE** (AM Dual Energy Algorithm)
  - *NO pre-correction of data*
  - *NO calibration data*
  - *NO regularization*



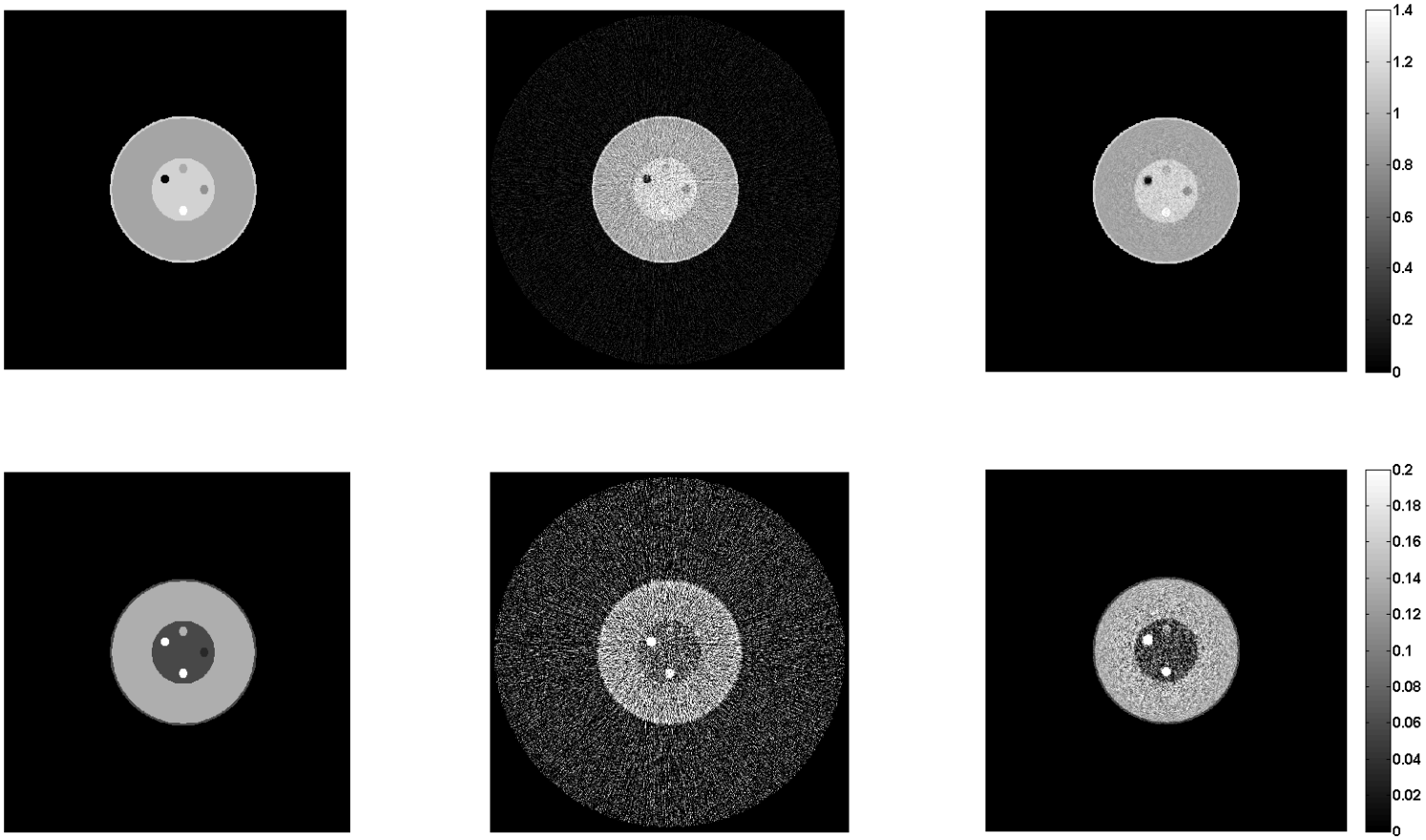
Materials Used - Fractions

Substance	Styrene Fraction	Ca. Chloride Fraction
Styrene	1	0
Ca. Chloride	0	1
Ethanol	0.79904	0.03369
Lucite	1.14	0.05834
Teflon	1.4194	0.48799
Water	0.90357	0.1357
Muscle	0.93995	0.13904
Substance X	0.03	2.8613

Large Phantom ($I_0=10^5$) – Component Images

Styrene

Calcium Chloride



Truth FBP-BVM AM-DE

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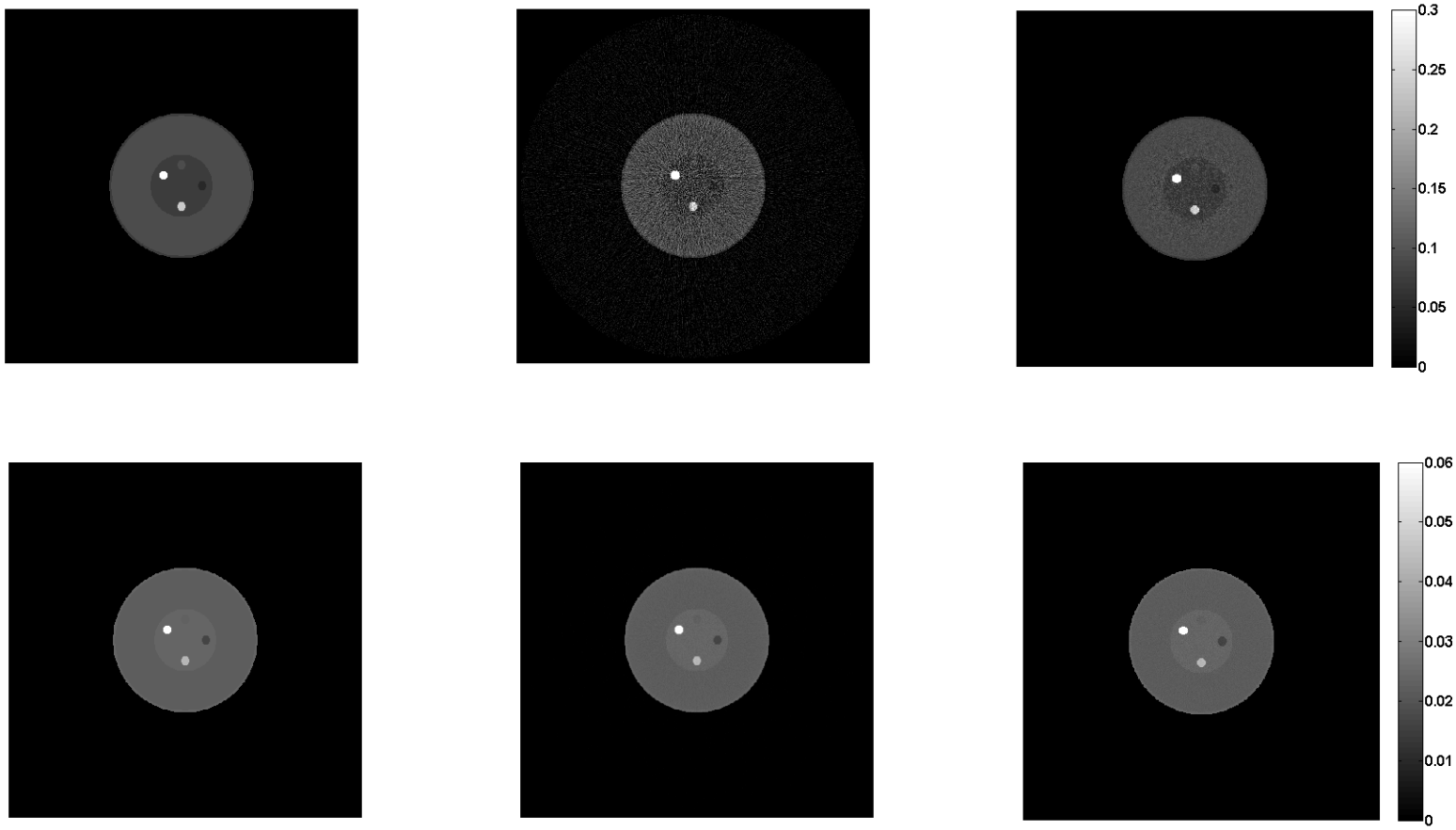
Large Phantom – Synthesized Images

20 keV

65 keV



Truth FBP-BVM AM-DE

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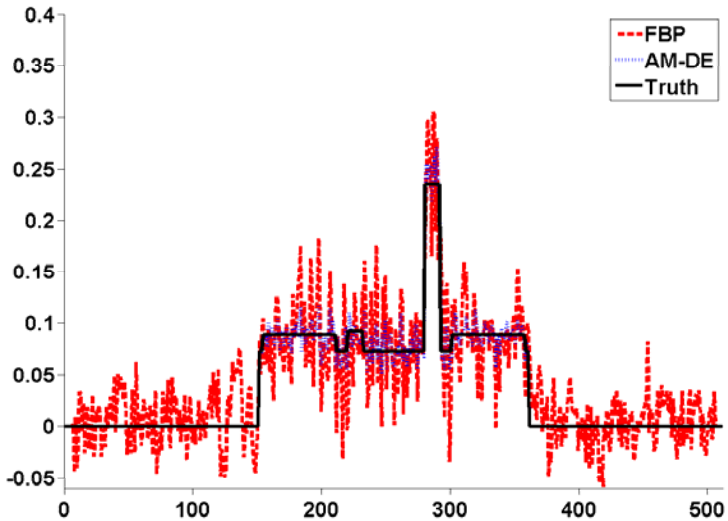
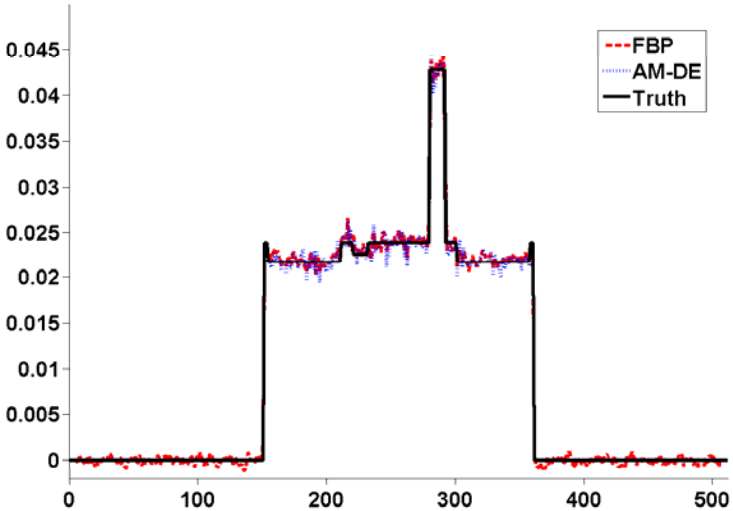
Large Phantom Profiles

Profiles through column 256 of 20keV image(left) and 65keV image(right)





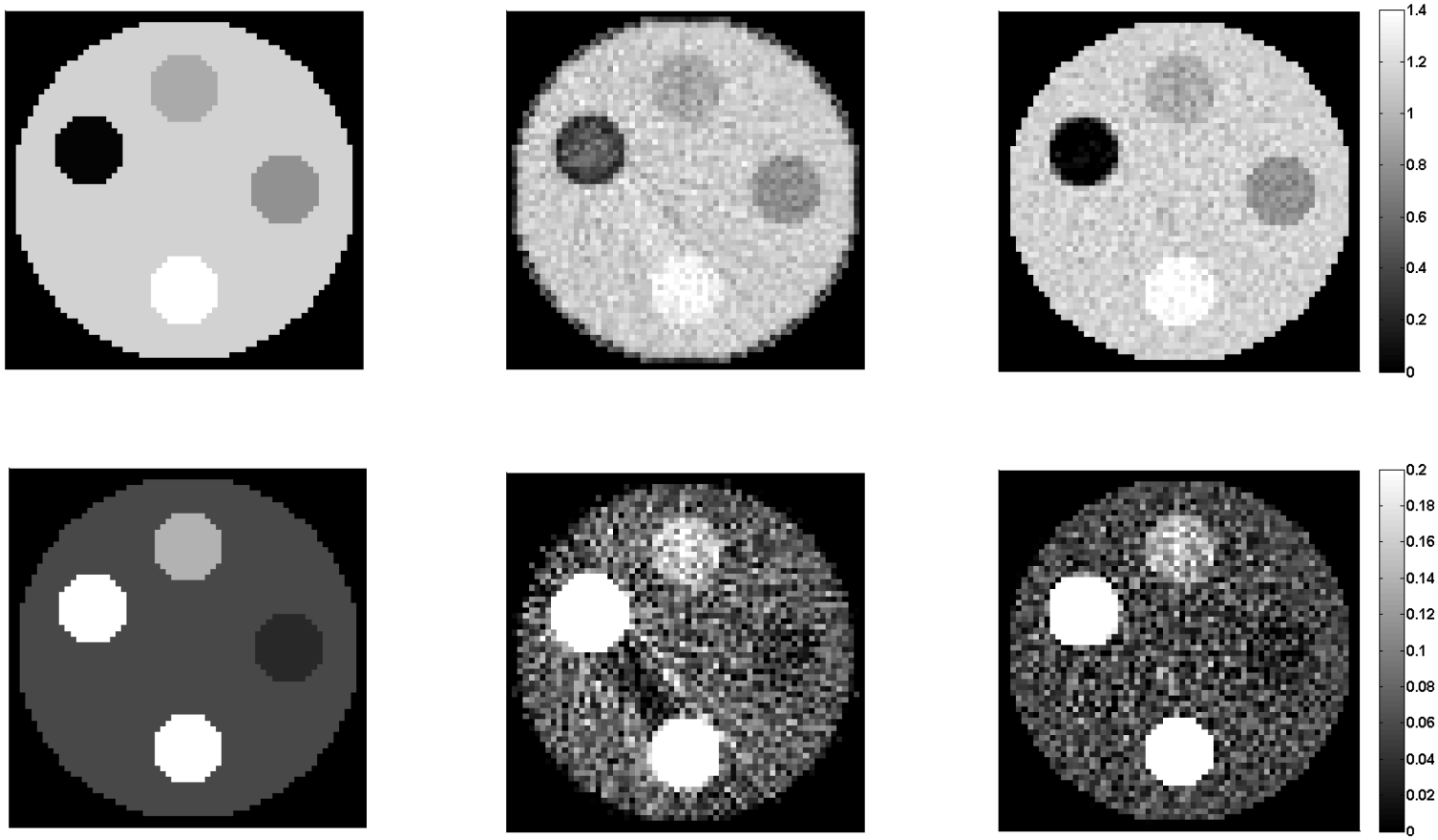
Mini CT – Small Phantom

- 360 Source positions
- 92 detectors
- Based on Siemens Somatom Plus 4 scanner
- 64 by 64 image

Small Phantom – Component Images

Styrene

Calcium Chloride



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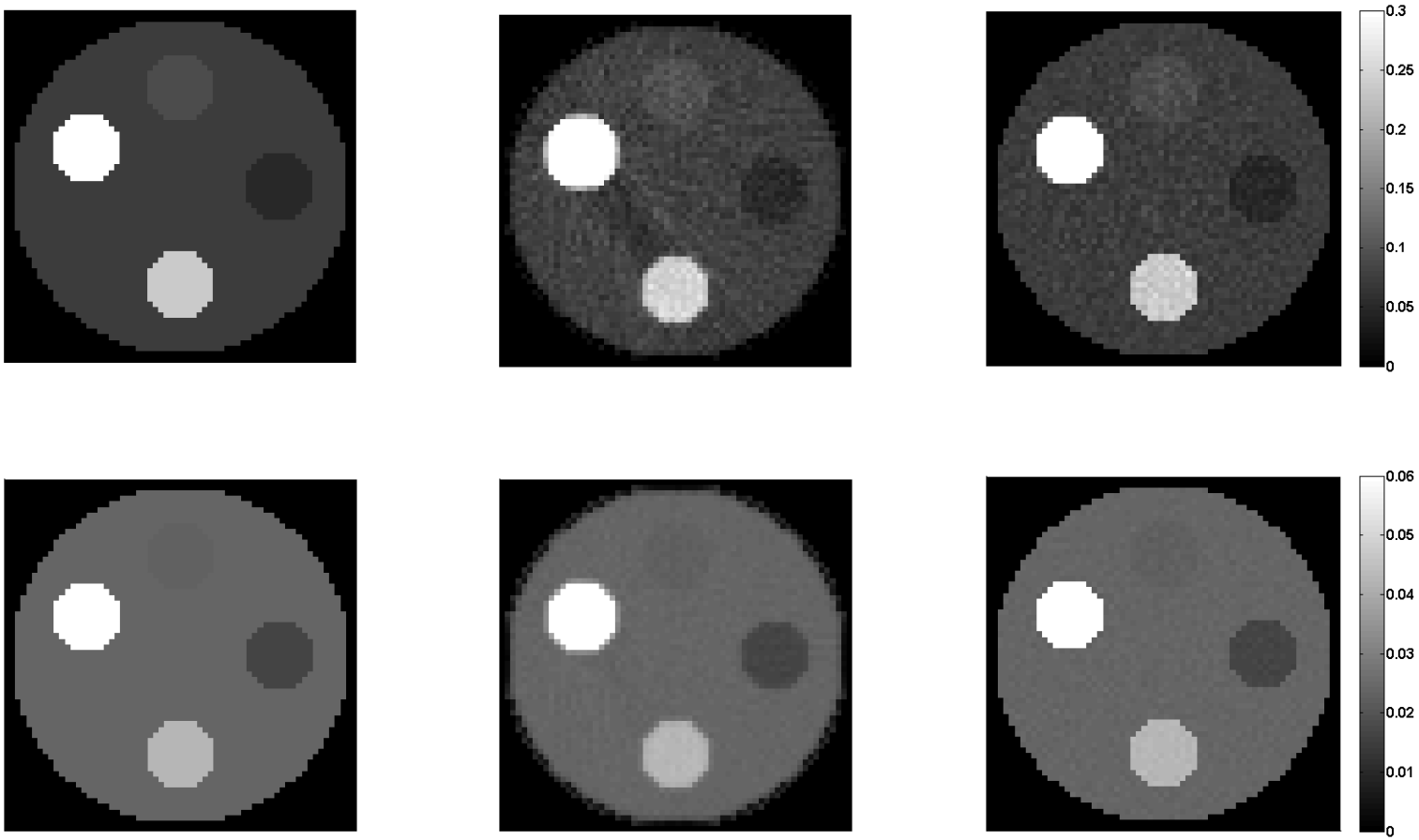
20 keV

65 keV



Truth

FBP-BVM

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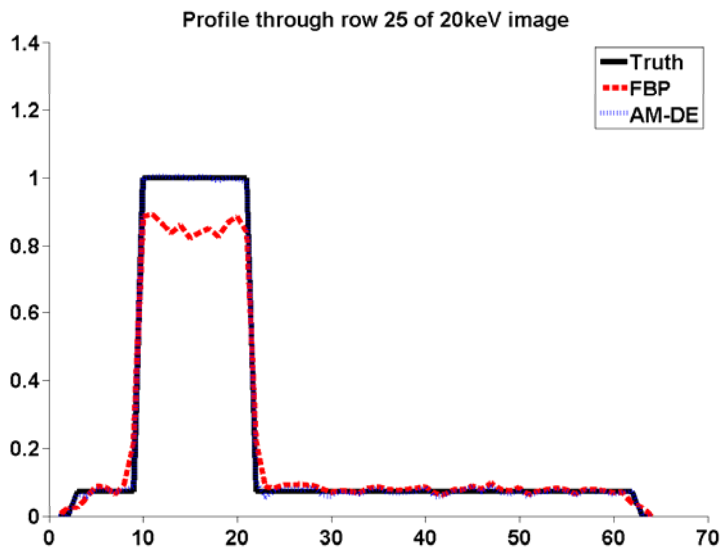
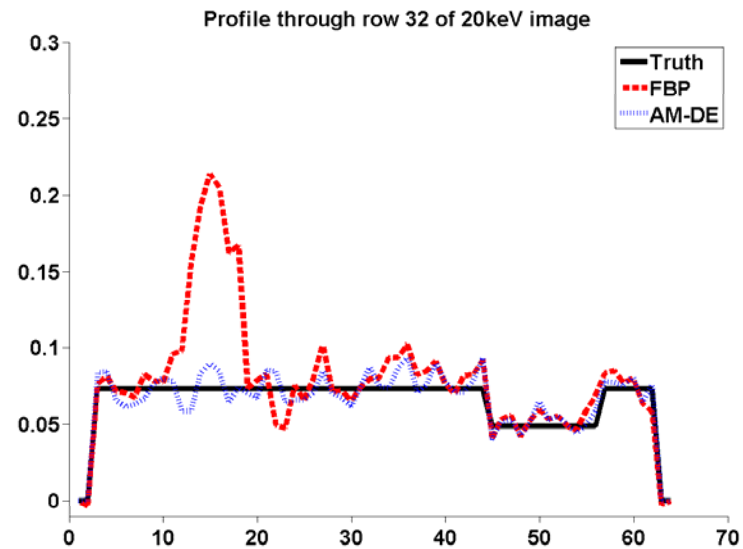
Small Phantom – 20keV profiles





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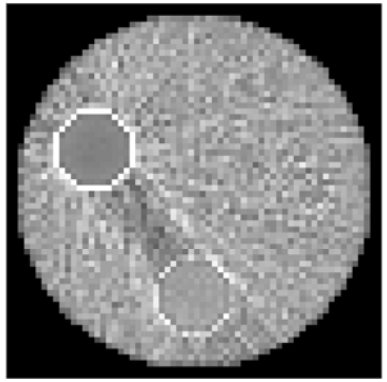
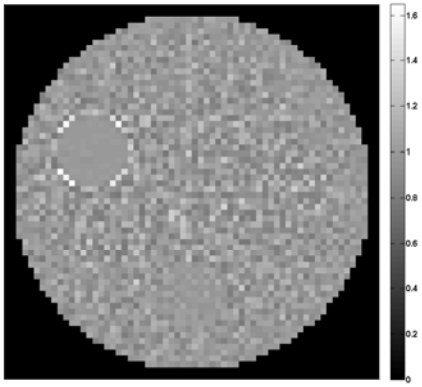
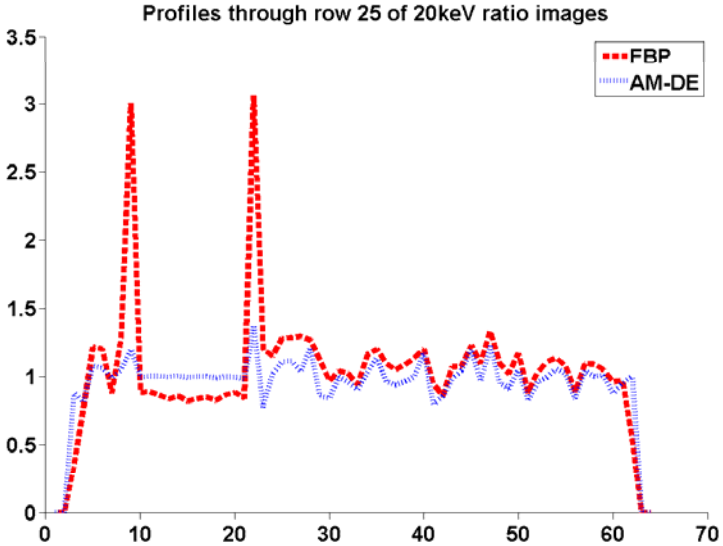
Small Phantom – 20keV ratio images and profiles

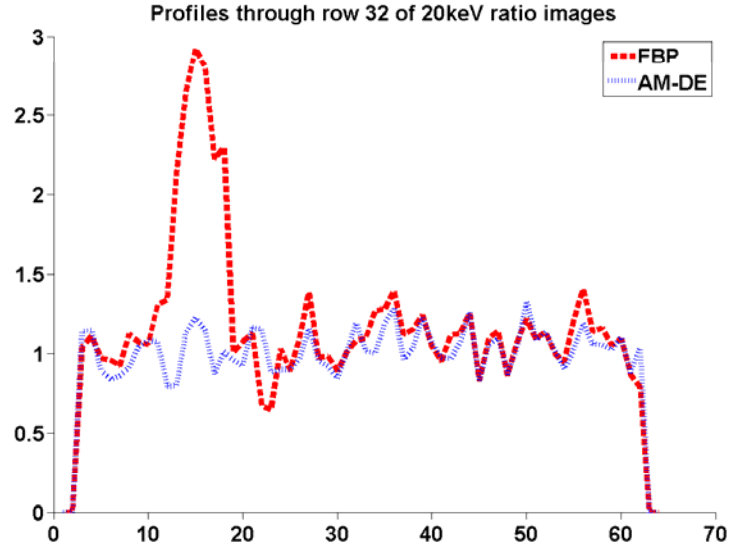
FBP-BVM



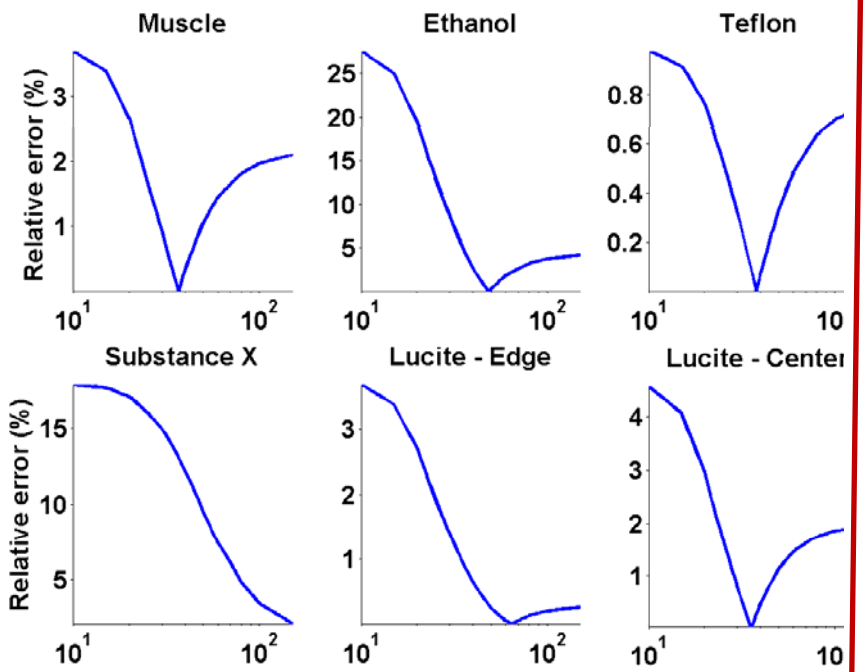


AM-DE

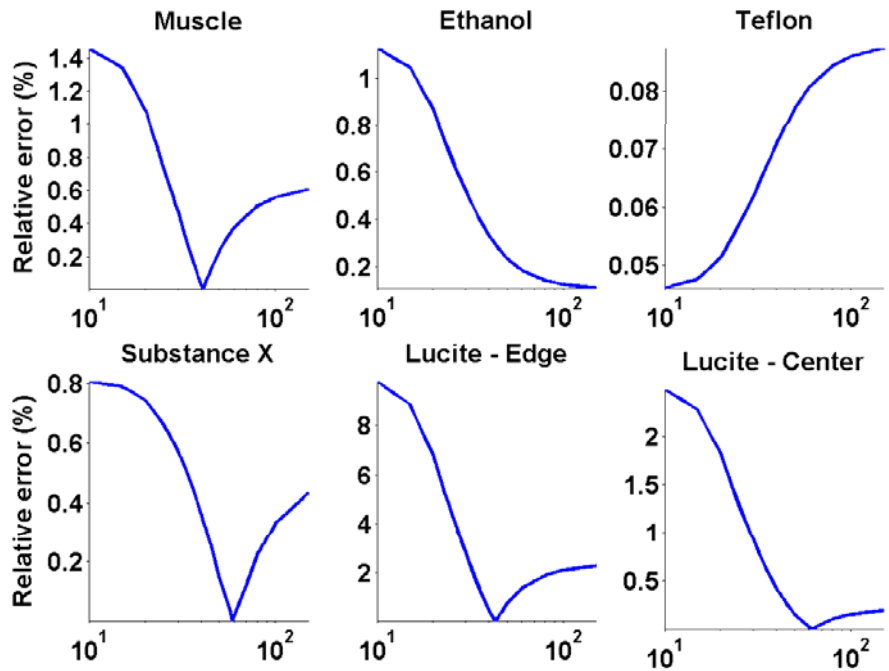




Small Phantom: Relative Errors
$I_0 = 10{,}000$



FBP-BVM



AM-DE

- Lucite – Center: A region of interest in the center of the phantom.
- Lucite – Edge: A region of interest near the edge of the phantom, between Substance X and Teflon.
- Relative error equals absolute difference divided by truth

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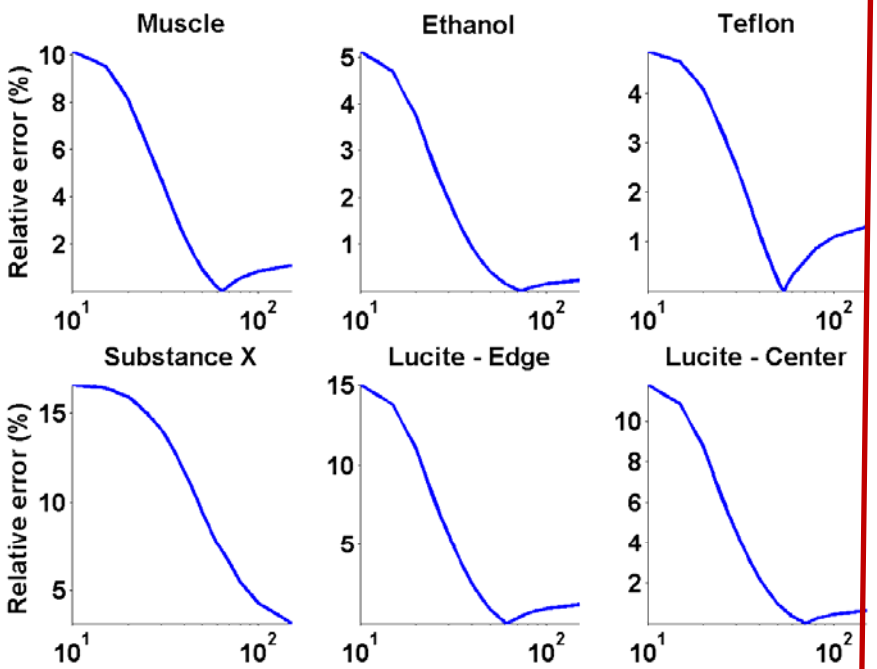
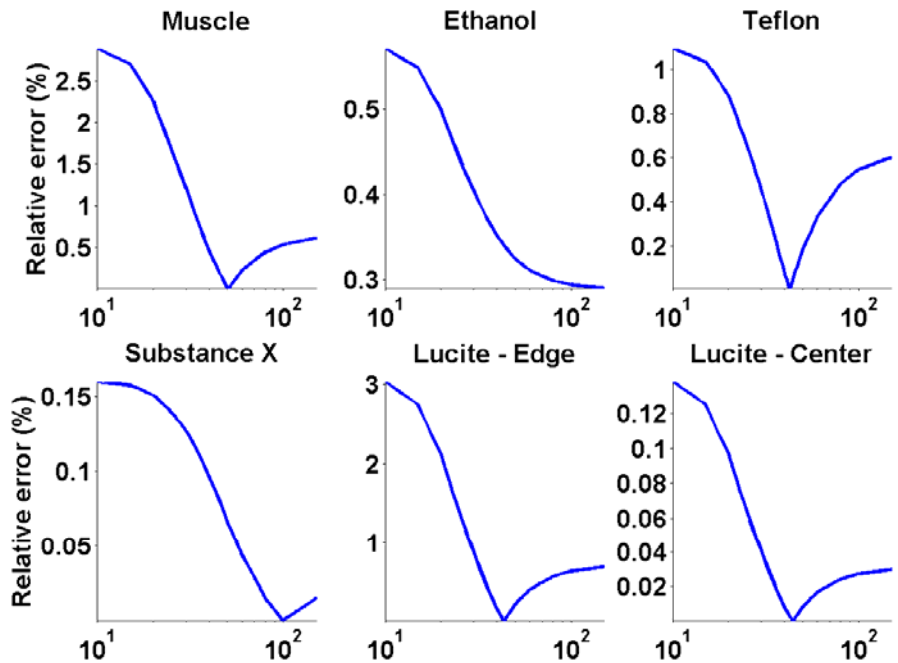
Small Phantom: Relative Errors
$I_0 = 100{,}000$

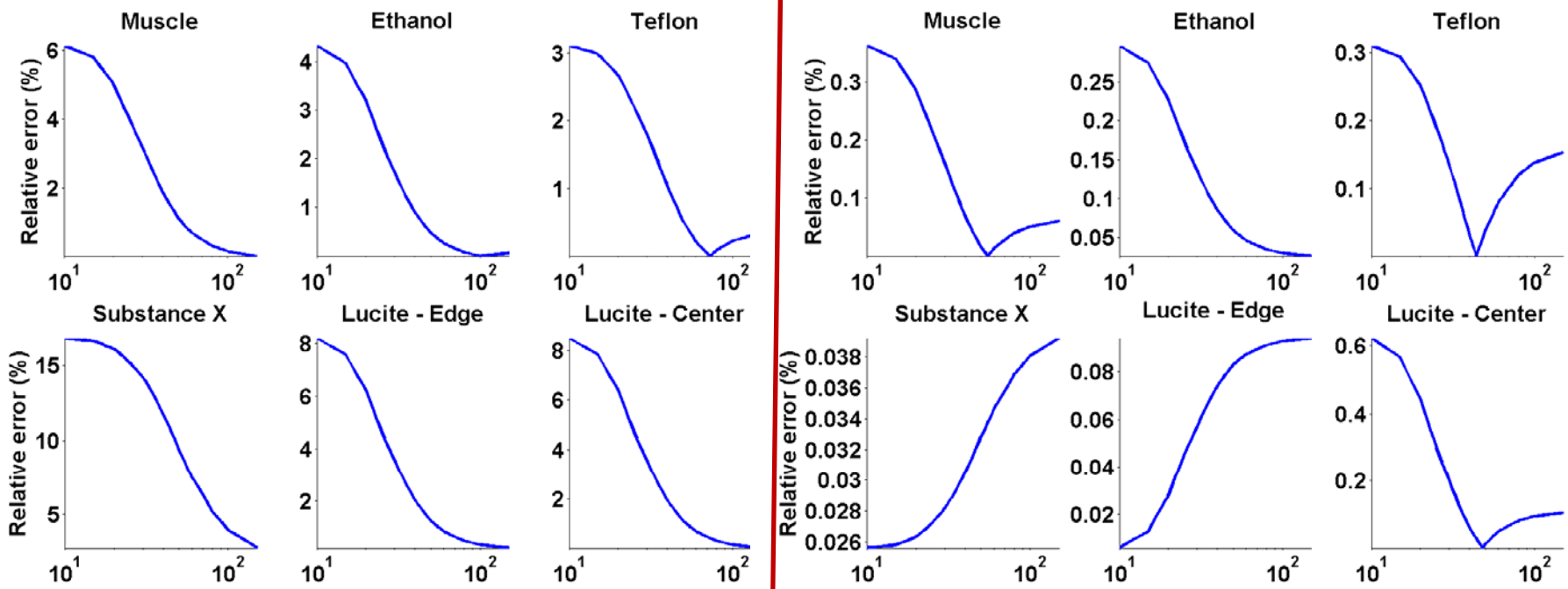


FBP-BVM



AM-DE

Small Phantom: Relative Errors
$I_0 = 1{,}000{,}000$



FBP-BVM

AM-DE

- AM-DE relative errors decrease consistently as the number of counts increases
- AM-DE performs at least an order of magnitude better

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Dual Energy CT Image Reconstruction

SOMATOM Definition CT Scanner
ccir.wustl.edu

- Data Models ➔ Reconstruction Algorithms
- Image Reconstruction Approaches
  - "Linear" Approaches
  - Statistical Iterative Reconstruction
- Simulation Study of the Dual Energy Alternating Minimization Algorithm
- Performance Quantification: the Cramer-Rao Lower Bound
- Conclusions

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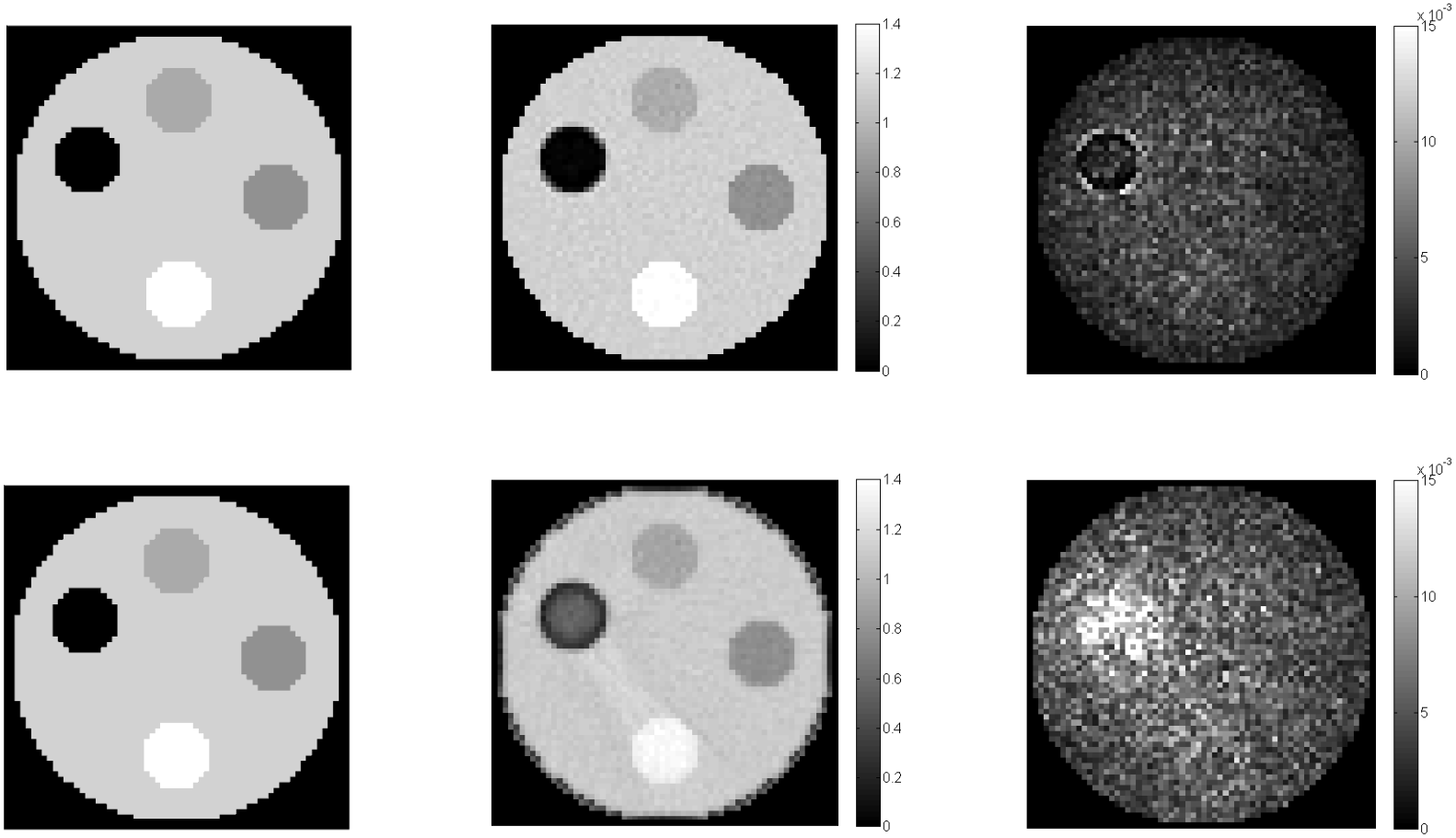
Small Phantom – Ensemble mean and variance (from 15 samples) I_0 = 100,000 *Styrene Component*

AM-DE

FBP-BVM



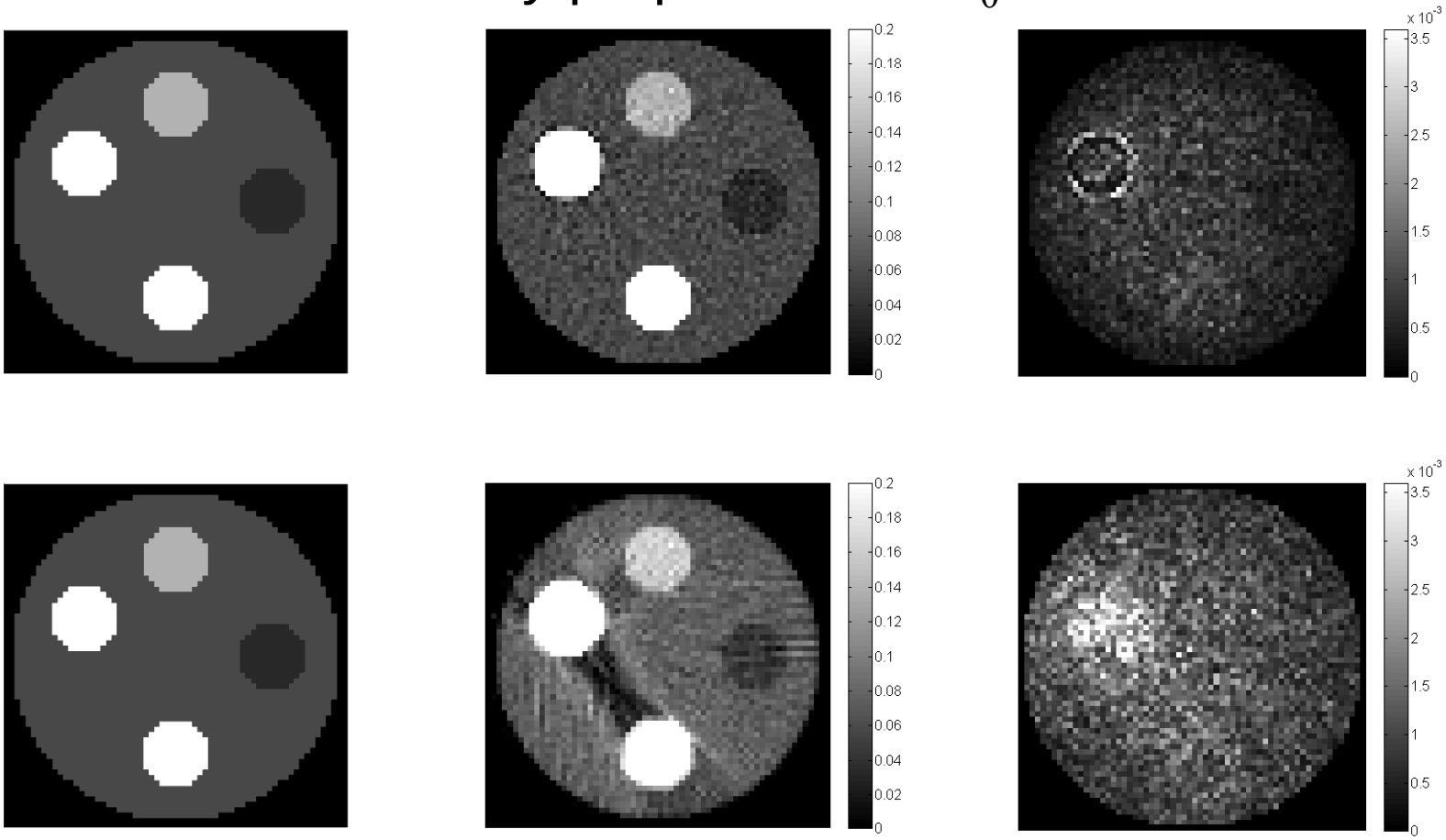
Truth Mean Variance

Small Phantom – Ensemble mean and variance (from 15 samples) I_0 = 100,000 *Ca. Chloride Component*

Variance is inversely proportional to I_0

AM-DE

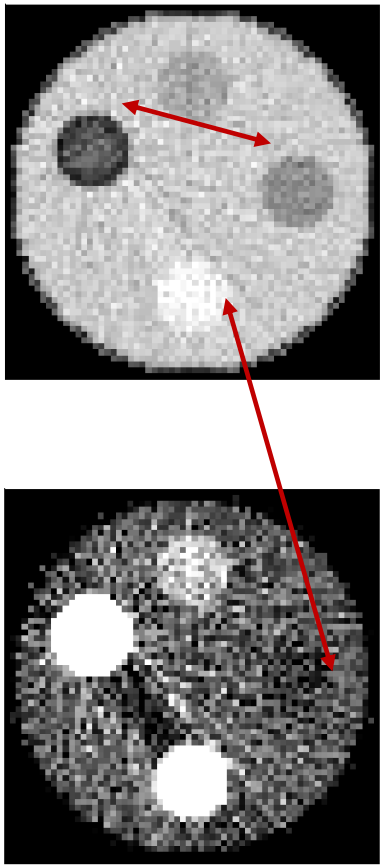
FBP-BVM



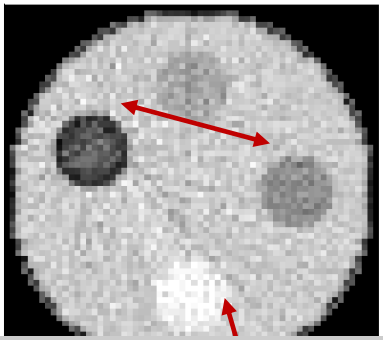
Truth | Mean | Variance

Predicting Performance: Fisher Information and the Cramer-Rao Bound

- The variance of an unbiased estimate is greater than or equal to the Cramer-Rao lower bound (CRLB)
- CRLB is conditioned on a model
- CRLB is independent of the algorithm
- AM-DE is biased (in part due to nonnegativity constraint)
- CRLB is derived from the inverse of Fisher information
- Fisher information measures the joint dependence of values within a component image and across images
- Fisher information is proportional to I_0

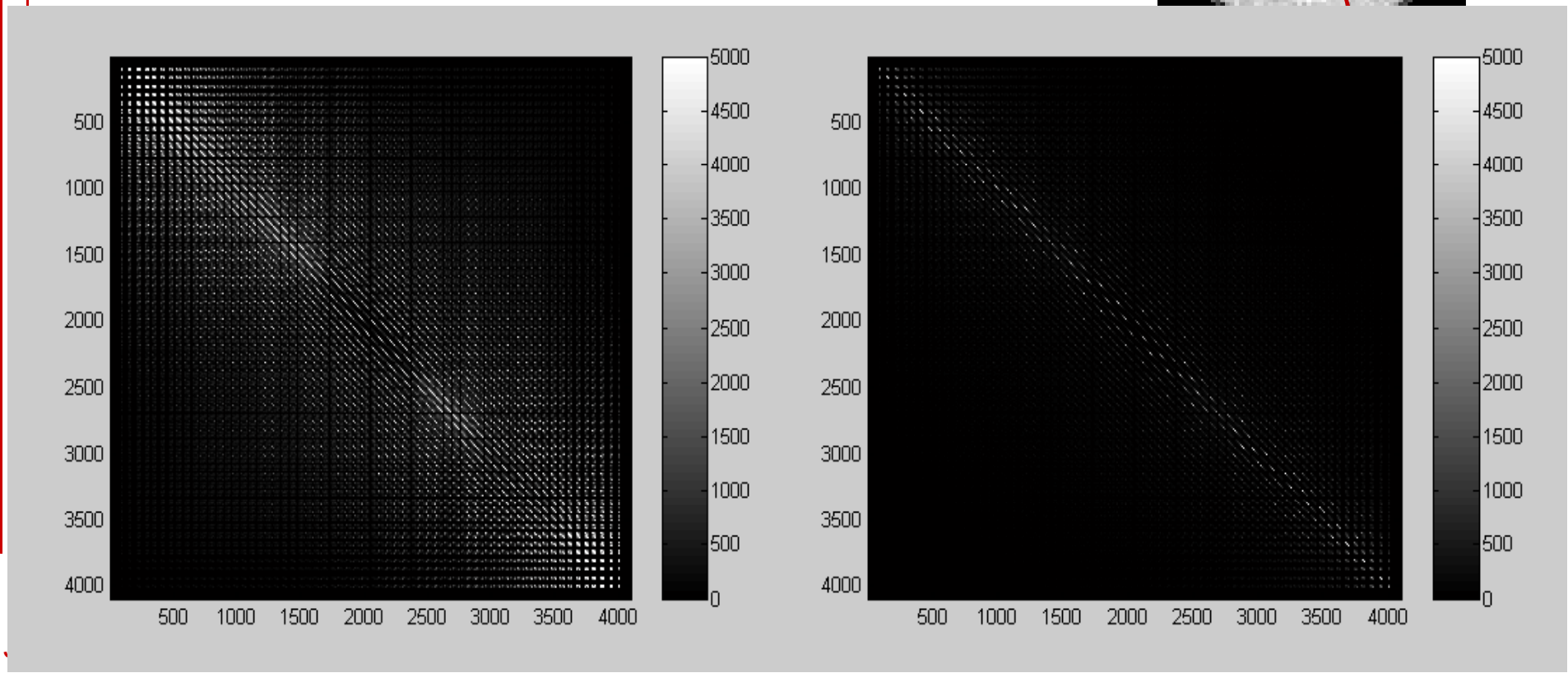


Fisher Information: Within a Component Image



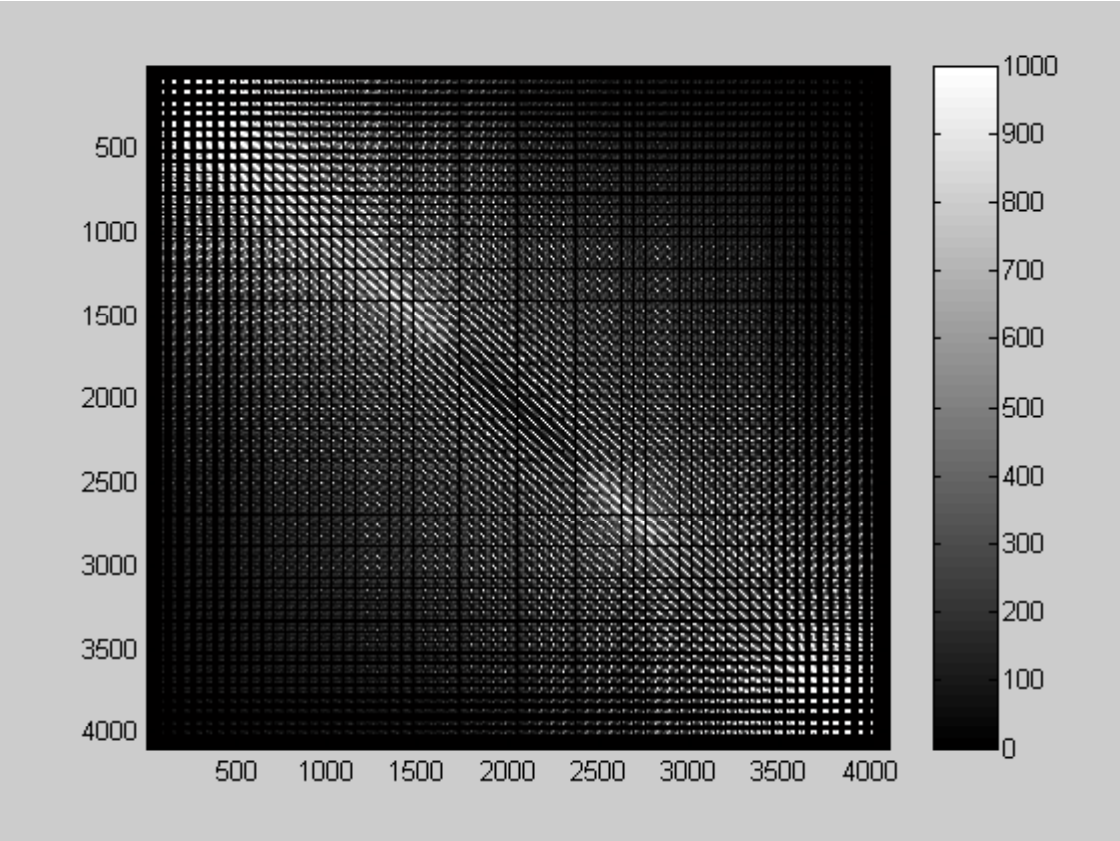
Calcium chloride
Maximum 1.2E5

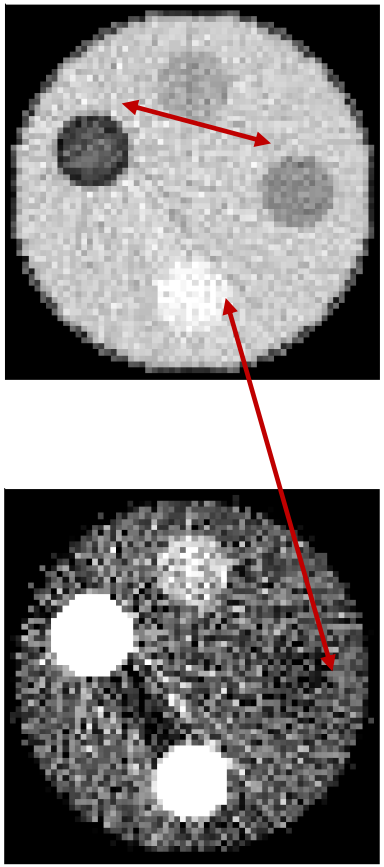
Styrene
Maximum 1.6E4



Fisher Information: Across Images

Maximum 4.2E4



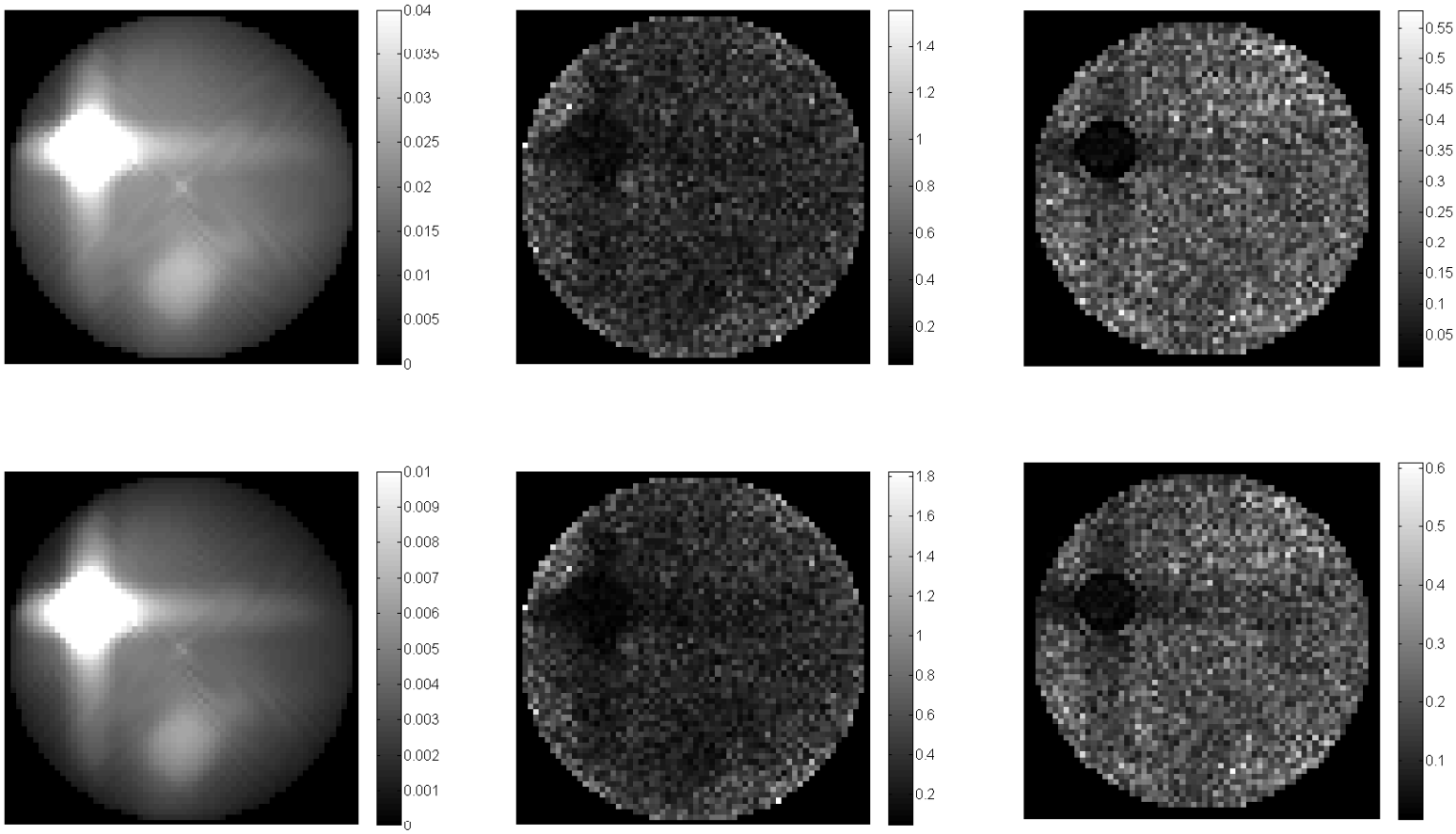


CRLB Images: Diagonals of Inverse of Fisher Information

Ratio of variance images to CRLB. Different display windows.

Styrene

Calcium Chloride



CRLB FBP-BVM AM-DE

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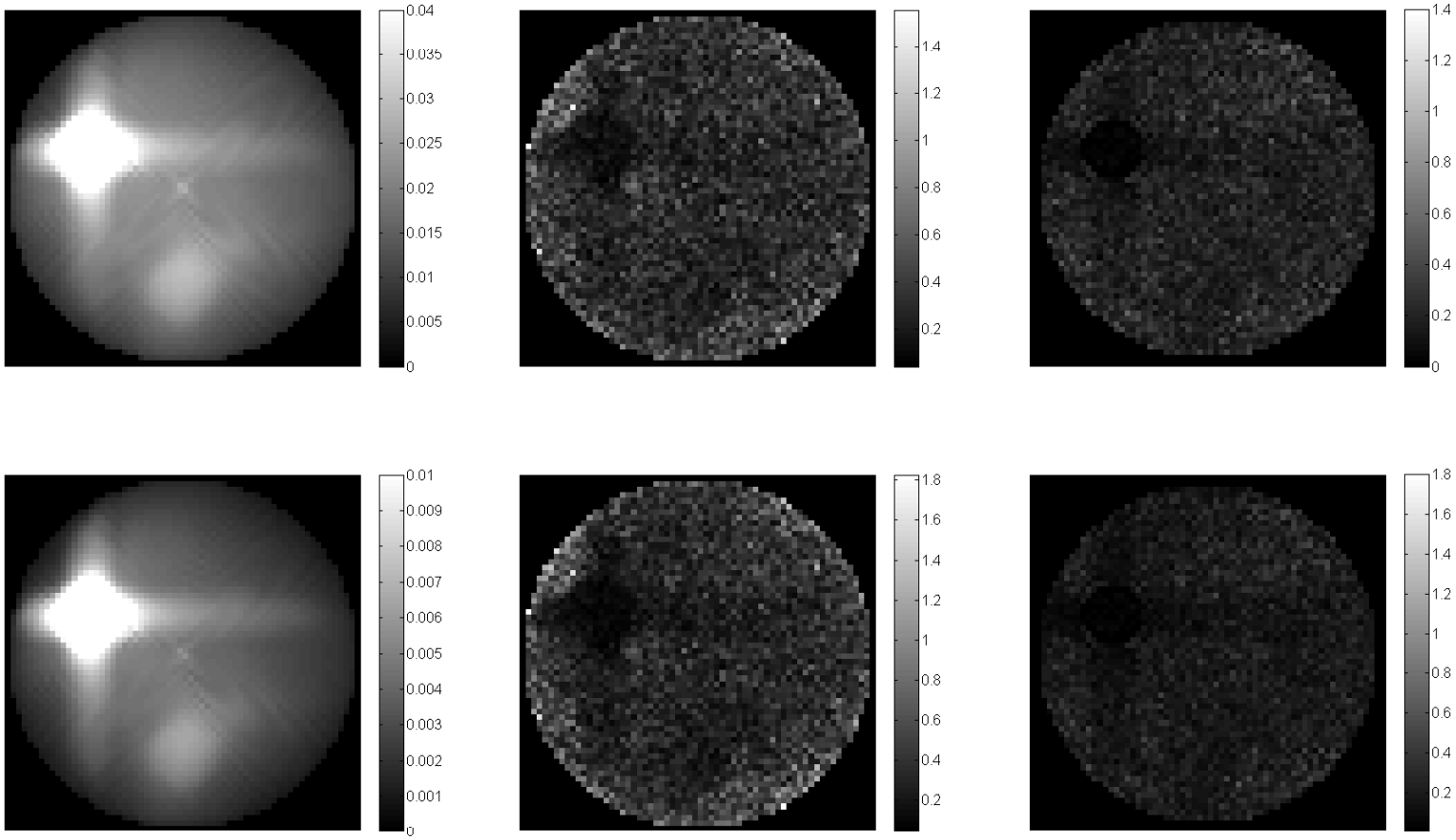
CRLB Images: Diagonals of Inverse of Fisher Information

Ratio of variance images to CRLB. SAME display windows.

Styrene

Calcium Chloride



CRLB FBP-BVM AM-DE

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Conclusions

- Reviewed Models for CT Image Reconstruction
- Outlined Various Approaches for Dual Energy CT Image Reconstruction
- Summarized a Simulation Study Showing Performance of a Dual Energy Alternating Minimization Algorithm
- Outlined and Applied a Method for Quantifying Achievable Performance Based on Fisher Information
- Quantitative Analysis Shows Benefit of AM-DE algorithm
- AM-DE Algorithm is Computationally Demanding
- Other Work: Quantify Impact of Modeling Errors

References

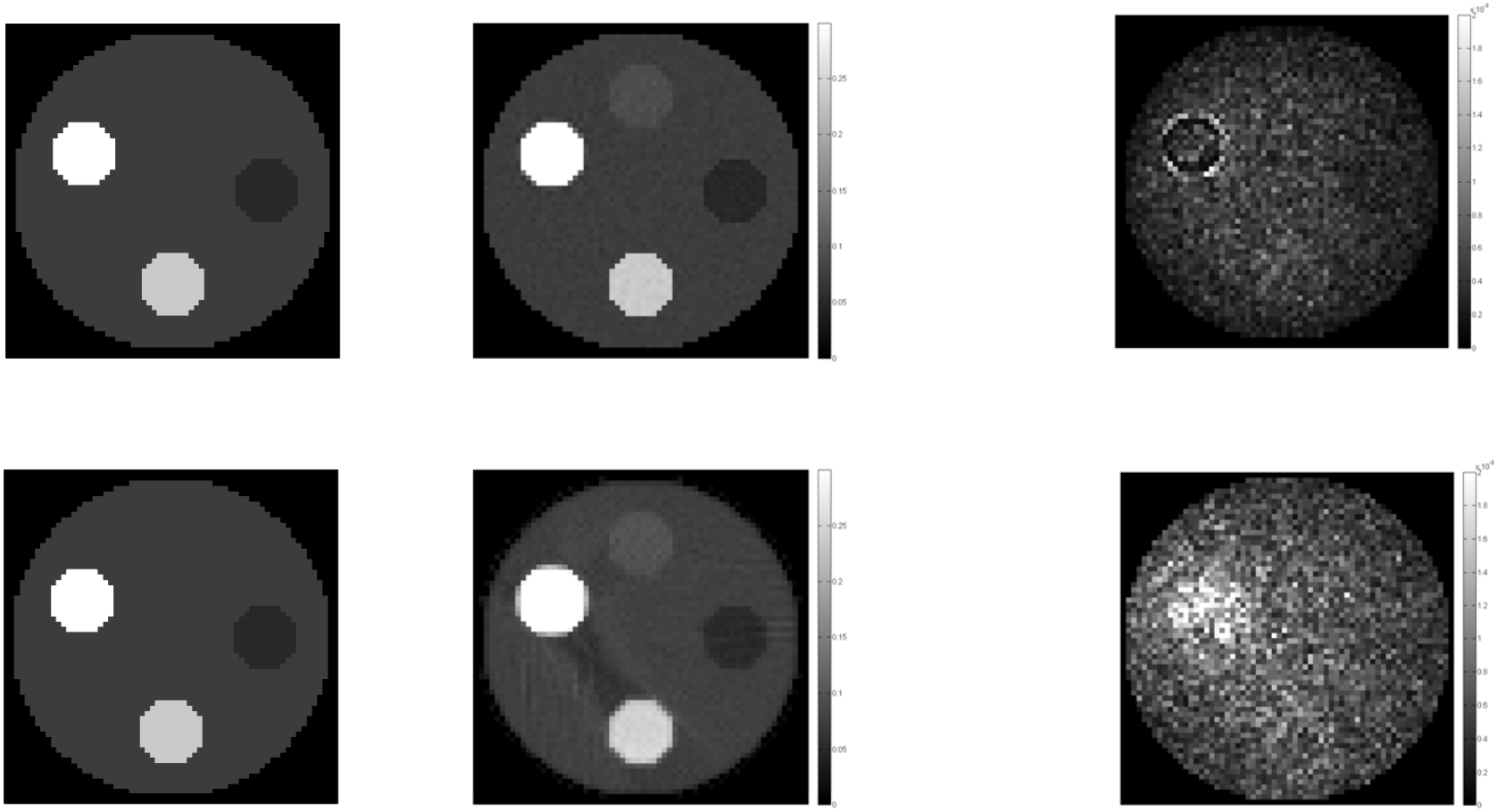
- J. F. Williamson et al., "Prospects for quantitative computed tomography imaging in the presence of foreign metal bodies using statistical image reconstruction," *Med. Phys.* 2910, 2404-2418 2002
- J. F. Williamson, S. Li, B. R. Whiting, and F. A. Lerma, "On two-parameter representations of photon cross section: Application to dual energy CT imaging," *Medical Physics*, vol. 33, pp. 4115-4129, November 2006
- J. A. O'Sullivan and J. Benac, "Alternating minimization algorithms for transmission tomography," *IEEE Transactions on Medical Imaging*, vol. 26, pp. 283-297, March 2007
- J. Benac, "Alternating minimization algorithms for x-ray computed tomography: multigrid acceleration and dual energy." *PhD thesis, Washington University in St. Louis,* St. Louis, 2005
- J. Benac, J. A. O'Sullivan, and J. F. Williamson, "Alternating minimization algorithm for dual energy X-ray CT," *in Proc. IEEE Int. Symp. Biomedical Imag.,* Arlington, VA, Apr. 2004, pp. 579582

Small Phantom – Ensemble mean and variance (from 15 samples) I_0 = 100,000

20keV Image

AM-DE

FBP-BVM



Truth

Mean

Variance

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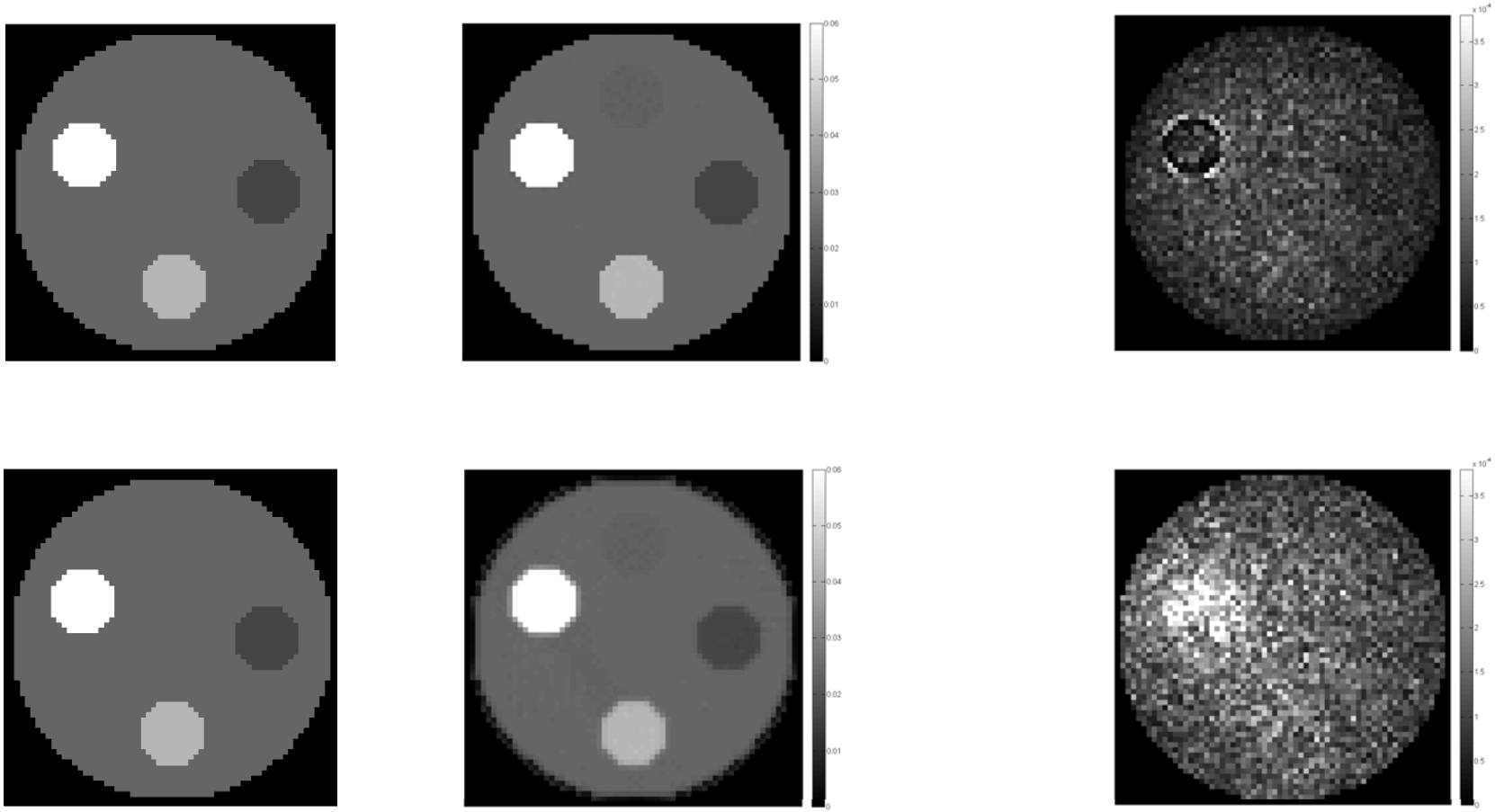
Small Phantom – Ensemble mean and variance (from 15 samples) I_0 = 100,000

65keV Image

AM-DE

FBP-BVM

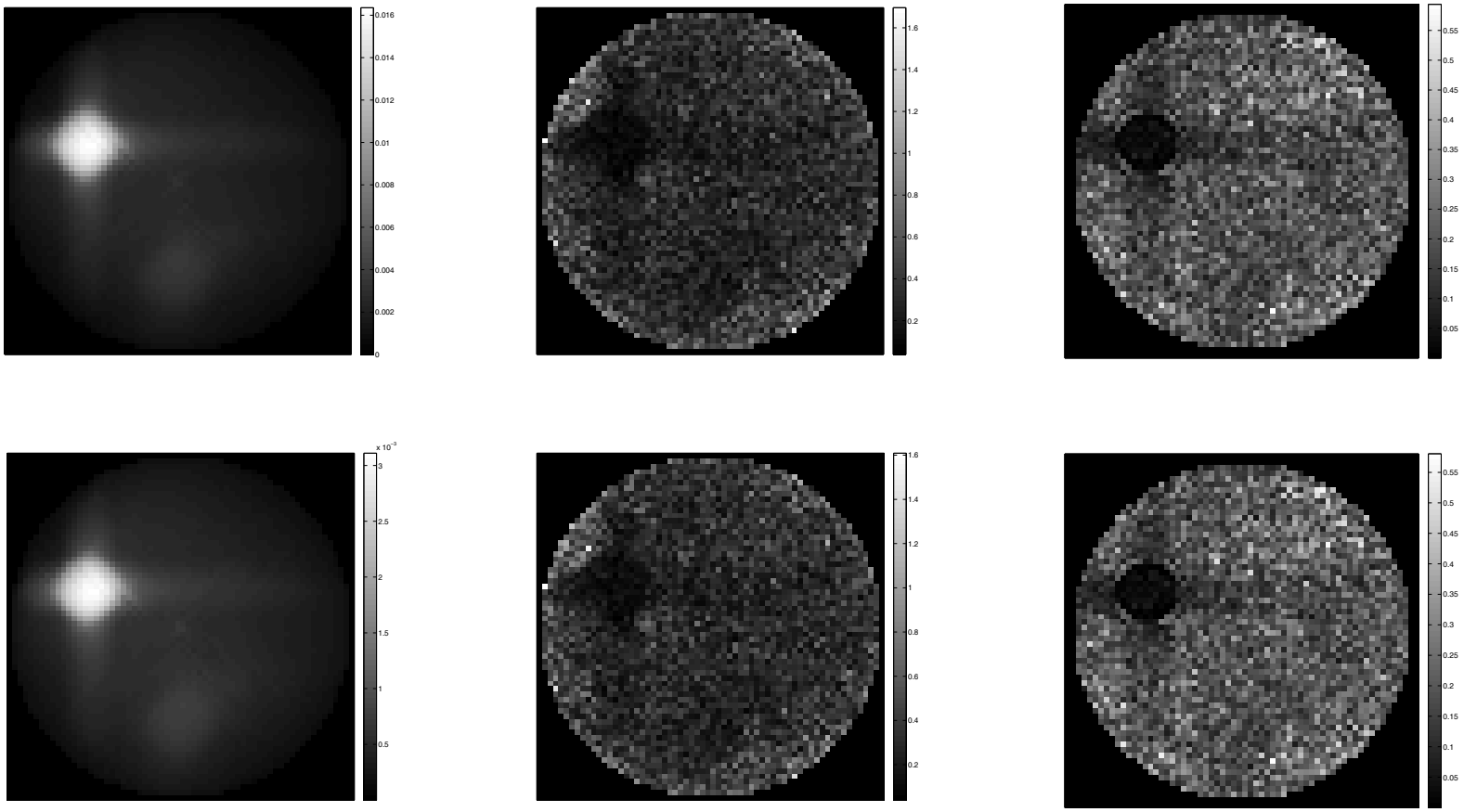


Truth

Mean

Variance

CRLB images for 20keV (above) and 65keV (below) images



CRLB FBP-BVM AM-DE

Var / CRLB

Model-Based Imaging: *Principled Algorithm Development*

- Model as much of the underlying physics, biology, chemistry as possible
- Derive an objective function based on physical model, problem definition, and implementation constraints (complexity, robustness) → loglikelihood function
- Derive algorithms to optimize objective function → maximum likelihood; mathematical considerations
- Predict and evaluate performance using simulations and phantom experiments
- Revisit physical models, algorithms
- Transition to clinical settings → improve algorithms, implementations, connect to imaging sensors or scanners, identify key applications