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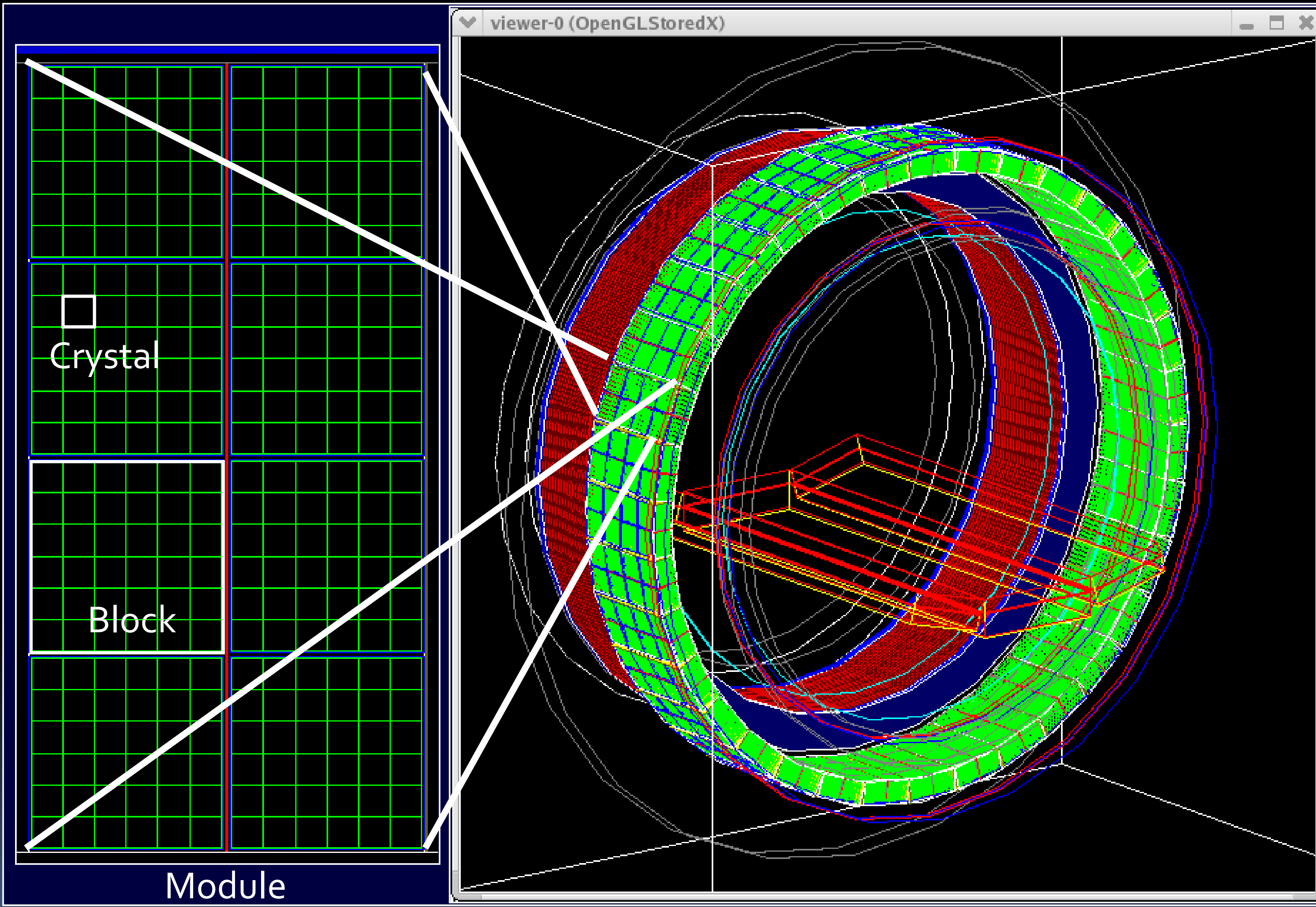
BASIC PET IMAGING PHYSICS: ILLUSTRATED VIA MONTE CARLO

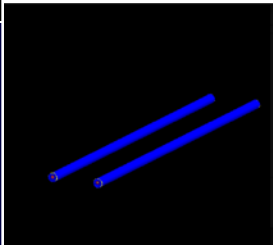
Overview:

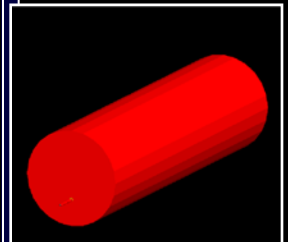
- Imaging system: camera overview
- Detection process
- Data representations
- Data corrections
- Image reconstruction

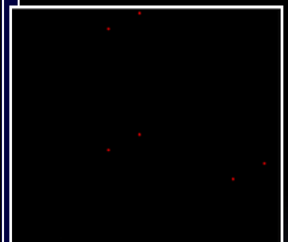
GATE GE Discovery DST



NEMA Phantoms


Sensitivity Phantom


Scatter Fraction Phantom


Spatial Resolution Phantom

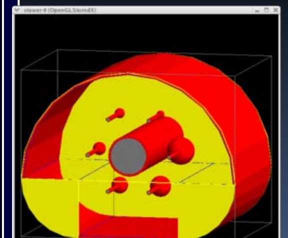


Image Quality

Images produced by the GATE code.

Signal path

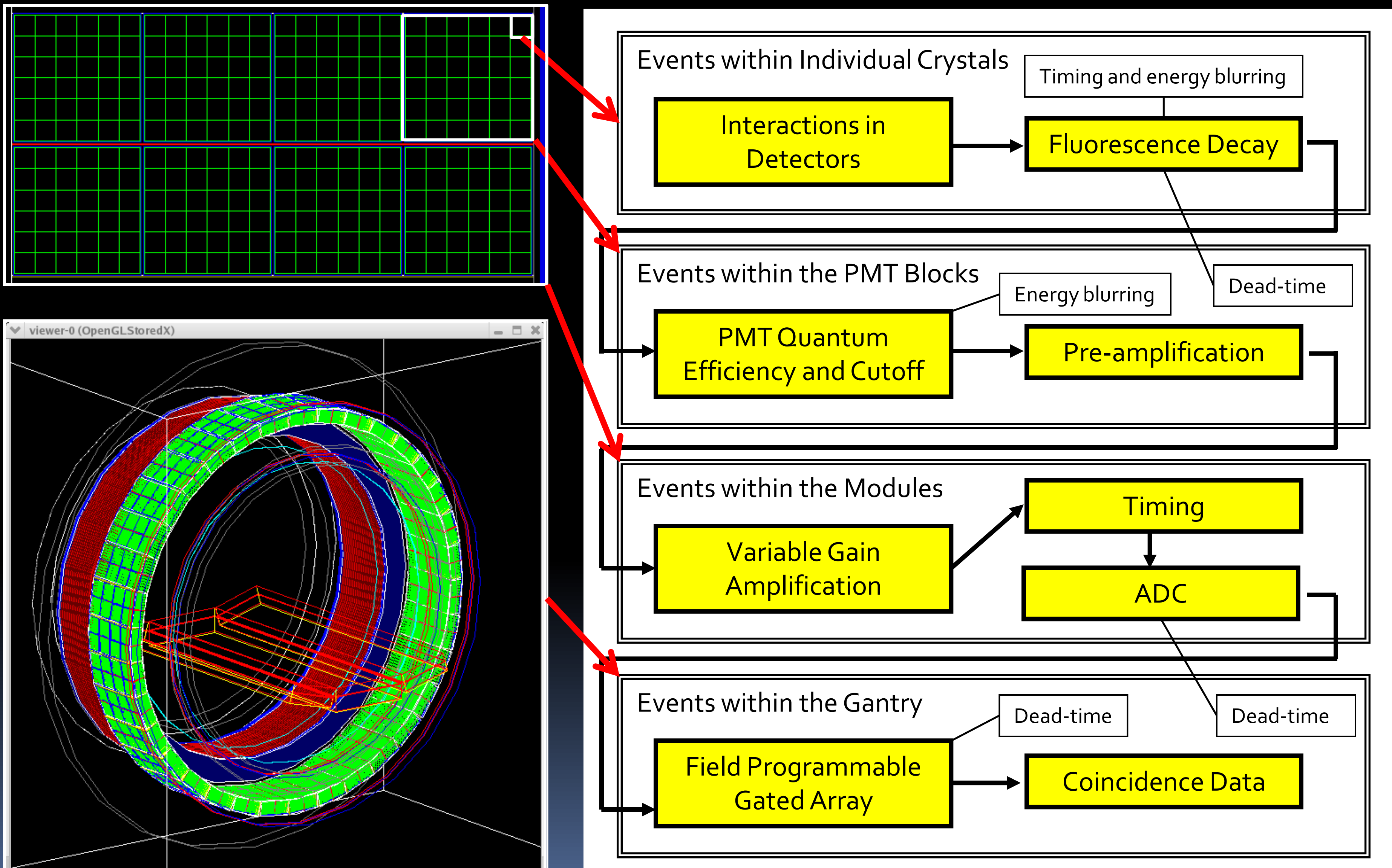
- Photon detector interactions
- Scintillation
- Signal amplification
  - PMT's, CCD's
  - Pre-amplification
- Crystal map
- MCA's
  - Energy window
- Coincidence timing

Block effect

Scatter counts

Random counts

Signal Processing



Description of the signal processing flow chart provided by A. Ganin GE Health Care.

GE Discovery LS / ST PET Cameras

- 18/24 Rings
- 35/47 Reconstructed Slices
- 672/420 BGO Crystals per Ring
- 12,096/10,080 Crystals Total
- Ring Diameter – 92.7/88.1 cm
- Axial Field of View – 15.2/15.2 cm
- Crystal Size –
  - DLS: 4 x 8 x 30 mm^3
  - DST: 6.3 x 6.3 x 30 mm^3
- Energy Window – 375 to 650 keV
- Coincidence Window – 12.5/11.7 ns

GE Discovery LS

GE Discovery ST

Positron tracers

Confounding factors

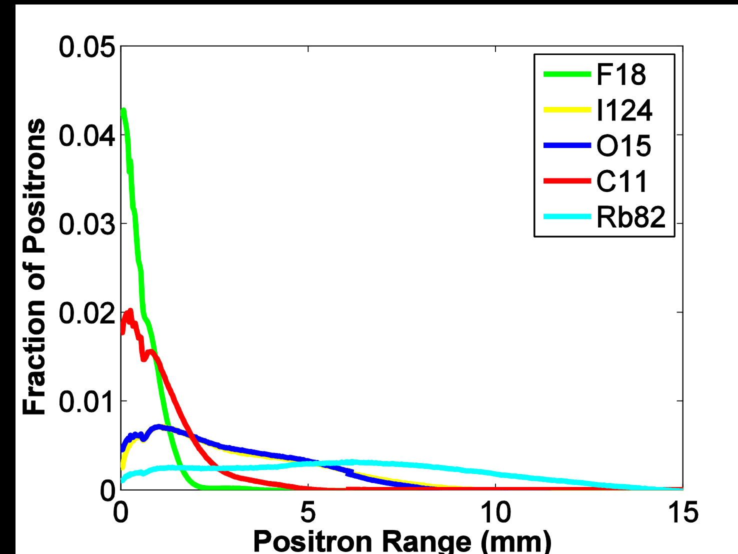
Isotope	Branching Ratio (%)	Half life	Mean Energy (keV)	Prompt Gammas
^{11}C	99.8	20.38 min.	385.5	-
^{13}N	99.8	9.965 min.	491.8	-
^{15}O	99.9	2.037 min	735.3	-
^{18}F	96.7	109.77 min.	249.8	-
^{22}Na	89.8+	2.602 years	215.4	1275(99.9%)
^{64}Cu	17.9	12.701 hrs.	278.1	1346(0.49%)
^{68}Ga	87.9/1.08	68.0 min.	835.8/352.6	1077(3.30%)+
^{68}Ge -> ^{68}Ga		271 days		-
^{82}Rb	83.3/11.6+	1.3 min.	1523/1157	776.5(13.4%)+
^{86}Y	32.3+	14.74 hrs.	213.1	many
^{124}I	11.2/11.2+	4.18 days	685.9/973.6	602.7(61.1%) 722.8(10.1%) 1691(10.6%)+
Many Others: $^{75,76}Br$, $^{93,94,94m}Tc$, ^{89}Zr,…				

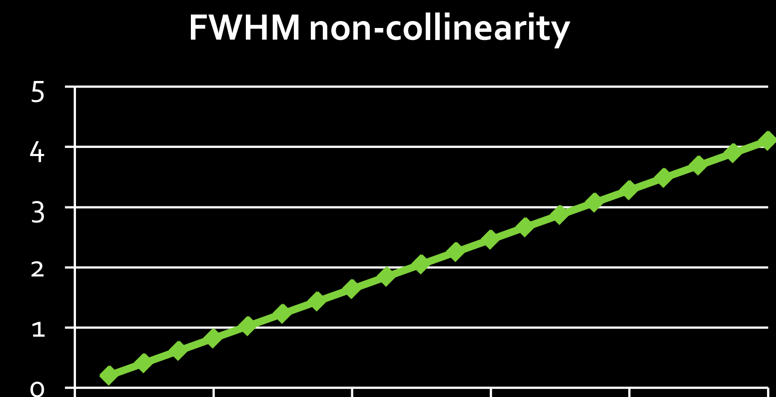
% of decays that contribute to image formation

Determines dose

Annihilation detection

- Positron Range
  - Dependent on decay energy spectrum
- Positron non-collinearity
  - 0.47 deg. FWHM
  - Related to positronium KE
    - (~ 8.6 eV)
- Detector size
  - d/2, for discrete detectors
- Block effects





$$FWHM = \sqrt{(d/2)^2 + b^2 + r^2 + (0.0022D)^2}$$

Detector Size

Block Effect

Positron Range

Non-collinearity

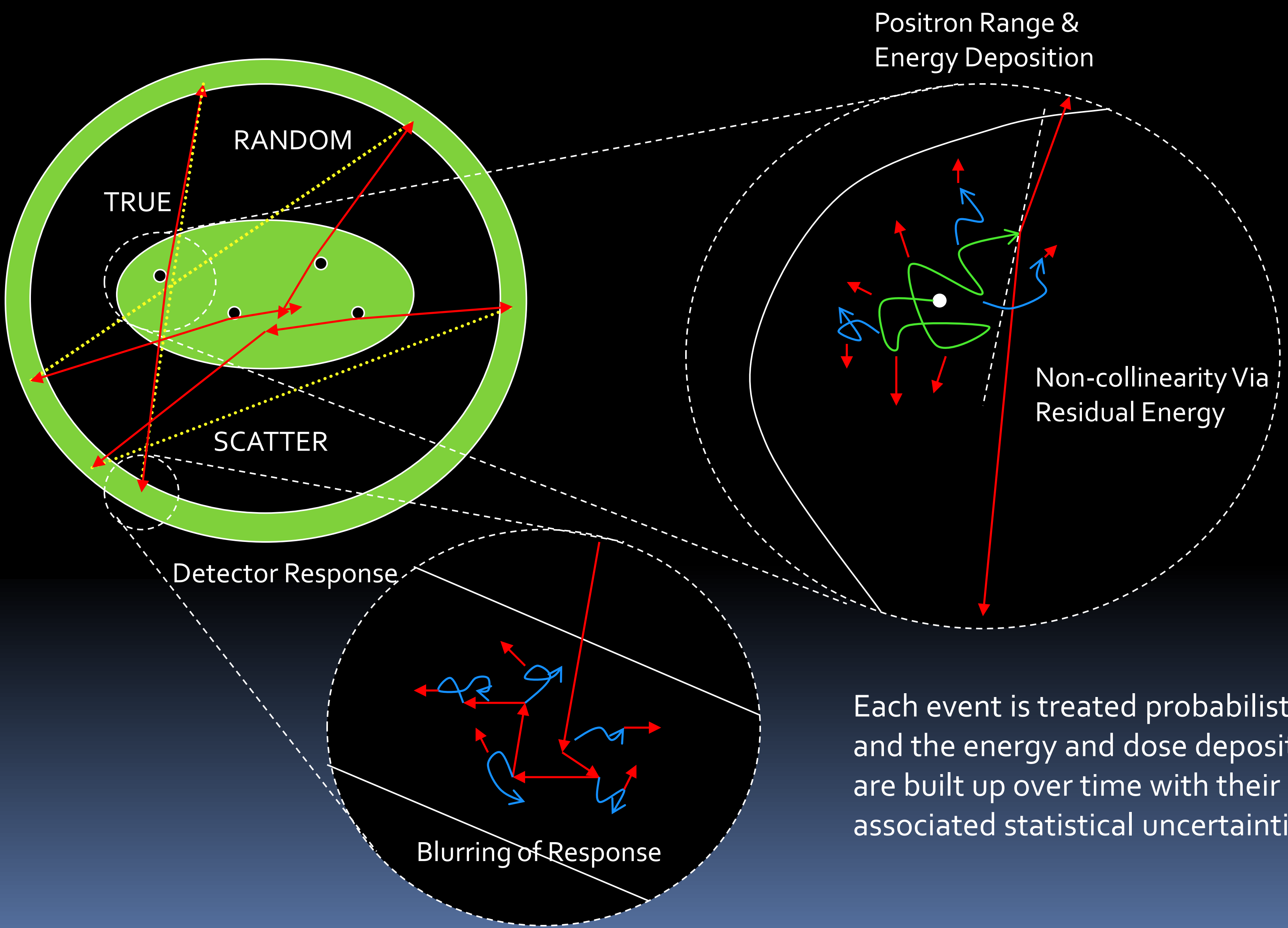
Coincidence detection

- Prompt coincidences
  - Trues
  - Scatters
  - Cascades
- Delayed coincidences
  - Randoms
- Singles
- Measures
  - Scatter Fraction (SF)
  - Noise Equivalent Count Rate (NECR)

$$SF = \frac{S}{T+S}$$

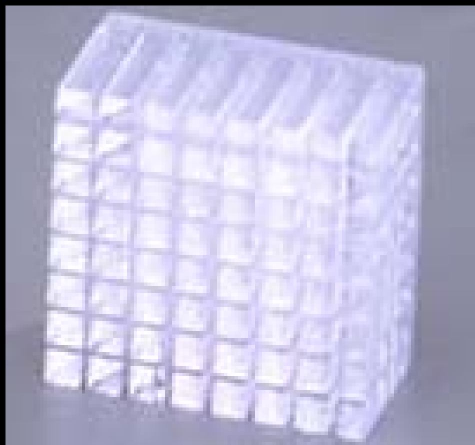
$$NECR = \frac{T^2}{T+S+R+C}$$

So how do these effects come together?

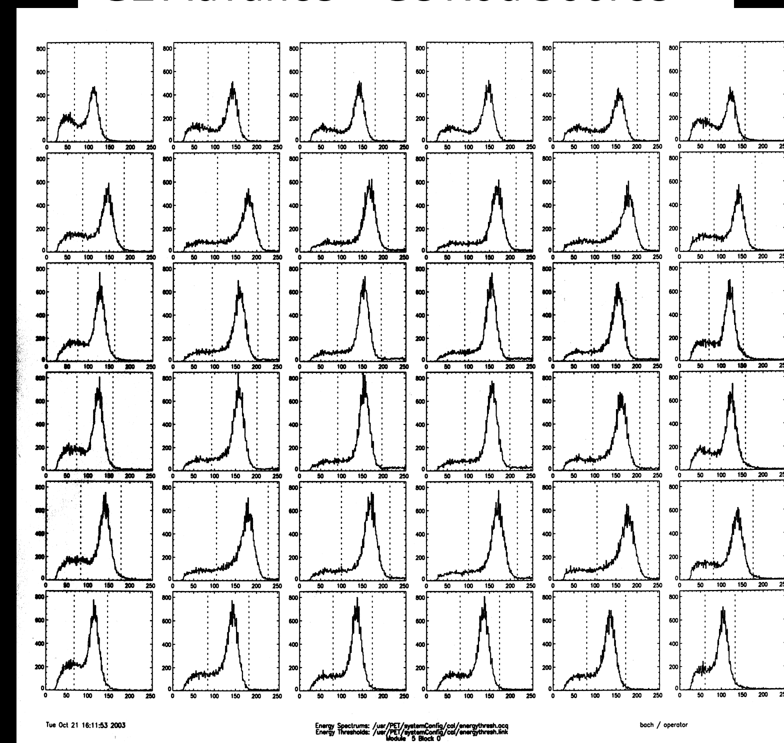


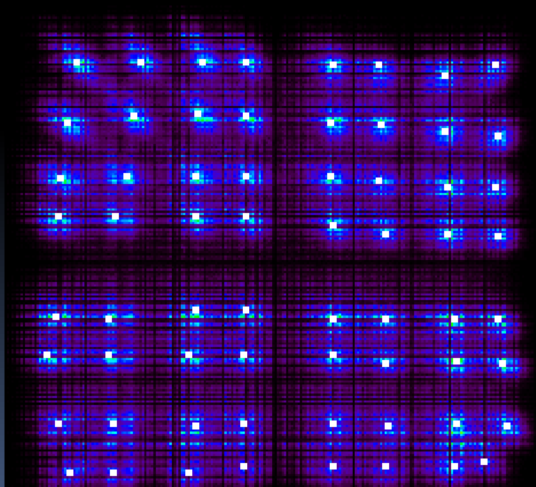
Each event is treated probabilistically and the energy and dose depositions are built up over time with their associated statistical uncertainties.

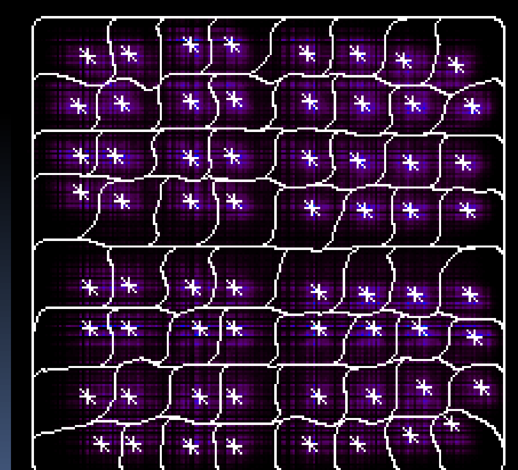
Signal in the blocks



GE Advance ^{68}Ge Rod Source







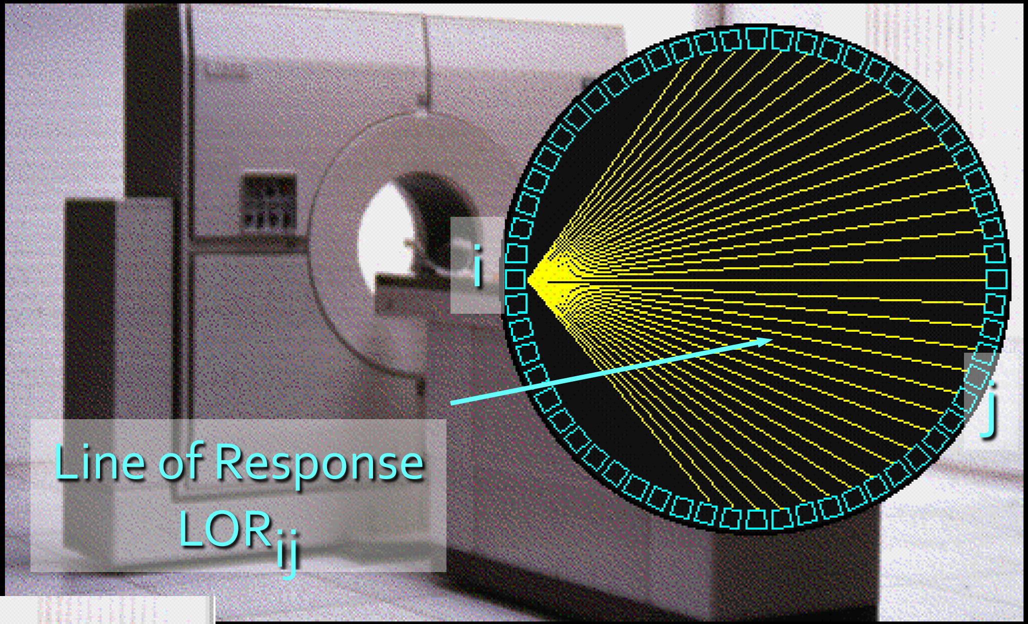
Raw data structures

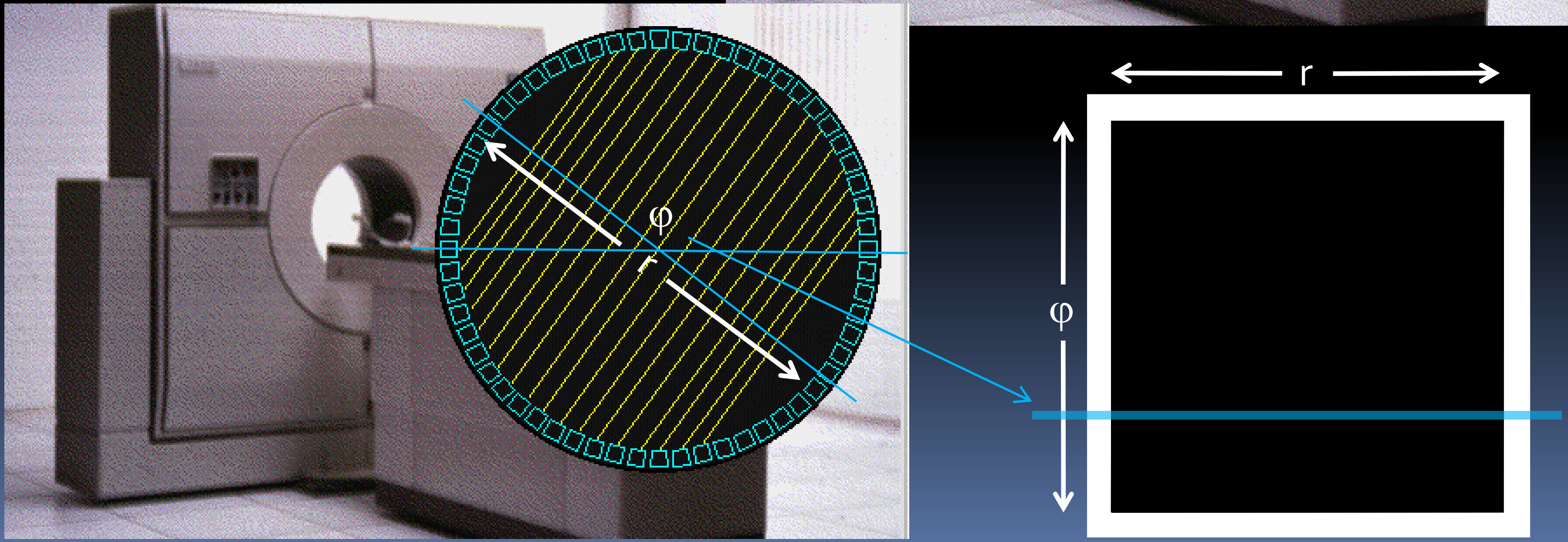
- Sinogram binning: r, φ
- 3D Data formats
  - Michelograms
  - Projection data

Coincidence binning

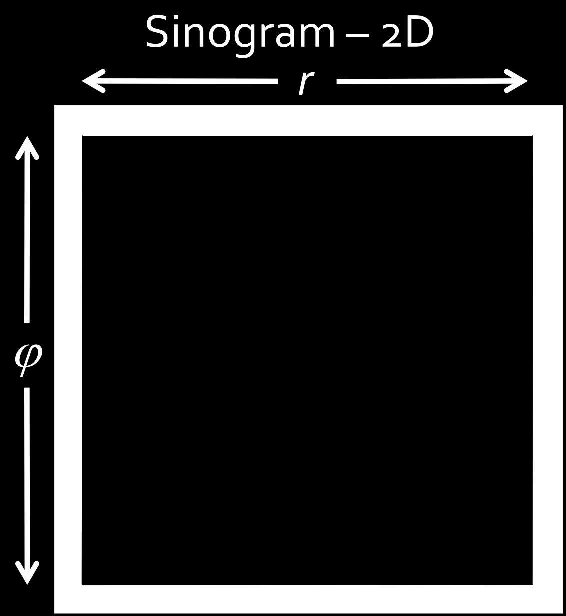
List-Mode Raw Data
List of # of events
for each detector-pair LOR

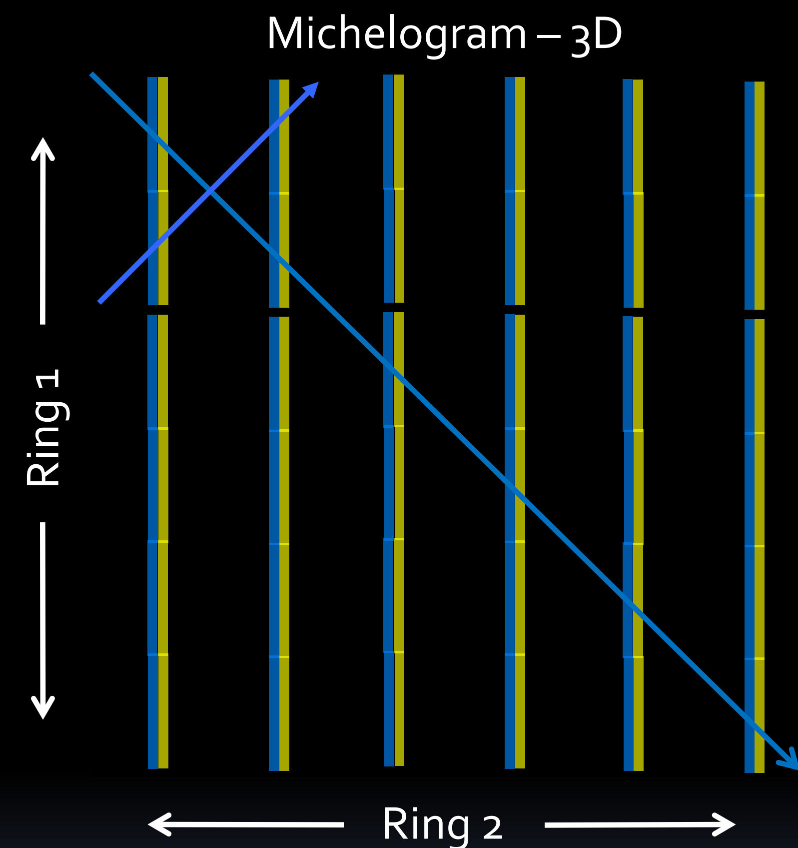
Re-binning
Sort raw data
into projection images





Data representation

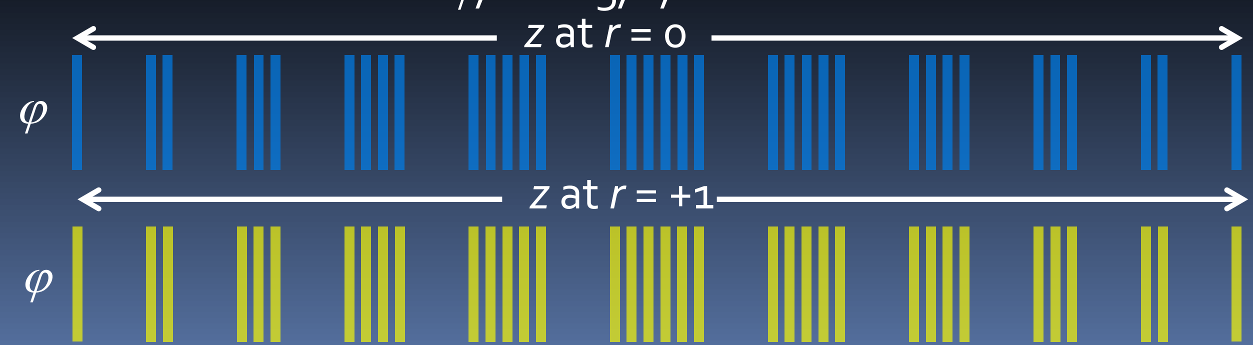




Generalized Projection formats

- Can bin by various index's
- These are:
  - φ & r
  - Ring#
  - z
  - ΔRing

In Particular: φ, ΔRing, z, r



PET Basics: data correction

- Normalization
- Attenuation correction
- Scatter correction
- Random correction
- Cascade correction
- Other corrections: →
  - Sampling correction
  - Recovery correction
  - Dead time correction
  - Decay correction

Normalization

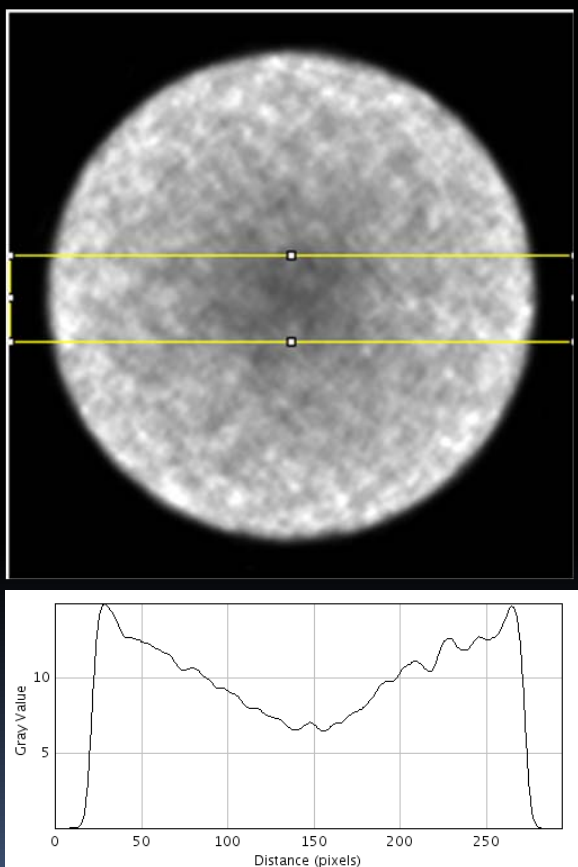
- Components accounted for in normalization
  - Individual detector efficiency: $\varepsilon_{\text{det}_1,\text{ring}_1,\text{det}_2,\text{ring}_2}$
  - Block profile factors: $B_{\text{det}_1,\text{ring}_1,\text{det}_2,\text{ring}_2}$
  - Geometric factors: $G_{\text{det}_1,\text{ring}_1,\text{det}_2,\text{ring}_2}$
  - Time-window alignment factor: $T_{\text{det}_1,\text{ring}_1,\text{det}_2,\text{ring}_2}$
  - Structural misalignment factor: $M_{\text{det}_1,\text{ring}_1,\text{det}_2,\text{ring}_2}$

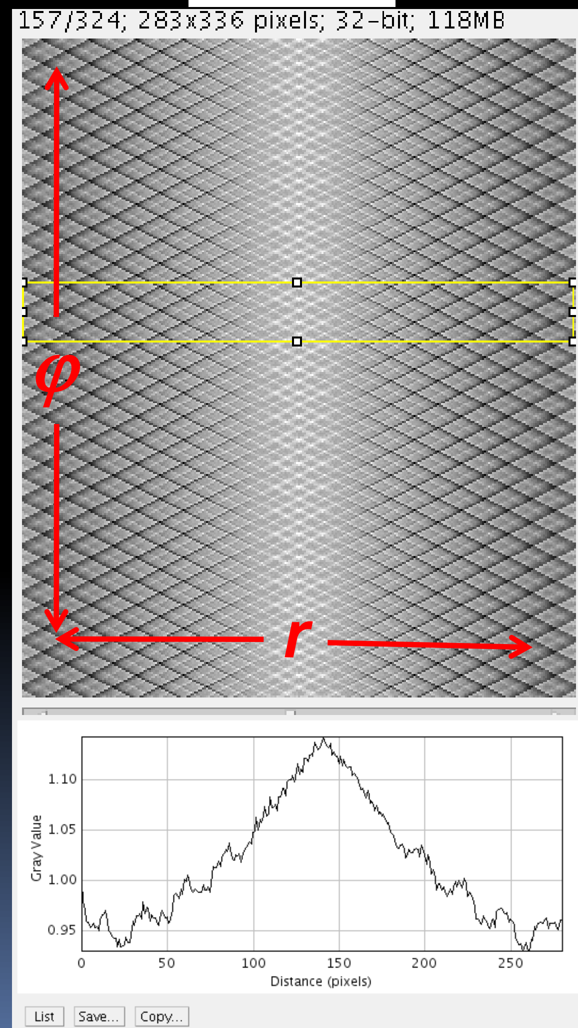
$$\eta_{1,2}^{true} = \varepsilon_{1,2} B_{1,2} G_{1,2} T_{1,2} M_{1,2}$$

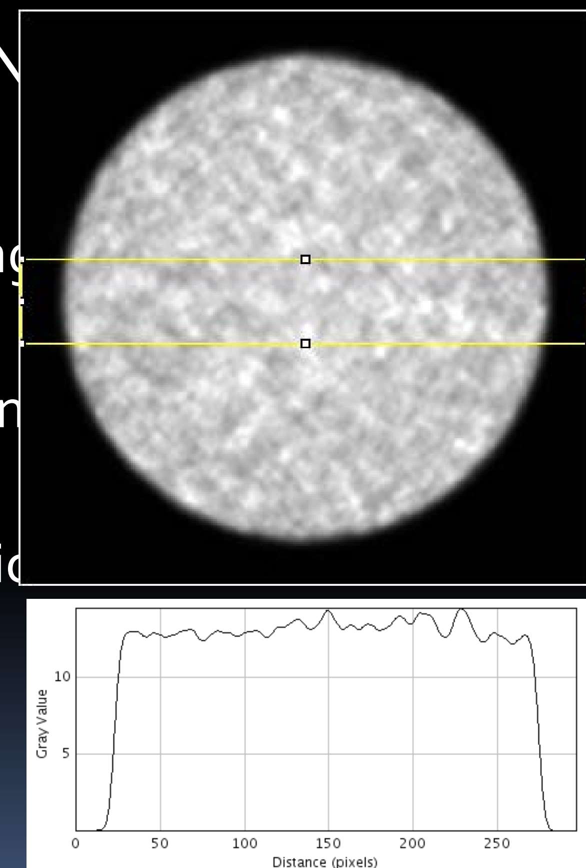
$$\eta_{1,2}^{scatter} = \varepsilon_{1,2} B_{1,2} T_{1,2}$$

Normalization ex:

$$1/\eta$$



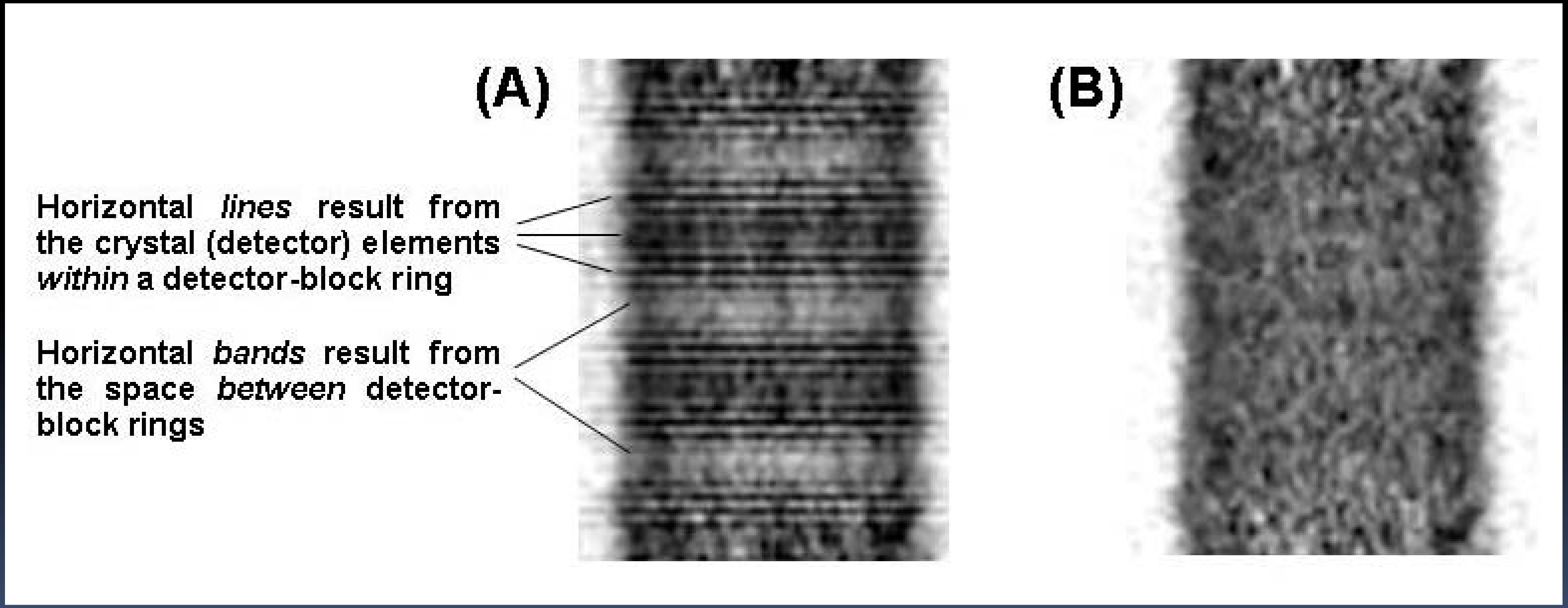




Normalization ex:

Uniformity ("Flood"/Sensitivity) Correction

Without Normalization *With Normalization*

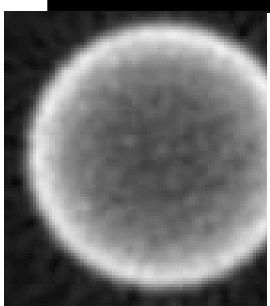


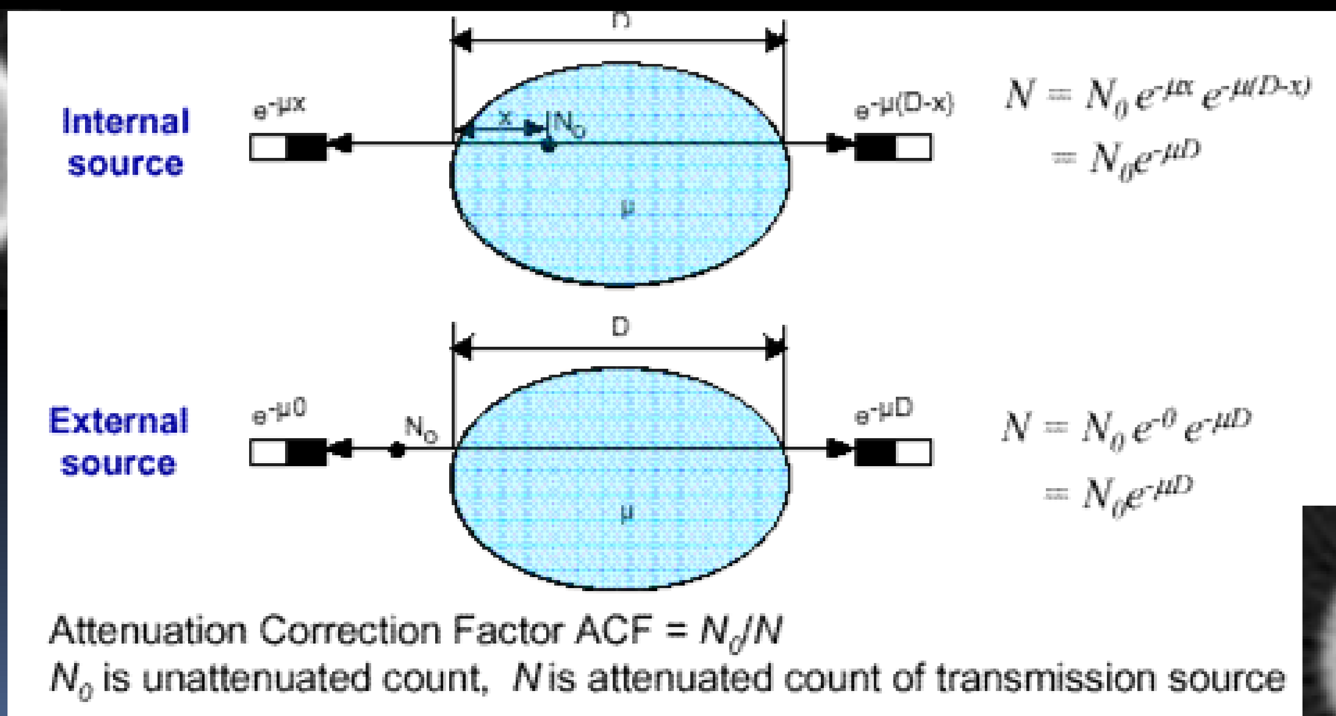
Coronal images of an F18-filled cylinder

Attenuation correction

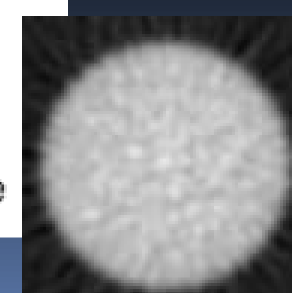
- Coincidence detection depends on the detection of two photons

Uncorrected





Corrected



Generating the correction

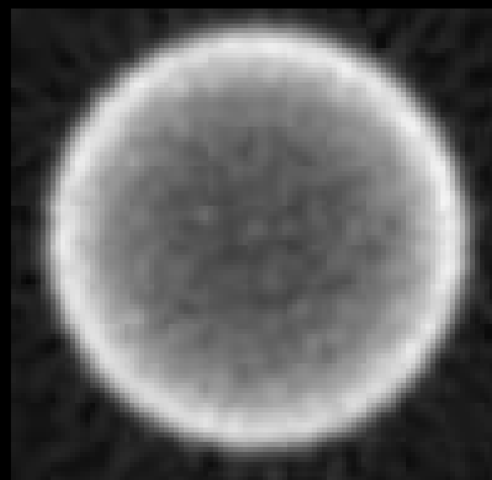
- Transmission scan
- Segmentation

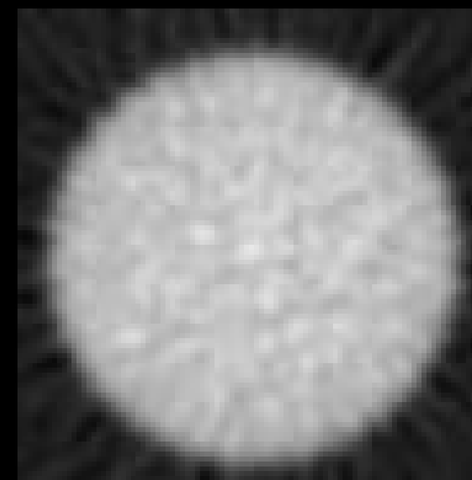
- CT scan
  - Conversion of Hounsfield units (~140-kVp attenuation coefficients) to 511-keV attenuation coefficients
- Issues
  - Motion inter/intra scan motion
    - Gated acquisitions: cardiac/respiratory
  - CT artifacts
    - Data truncation
    - High-Z objects: contrast/implants

Attenuation correction ex:

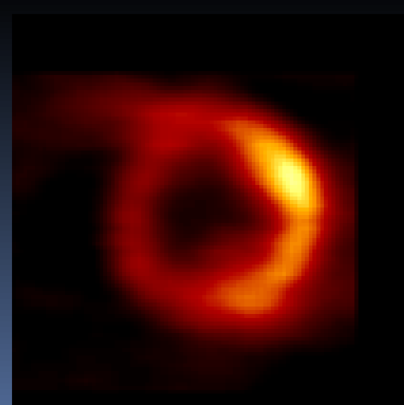
Uncorrected | *Corrected*

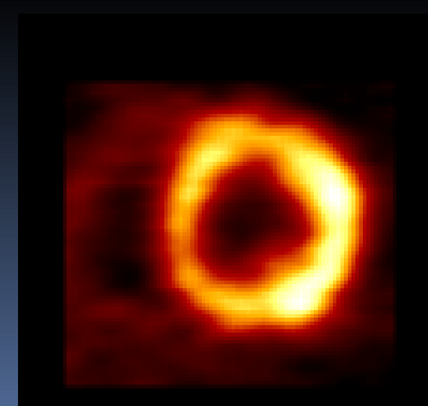
Cylinder uniformly filled with F18
Transverse image





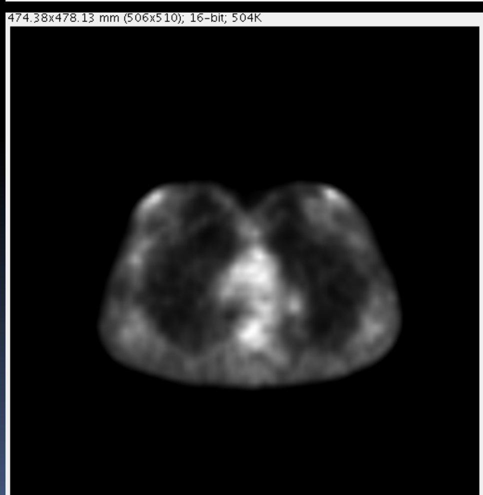
Rb82 Myocardial Perfusion Study
Short-axis image of left ventricle



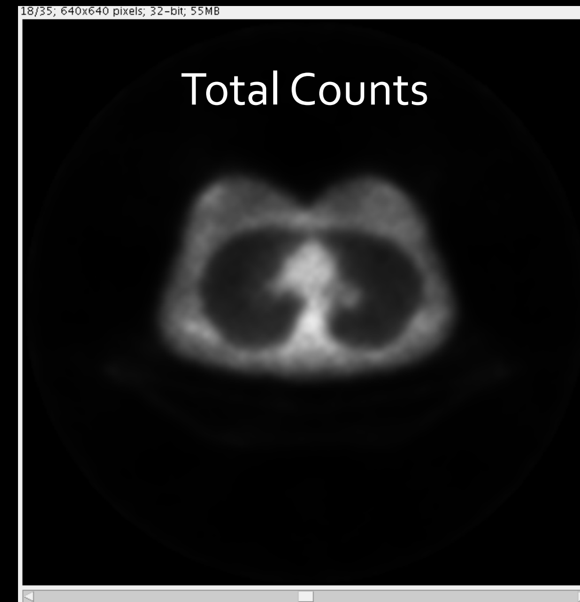


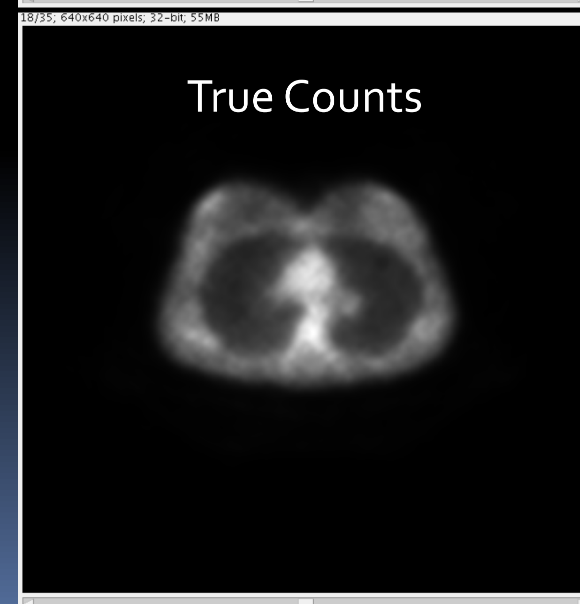
Scatter and random effects

Reference Images



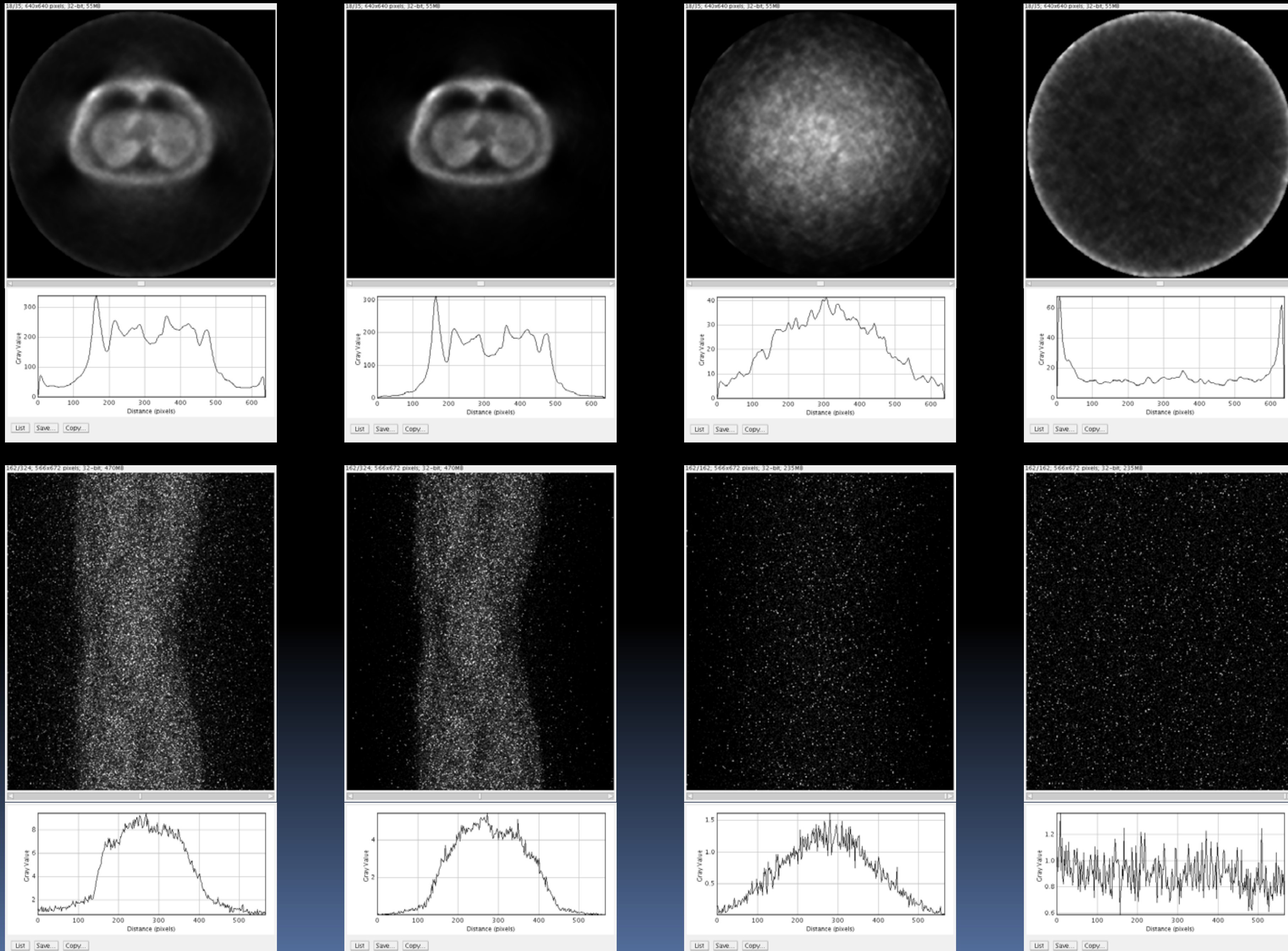
Attenuation Image
For Monte Carlo





Kang H *et al*, *Med. Phys. 36, 6,p 2468*, 2009

Total Counts | True Counts | Scatter Counts | Random Counts

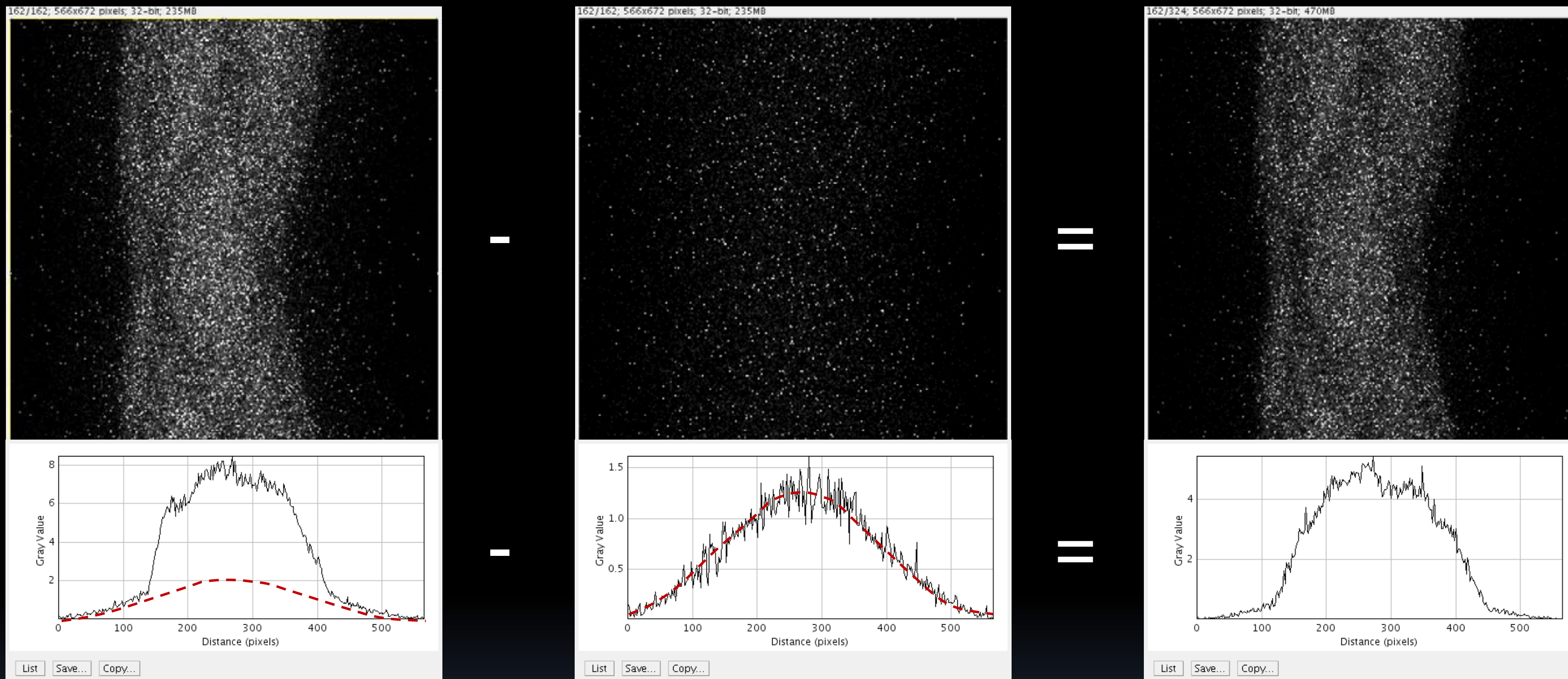


Scatter correction

- Multiple approaches:
  - Fitting scatter tails
  - Multiple energy windows
  - Convolution
  - Scatter simulation

Scatter correction:

Tail Fit:



Convolution:

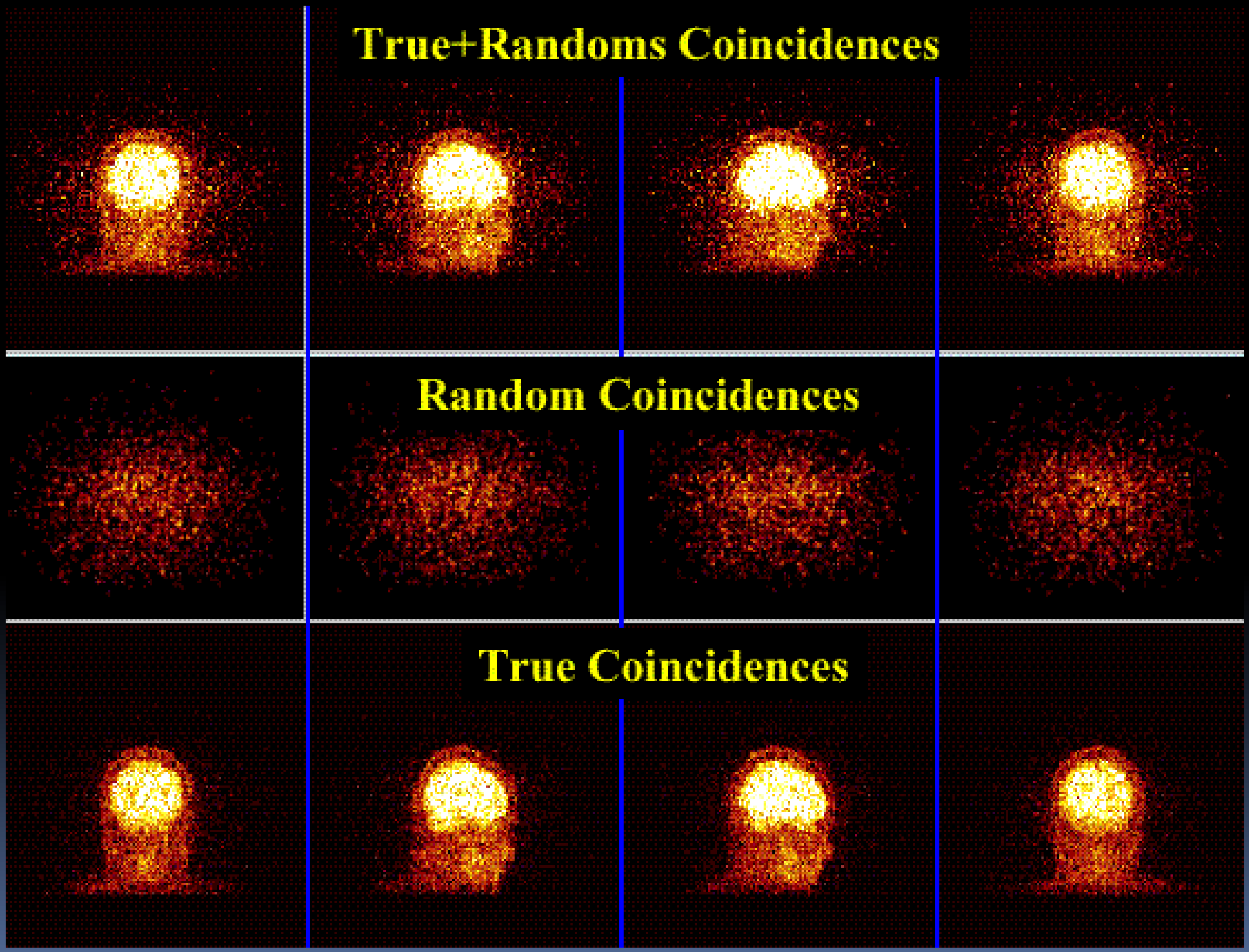
$$T^{k+1} = [P - \widetilde{R}] - SF\left(T^{k} \otimes K_{scatter}\right)$$

Random Corrections

- Real time subtraction
  - Doubles the noise from the randoms
  - Can result in negative counts
- Delayed event subtraction
  - Requires a second coincidence window (delayed)
  - Adds dead-time
  - Smooth the delayed projection data: little added noise
- Randoms by singles
  - May not account for coincidence dead-time
  - Smooth the projection data: little added noise

$$r_{i,j} = 2\tau s_i s_j$$

Random Correction ex:



Cascade Corrections

- Is scatter like in that:
  - It is correlated in time with the annihilation photons
- Is random like in that:
  - It contains very little spatial information
- Its correction is:
  - Performed in projection space

1. Fit the distribution shape:
   Convolve the corrected data with the cascade coincidence kernel

$$P_{cc} = \left(M \cdot P_{std}\right) \otimes K_{cc}$$

2. Scale the fit:
   Tail fit the estimate to get a scaling factor

3. Correct the data:
   Subtract the scaled fit from the corrected data

$$P_{corr} = \left[\left(P_{std} \cdot Ac^{-1}\right) - \alpha P_{cc}\right] \cdot Ac$$

Other corrections

- Sampling corrections:
  - Non uniform projection spacing
    - Arc correction by interpolating the data
    - Accounted for in the projection model
- Dead-time corrections
  - Paralyzable / Non- paralyzable:
  - Block polling to estimate “live”-time

$$m_p = ne^{-n\tau}$$

$$m_{np} = \frac{n}{1-n\tau}$$

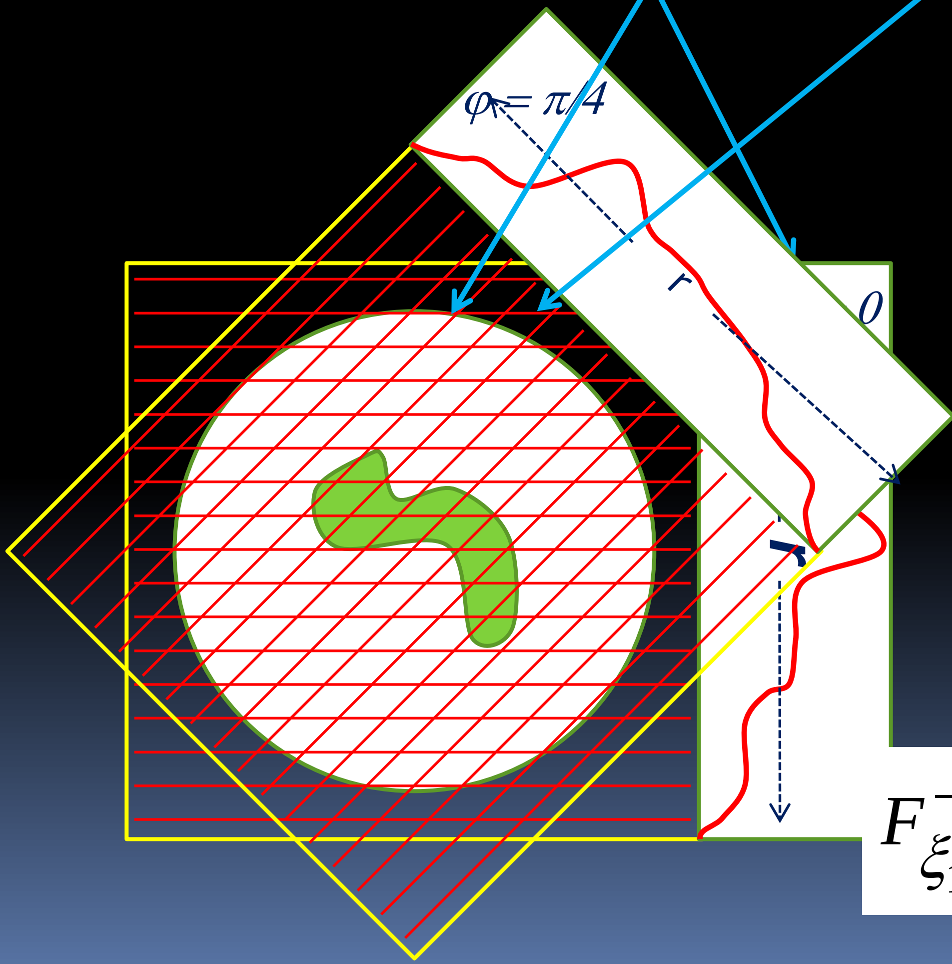
- Count recovery correction
  - Dilution of a known activity to generate a correction factor
- Decay corrections
  - Scales the counts to the start of the scan

Basic Image Reconstruction

- Forward and Back-Projection
  - The data represents a forward-projection from the object with the camera as the projector
  - The trick is to generate a back-projection
  - But the devil's in the details…
    - The fidelity of the projector pair is important
    - The data is count limited
    - The data is contaminated with noise
- Reconstructing the Image
  - Deterministic methods (FBP)
  - Statistical methods (MLEM, OSEM, …)

The Projection Slice Theorem

$$p(r,\varphi) = Rf(r,\varphi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(r - x\cos\varphi - y\sin\varphi)\,dxdy$$



$$F_r\,p(\rho,\varphi) = \int_{-\infty}^{\infty} e^{-2\pi i\rho r}\,Rf(r,\varphi)\,dr$$

$$F_{x,y}\,f(\xi_1,\xi_2) = F_r\,p(\rho,\varphi)$$

$$F^{-1}_{\xi_1,\xi_2}\left(F_{x,y}\,f(\xi_1,\xi_2)\right)(x,y) = f(x,y)$$

Filtered Back-Projection

$$f(x,y) = F^{-1}_{\xi_1,\xi_1}\left[Q\upsilon_\rho F_r\, p(\rho,\varphi)\right](x,y)$$

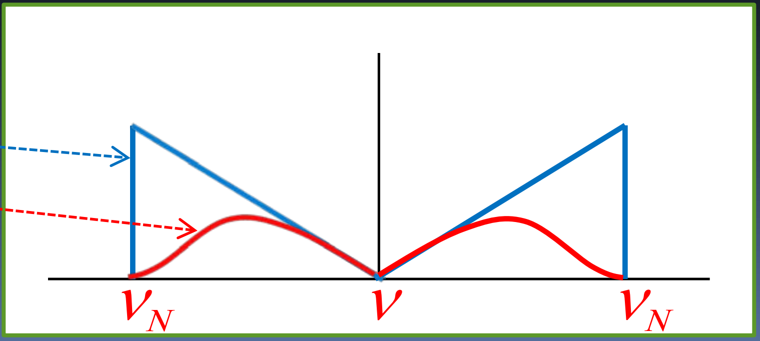
$$\xi_1 = \rho\cos\varphi, \quad \xi_2 = \rho\sin\varphi,$$

- Intrinsic limitations
  - Sampling:

    The data is sampled within discrete detectors and is therefore limited by the Nyquist frequency, $\nu_N = 1/2\Delta x$.
  - Noise:

    The signal's power spectrum drops faster than the noise's power spectrum.
  - Modeling:

    Cannot account for system model information or prior knowledge of the object.
- Filter choices
  - Ramp:
  - Hanning:
  - Many more…



FBP example:

MATLAB code:

- IM_REF = phantom(128);
  - Shepp-Logan Phantom
- phi = 0:(180/128):(180-1/(128+1));
- [IM_FW,xp] = radon(IM_REF,phi);
  - Forward projection
- Noise_1 = poissrnd(IM_FW);
  - Noisy Poisson Realization
- IM_BK_filtered = iradon(IM_FW,phi,'Shepp-Logan',1,128);
  - Reconstruction of un-noised phantom
- IM_BK_Noise = iradon(Noise_1,phi,'Shepp-Logan',1,128);
  - Reconstruction of noised phantom

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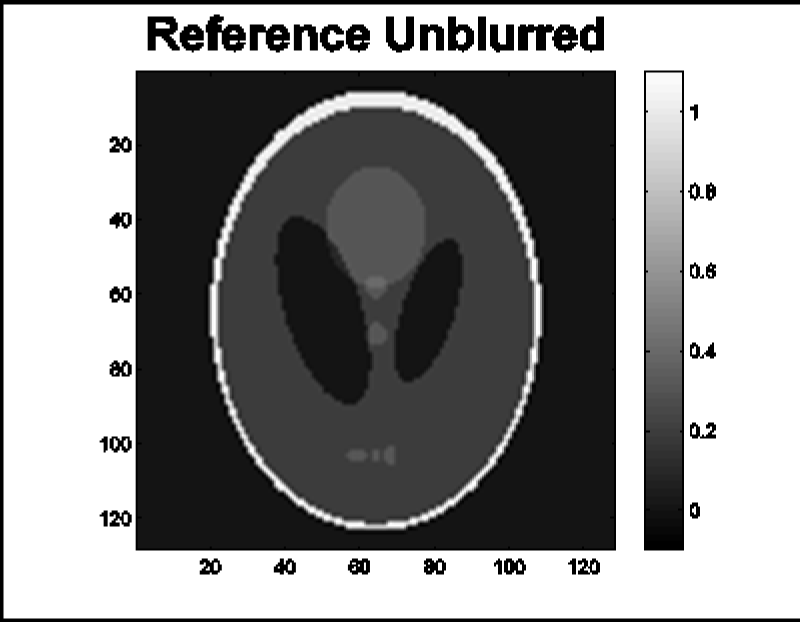

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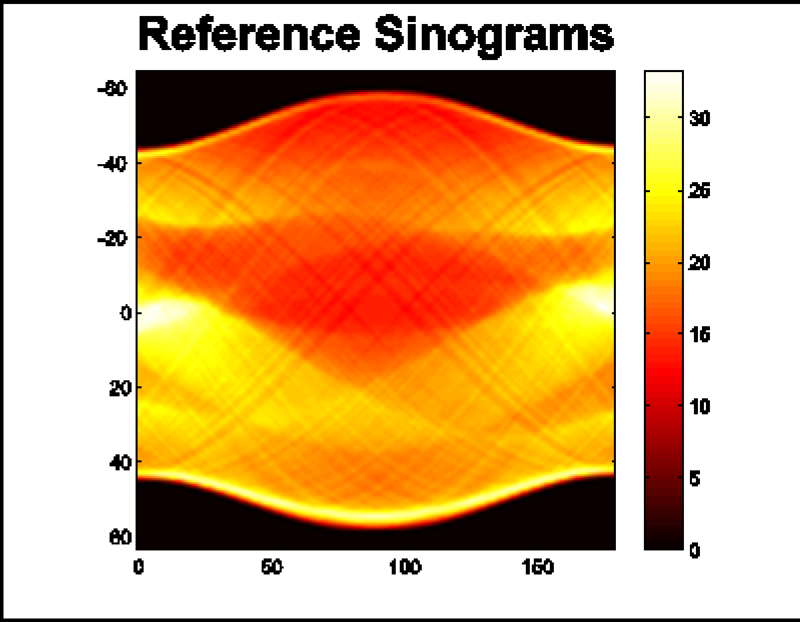

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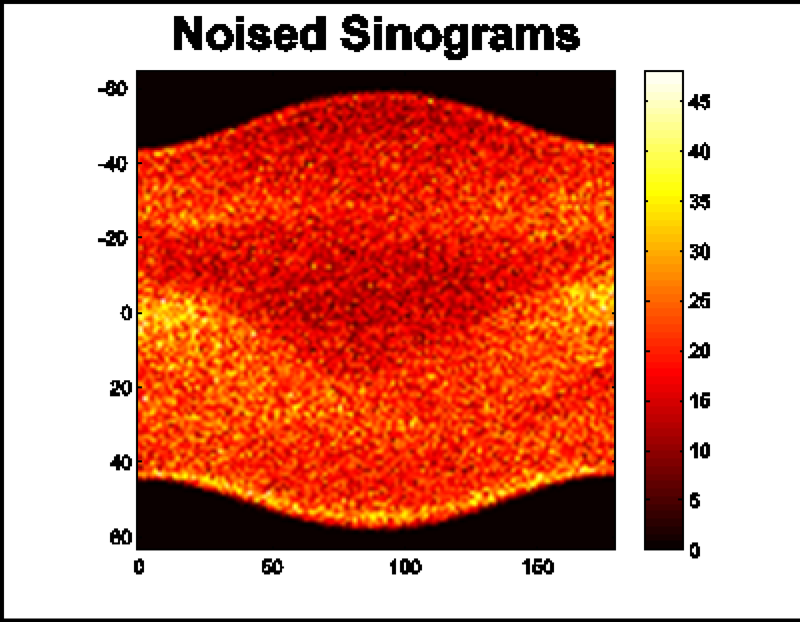

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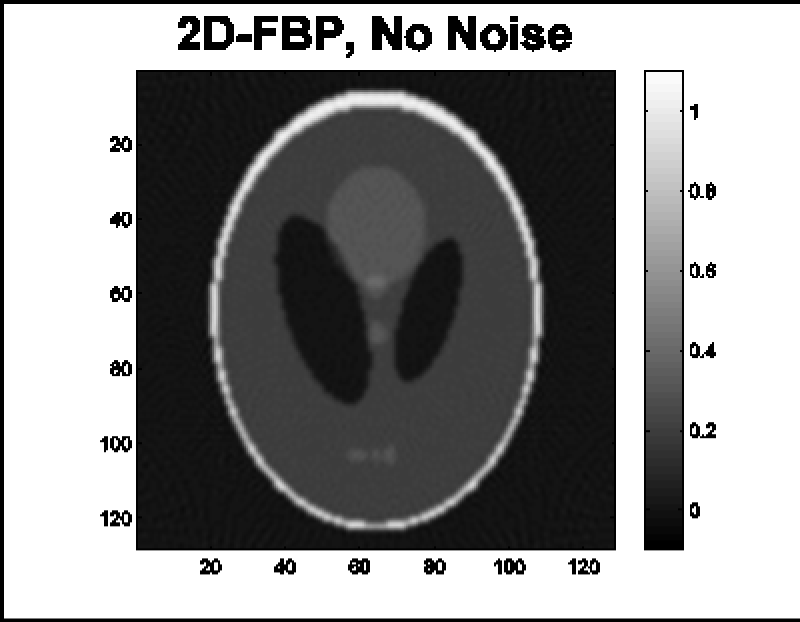
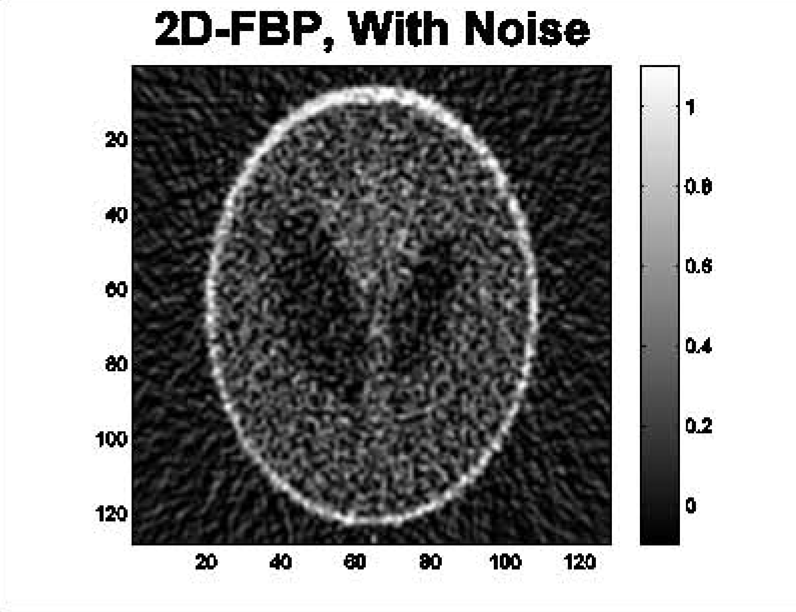
- IM_BK_Noise = iradon(Noise_1,phi,'Shepp-Logan',1,128);
  - Reconstruction of noised phantom



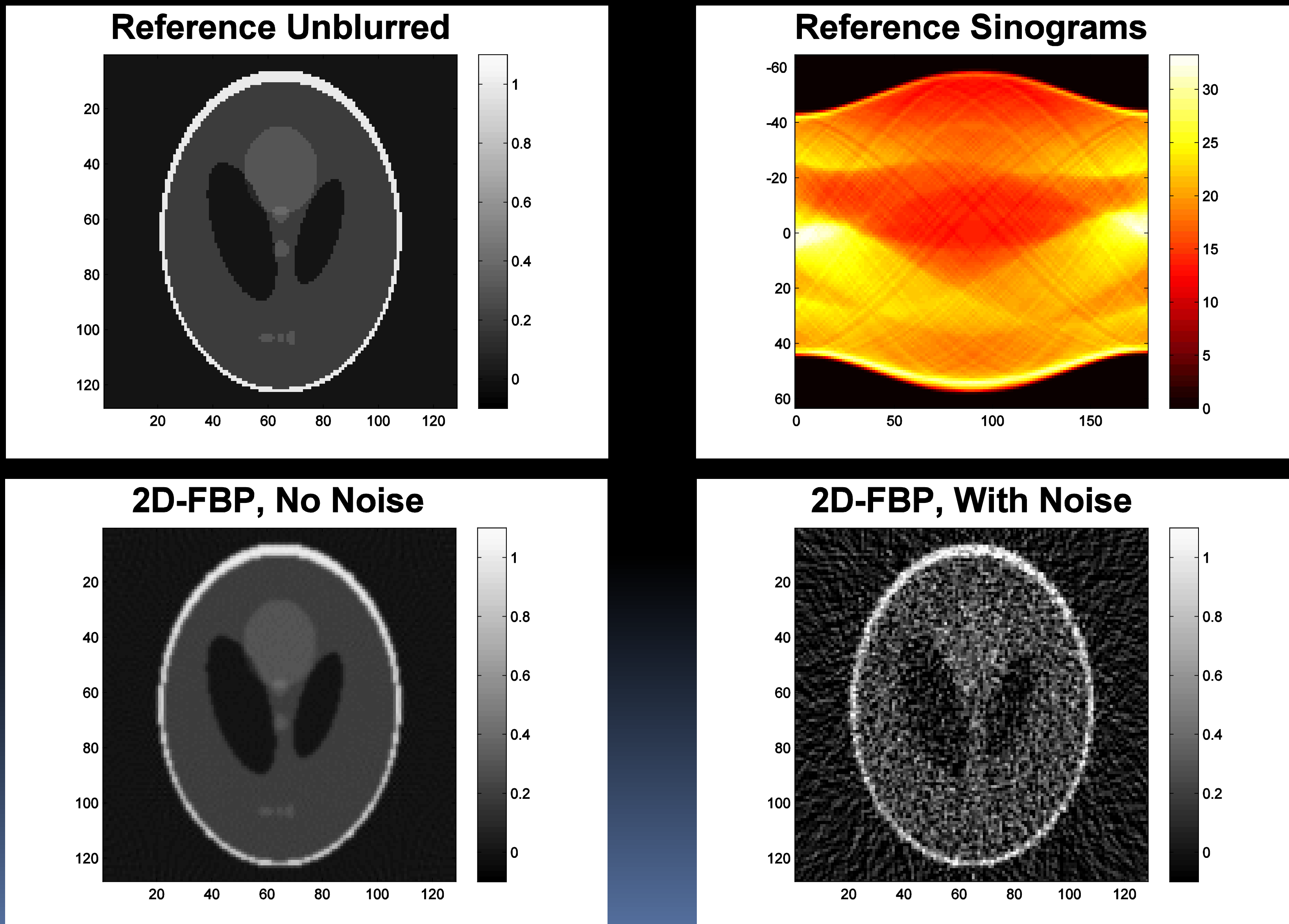
FBP example:



MATLAB code:

- IM_REF = phantom(128);
  - Shepp-Logan Phantom
- phi = 0:(180/128):(180-1/(128+1));
- [IM_FW,xp] = radon(IM_REF,phi);
  - Forward projection
- Noise_1 = poissrnd(IM_FW);
  - Noisy Poisson Realization
- IM_BK_filtered = iradon(IM_FW,phi,'Shepp-Logan',1,128);
  - Reconstruction of un-noised phantom
- IM_BK_Noise = iradon(Noise_1,phi,'Shepp-Logan',1,128);
  - Reconstruction of noised phantom

FBP example:


Reference Unblurred
20
40
60
80
100
120
1
0.8
0.6
0.4
0.2
0
Reference Sinograms
-60
-40
-20
0
20
40
60
0
50
100
150
30
25
20
15
10
5
2D-FBP, No Noise
2D-FBP, With Noise

Maximum Likelihood Expectation Maximization

Emission from object, $<f_n>$, to detection, $< g_m >$

$$\langle g_m \rangle = \sum_{n=1}^{N} H_{mn} \langle f_n \rangle = H\left[\langle f_n \rangle\right]$$

Poisson probability that an event, g_m, is drawn from $< g_m >$

$$p\left(g_m \mid \langle f \rangle\right) = \frac{e^{-\langle g_m \rangle} \langle g_m \rangle^{g_m}}{g_m!}$$

Maximizing the log-likelihood of the detections, g, on object, $<f>$

$$\frac{\partial \log\left[p\left(g \mid \langle f \rangle\right)\right]}{\partial f} = H^T \left[\frac{g}{H[f]} - 1\right] = 0$$

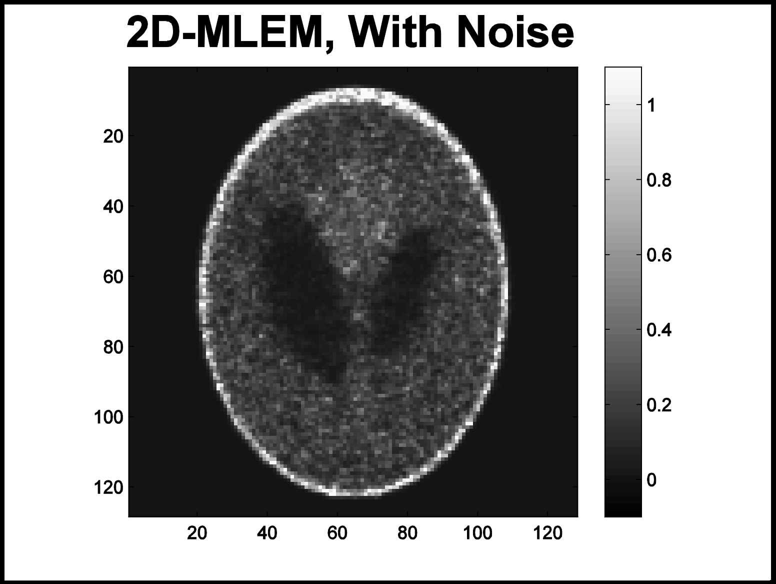
Maximum Likelihood Expectation Maximization

$$f^{k+1} = \frac{f^k}{s} H^T \left[\frac{g}{H[f^k]}\right], \qquad s = H^T[1]$$

ML-EM example:

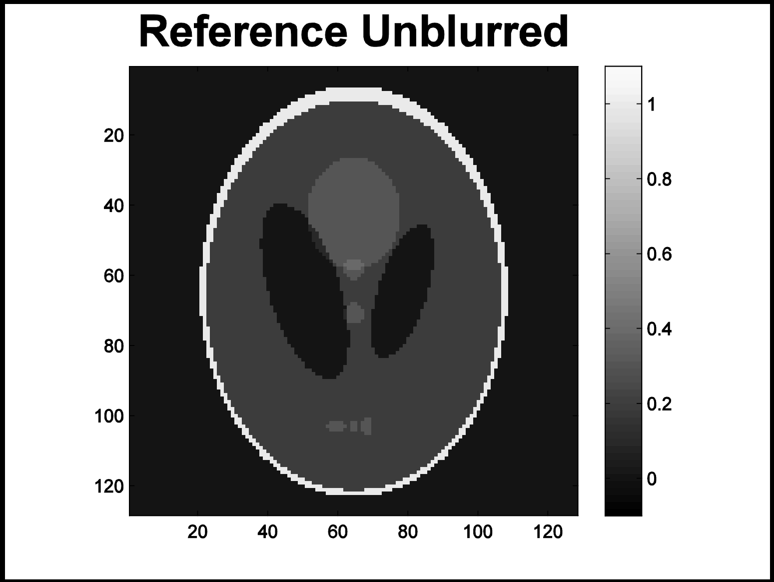
MATLAB code:

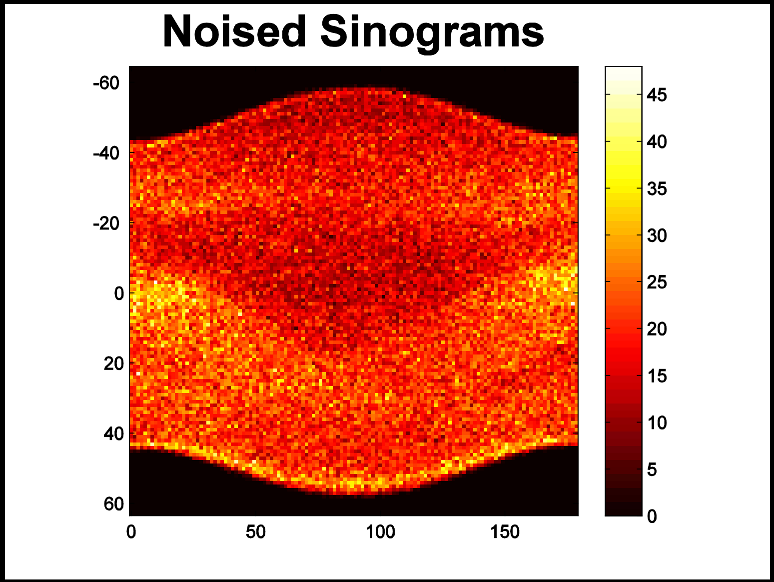
- IM_REF = phantom(128);
  phi = 0:(180/128):(180-1/(128+1));
  [IM_FW,xp] = radon(IM_REF,phi);
  G = poissrnd(IM_FW);
- s = radon(ones(128,128),phi);
- F(:,:) = ones(128,128);
- for loop = 1:ITER
  - [FW,xp] = radon(F,phi);
  - F = F ./s.*iradon(G/FW,phi,'None',128);
- end

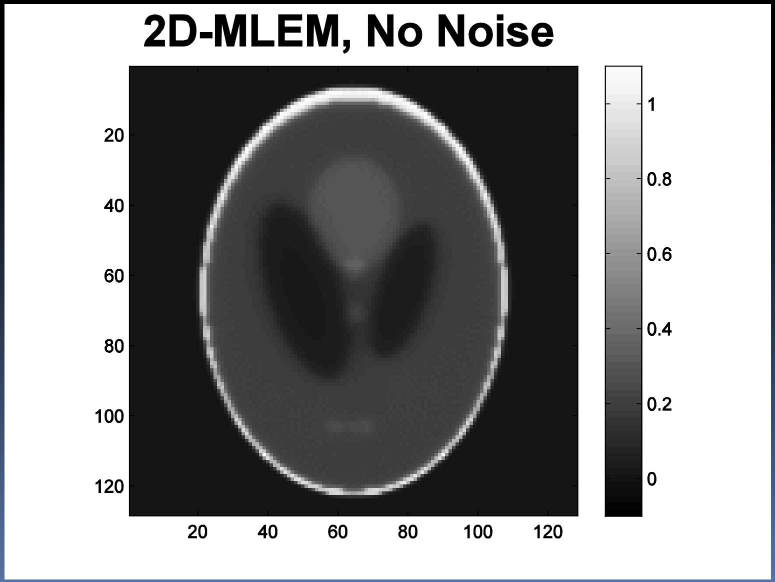


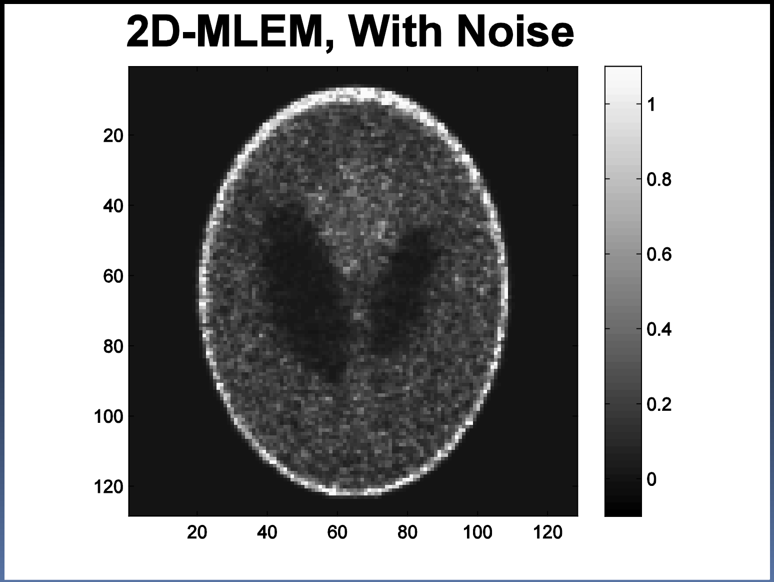
$$f^{k+1} = \frac{f^k}{s} H^T \left[\frac{g}{H[f^k]} \right]$$

ML-EM example:

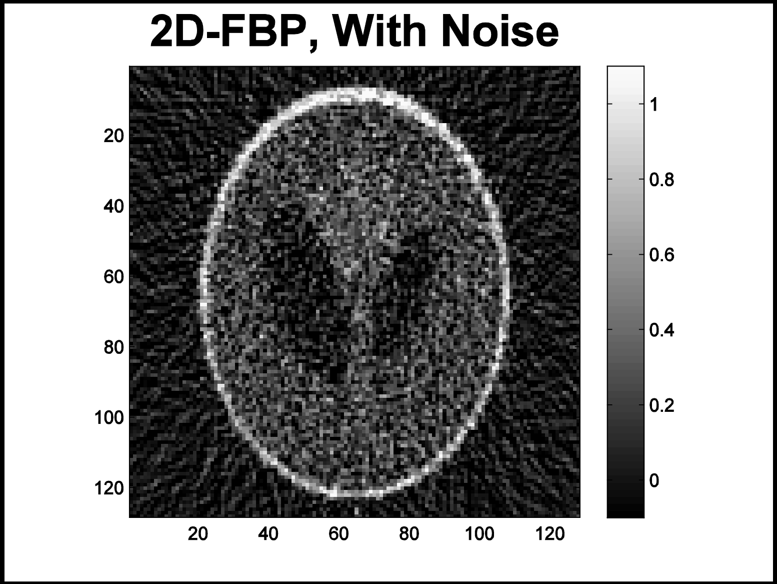


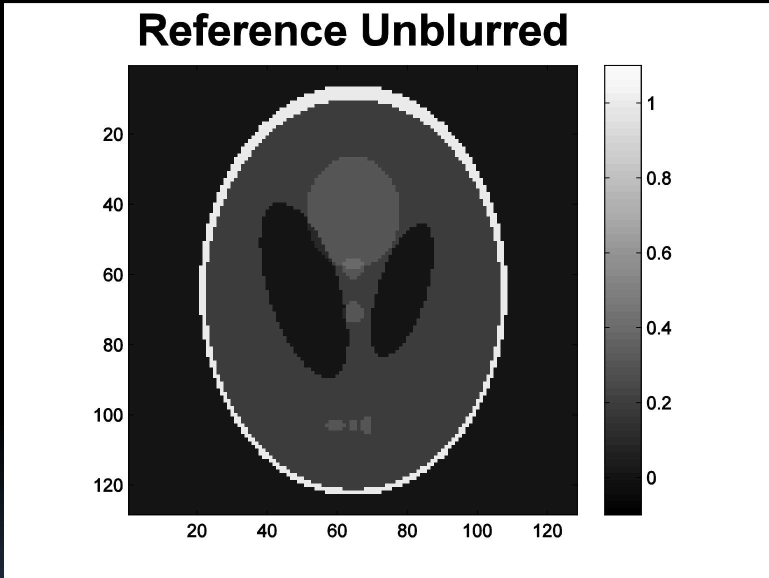


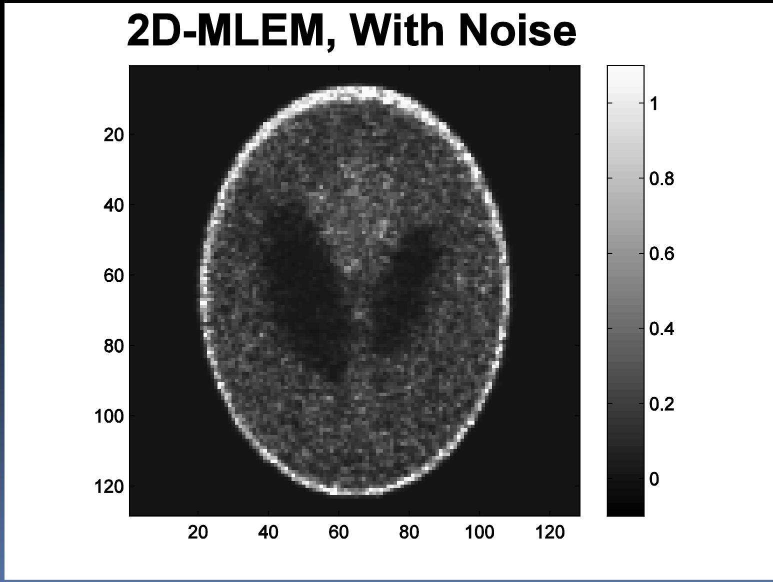




Comparison of FBP MLEM:





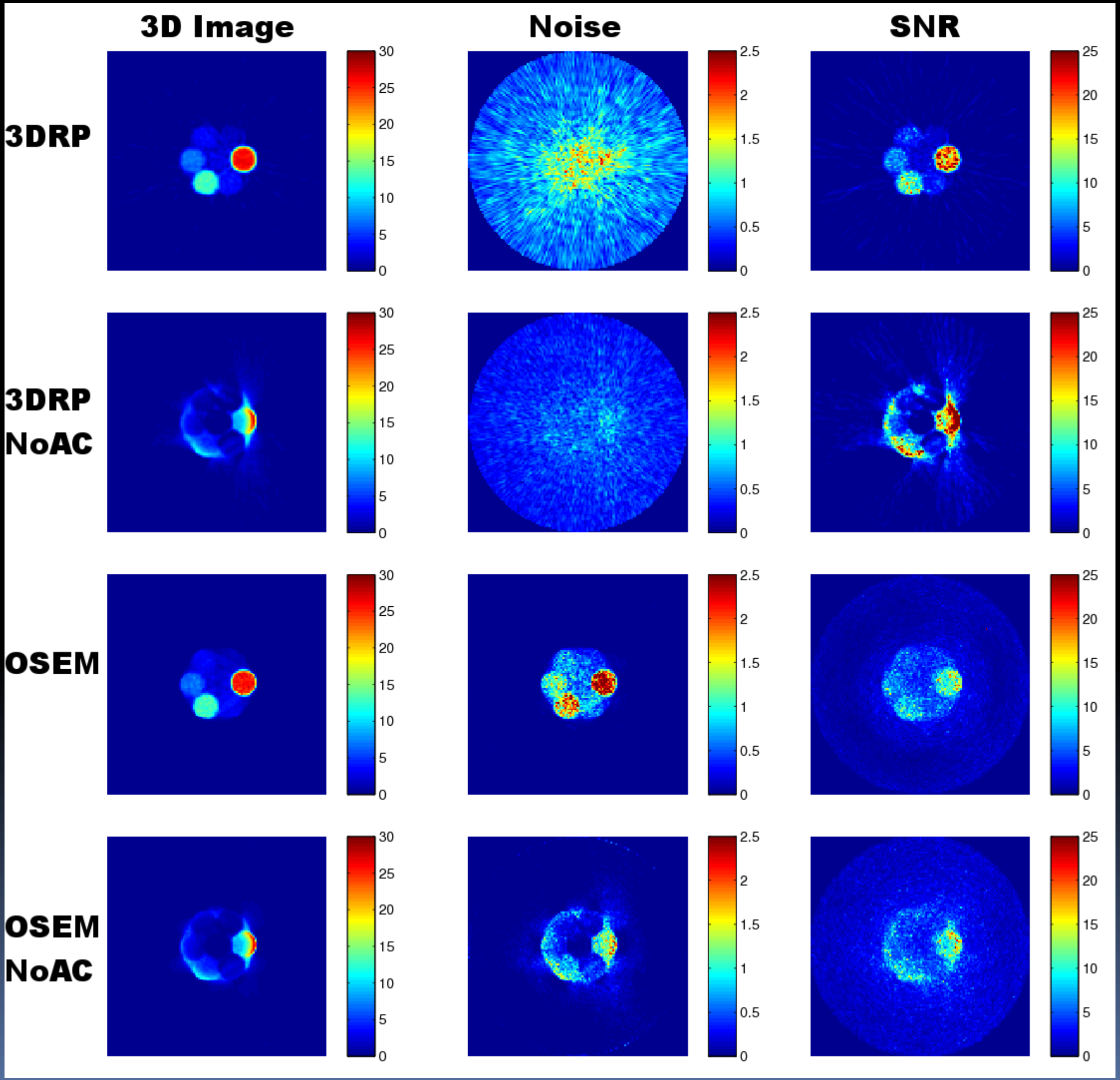


Noise Properties of Image Reconstruction

- FBP
  - Scales linearly with counts,
  - Before attenuation correction uniform
  - After attenuation correction azimuthally invariant

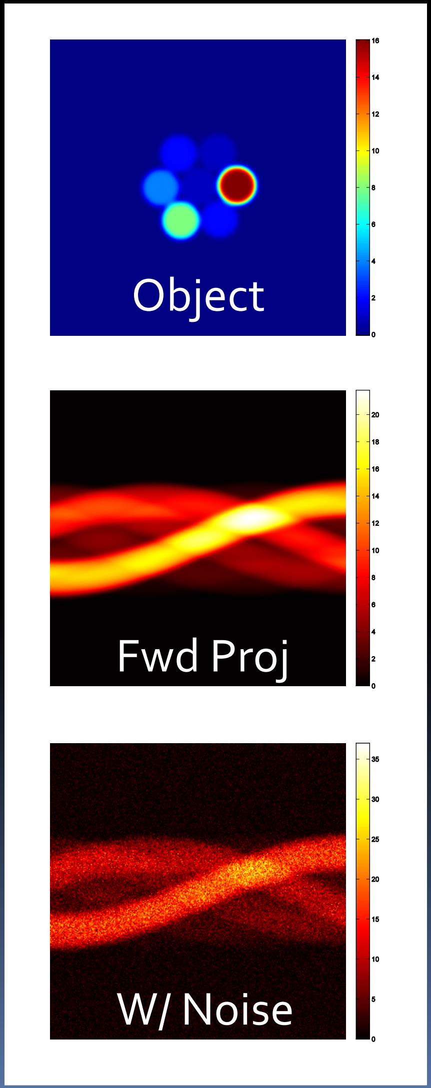
- OSEM
  - Does not scale linearly with counts
  - More proportional to image intensity
  - Better SNR for low uptake regions
  - Worse SNR for high uptake regions

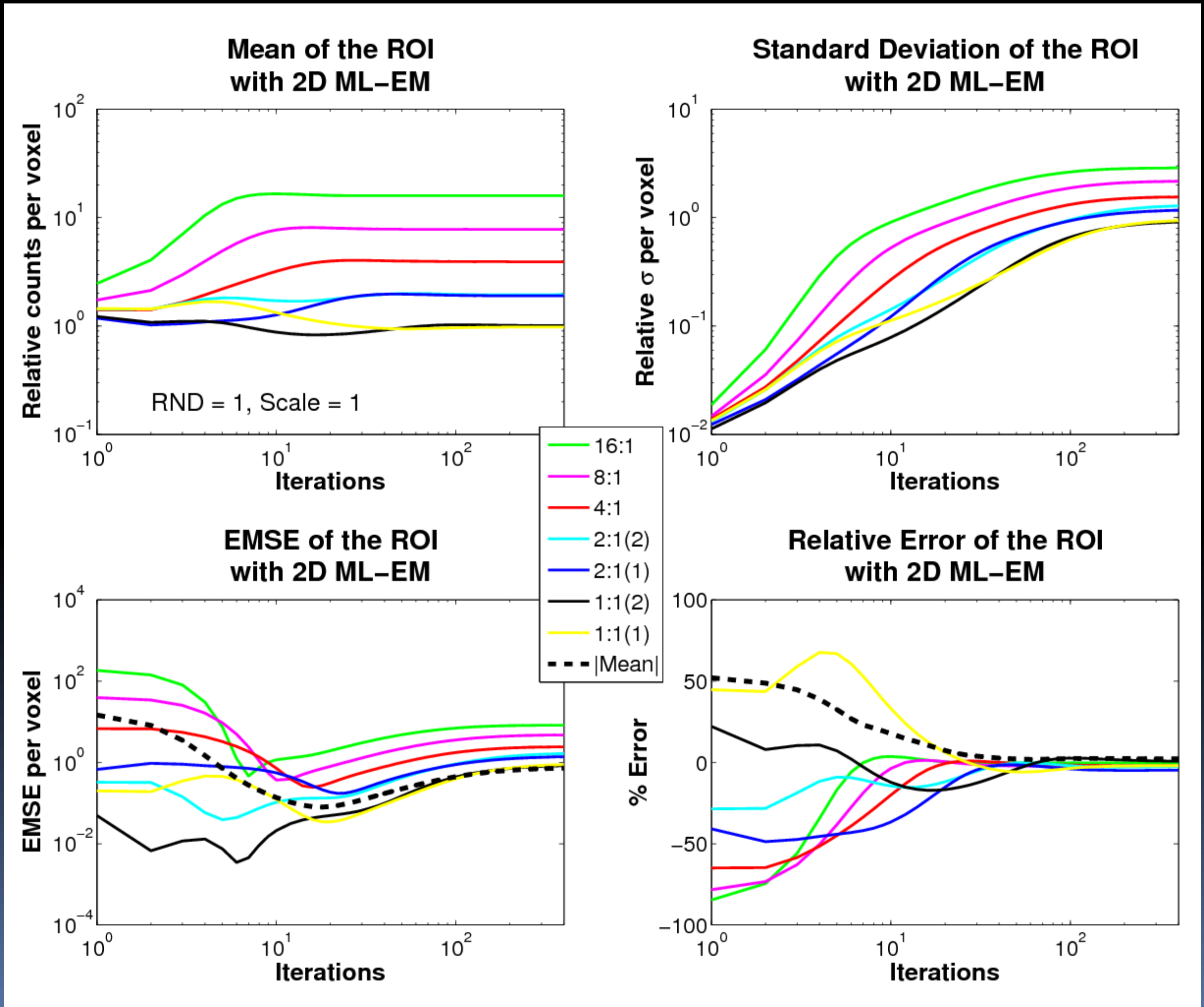
Noise images:



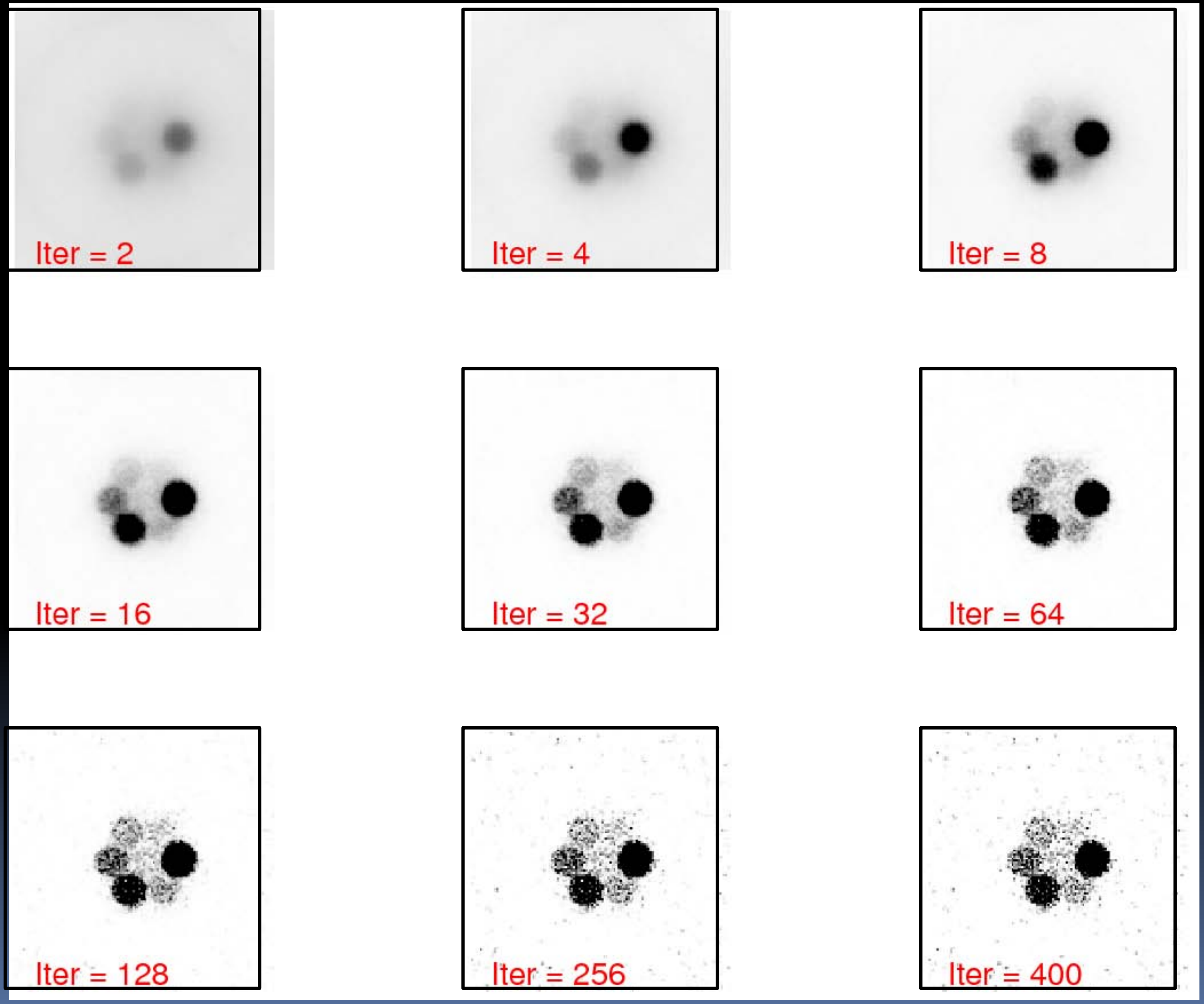
Noise as a function of iteration:

$$f^{k+1} = \frac{f^k}{s} H^T \left[\frac{g}{H[f^k] + \tilde{R}} \right]$$





Noise as a function of iteration:



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Rashid Ghani

Cascade Corrections

- Beattie's method:
  - Performed in projection space
  - Can be extended to be performed in the reconstruction loop
  - Procedure:
    1. Correct the projection data
    2. Convolve the corrected data inside the object with a cascade kernel
    3. Tail fit the estimate with the non-attenuation corrected projection data outside the object to get a scaling factor
    4. Correct the data

$$P_{std} = P \cdot C(S, R, Ac, DT, Norm, \ldots)$$

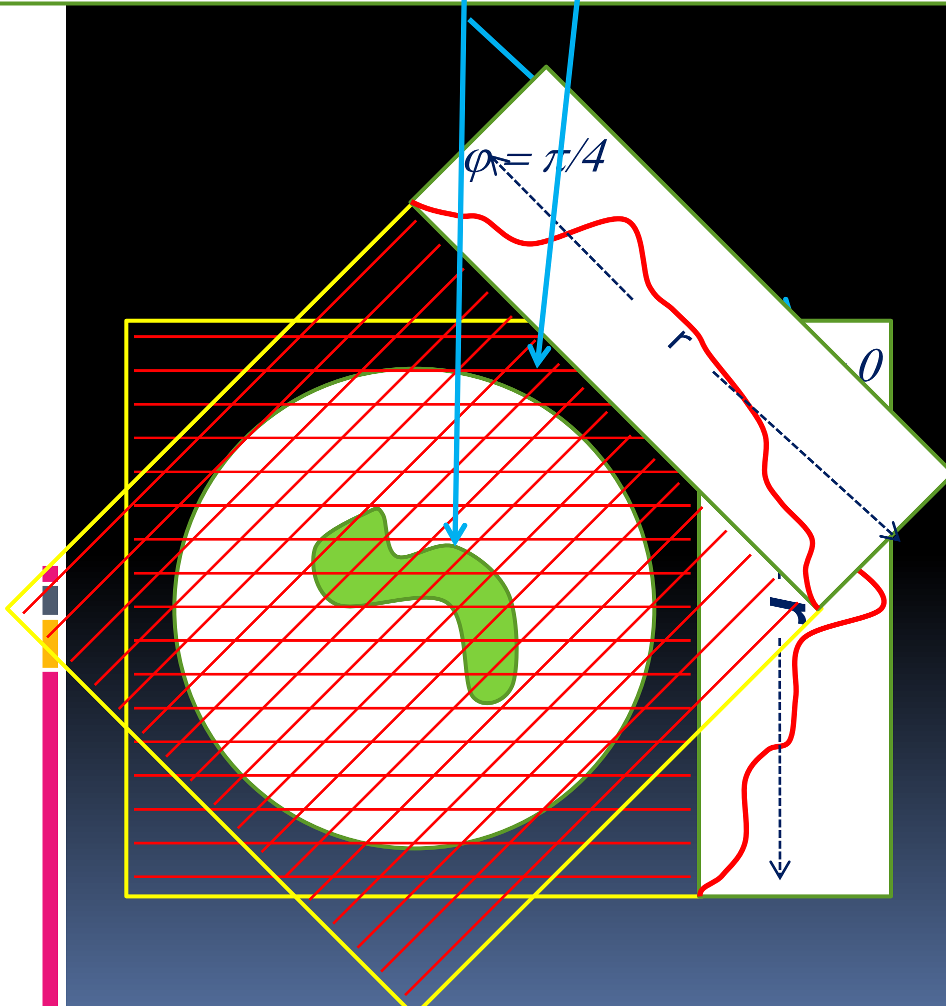
$$P_{cc} = (M \cdot P_{std}) \otimes K_{cc}$$

$$\alpha = \frac{M^{-1} \cdot (P_{std} \cdot Ac^{-1})}{M^{-1} \cdot P_{cc}}$$

$$P_{corr} = \left[(P_{std} \cdot Ac^{-1}) - \alpha P_{cc} \right] \cdot Ac$$

The Projection Slice Theorem

$$p(r,\varphi) = Rf(r,\varphi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(r - x\cos\varphi - y\sin\varphi)\,dxdy$$



$$F_r\, p(\rho,\varphi) = \int_{-\infty}^{\infty} e^{-2\pi i\rho r} Rf(r,\varphi)\,dr$$

$$= \int_{-\infty}^{\infty} e^{-2\pi i\rho r} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(r - x\cos\varphi - y\sin\varphi)\,dxdydr$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\left(\int_{-\infty}^{\infty} e^{-2\pi i\rho r}\,\delta(r - x\cos\varphi - y\sin\varphi)\,dr\right)dxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,e^{-2\pi i\rho(x\cos\varphi + y\sin\varphi)}\,dxdy$$

$$\xi_1 = \rho\cos\varphi, \quad \xi_2 = \rho\sin\varphi,$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,e^{-2\pi i(x\xi_1 + y\xi_2)}\,dxdy$$

$$F_{x,y} f(\xi_1,\xi_2) = F_r\, p(\rho,\varphi)$$

$$F^{-1}_{\xi_1,\xi_2}\left(F_{x,y} f(\xi_1,\xi_2)\right)(x,y) = f(x,y)$$

Maximum Likelihood Expectation Maximization

$$\langle g_m \rangle = \sum_{n=1}^{N} H_{mn} \langle f_n \rangle = H[\langle f_n \rangle] \qquad p(g_m \mid \langle f \rangle) = \frac{e^{-\langle g_m \rangle} \langle g_m \rangle^{g_m}}{g_m!}$$

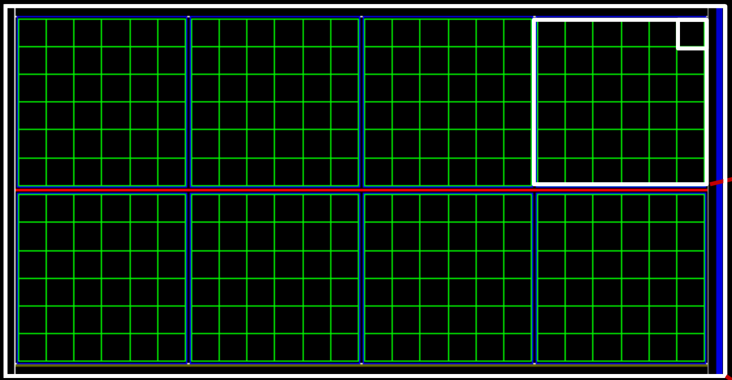
$$p(g \mid \langle f \rangle) = \prod_{m=1}^{M} p(g_m \mid \langle f \rangle) = \prod_{m=1}^{M} \frac{\exp(-H[\langle f_n \rangle])(H[\langle f_n \rangle])^{g_m}}{g_m!}$$

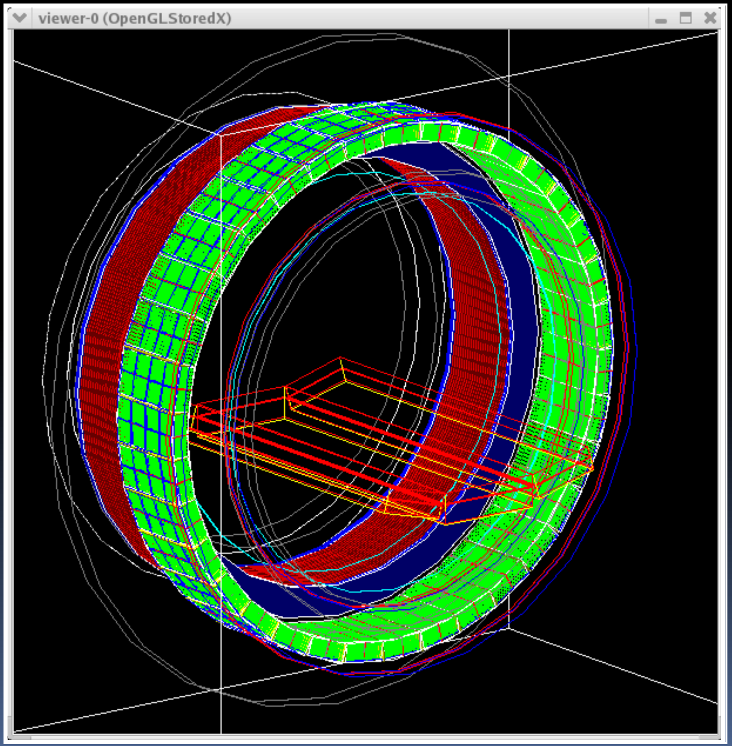
$$\log[p(g \mid \langle f \rangle)] = \sum_{m=1}^{M} \log[p(g_m \mid \langle f \rangle)] \qquad \hat{f} = \arg\max_{f} (\log[p(g \mid f)])$$

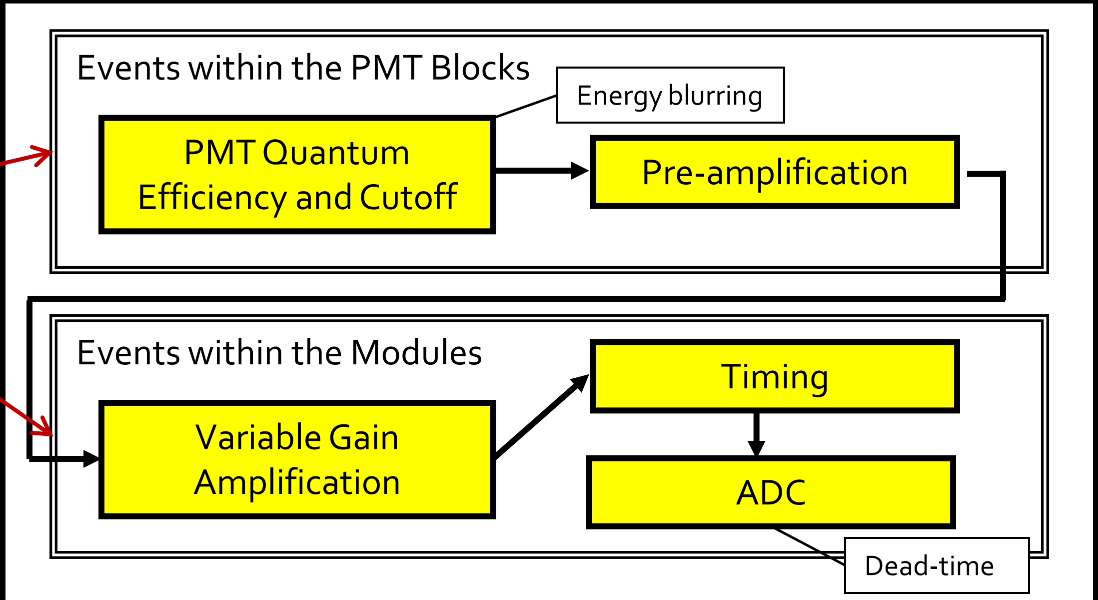
$$\frac{\partial \log[p(g \mid \langle f \rangle)]}{\partial f} = H^T \left[\frac{g}{H[f]} - 1 \right] = 0 \longrightarrow fH^T \left[\frac{g}{H[f]} \right] - fH^T[1] = 0$$

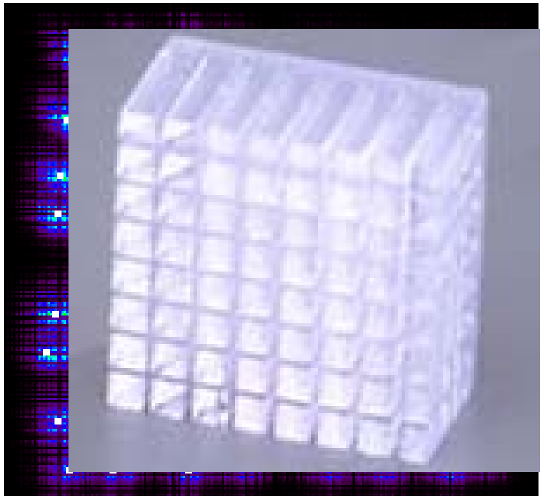
$$f = \frac{f}{H^T[1]} H^T \left[\frac{g}{H[f]} \right], \qquad s = H^T[1] \longrightarrow \boxed{f^{k+1} = \frac{f^k}{s} H^T \left[\frac{g}{H[f^k]} \right]}$$

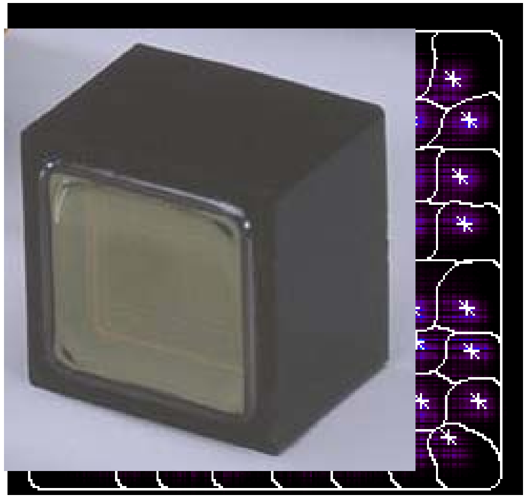
Signal in the blocks











Description of the signal processing flow chart provided by A. Ganin GE Health Care.

Question: What are the four <u>main</u> factors in PET that degrade spatial resolution?

0%
1. Positron Range, annihilation photon non-collinearity, PMT light sharing, and detector size.

0%
2. Positron range, annihilation photon non-collinearity, depth of interaction, and detector electron stopping power.

0%
3. Positron range, photon yield, depth of interaction, and detector size.

0%
4. Positron range, annihilation photon non-collinearity, block effect, and detector size.

0%
5. Positron range, annihilation photon non-collinearity, Compton scatter, and detector size.

10

Answer:

What are the four <u>main</u> factors in PET that degrade spatial resolution?

1. Positron Range, annihilation photon non-collinearity, PMT light sharing, and detector size.
2. Positron range, annihilation photon non-collinearity, depth of interaction, and detector electron stopping power.
3. Positron range, photon yield, depth of interaction, and detector size.
4. Positron range, annihilation photon non-collinearity, block effect, and detector size.
5. Positron range, annihilation photon non-collinearity, Compton scatter, and detector size.

Ref.: Cherry, Sorenson, and Phelps, Physics in Nuclear Medicine, 3rd ed., Saunders ,2003, pages 328-337.