

# *Progress in calculations of $k_Q$ for TG-51*

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# Acknowledgements

- most of this talk is based on the work of several graduate students who have worked with me over the last few years:
- Lesley Buckley ( $P_{\text{wall}}$ ),
- Lilie Wang ( $P_{\text{repl}}$ )
- Bryan Muir (Monte Carlo calculations of  $k_Q$ )  
critical to central part of this talk



# TG-51 for photon beams

$$D_w^Q = M N_{D,w}^Q$$

defines: chamber's **absorbed dose calibration coefficient**

$$N_{D,w}^Q = k_Q N_{D,w}^{60Co}$$

defines  $k_Q$ : chamber specific **beam quality conversion factor**

-accounts for  $N_{D,w}$  variation with Q

$$D_w^Q = M k_Q N_{D,w}^{60Co}$$

# Where does $k_Q$ come from?

Basically - same physics as TG-21,  
ie Spencer-Attix cavity theory

$$D_{\text{med}} = D_{\text{air}} \left( \frac{\bar{L}}{\rho} \right)_{\text{air}}^{\text{med}} P_{\text{wall}} P_{\text{fl}} P_{\text{gr}} P_{\text{cel}} K_h$$

$P_{\text{wall}}$  corrects for the wall **not being same** as med

$P_{\text{cel}}$  corrects for central electrode **not being wall material**

$K_h$  accounts for measurements being in **humid air**

- all factors refer to **dry air** ( $K_h = 0.997$ )

$$P_{repl} = P_{gr} P_{fl}$$

$P_{repl}$  accounts for **effects of** cavity on electron spectrum at point of measurement

$P_{gr}$ : that part of  $P_{repl}$  which accounts for less attenuation in cavity than in phantom.

- usually only applied to **cylindrical chambers**

- depends on local gradient** => no effect at  $d_{max}$

- handled by: -**effective point of measurement**

when measuring depth-dose ( $0.6 r_{cav}$  offset)

- **multiplicative results** used in  $k_Q$  calculations

$$P_{repl} = P_{gr} P_{fl} \text{ (cont)}$$

$P_{fl}$ : that part of  $P_{repl}$  which accounts for other changes in spectrum in cavity.

Photon beams

$P_{fl}$  not required past  $d_{max}$  because of transient charged particle equilibrium.

Fano theorem tells us electron spectrum is independent of density for cpe and to extent that water is like air, theorem applies.

## Equation used by TG-51 for $k_Q$

$$k_Q = \frac{\left[ \left( \frac{\bar{L}}{\rho} \right)_{\text{air}}^{\text{w}} P_{\text{wall}} P_{\text{fl}} P_{\text{gr}} P_{\text{cel}} \right]}{\left[ \left( \frac{\bar{L}}{\rho} \right)_{\text{air}}^{\text{w}} P_{\text{wall}} P_{\text{fl}} P_{\text{gr}} P_{\text{cel}} \right]}$$

This eqn assumes  $(W/e)_{\text{air}}$  (relating charge measured to dose to the air in cavity) is independent of beam quality

For a detailed derivation, see Ch 9 in 2009 AAPM Summer School book

# Physical data sets in TG-51

Much of data comes directly from TG-21 and/or IAEA's TRS-277 (1987 Code of Practice).

TG-51 consistently uses **ICRU Report 37** stopping powers (TG-21 used 35 and 37 values).

For photon beams, TG-51 **stopping-power ratios** are based on Monte Carlo calc'ns for 25 beam qualities

$$\left(\frac{\bar{L}}{\rho}\right)_{\text{air}}^{\text{water}} = 1.275 - 0.00231(\%dd(10)_x) \quad \%dd(10)_x \geq 63.35\%$$

Values from Rogers and Yang Med Phys 26 (1999) 536

# stopping-power ratios: state-of-the-art

Uncertainties are **uncertainties in stopping powers**  
-recent I-value measurement  $\Rightarrow$  6% change  
for  $H_2O \Rightarrow$  **0.1 % to 0.4 % change in  $k_Q$ .**

Using full photon beam phase-space (with **horns and varying energy across beam**) rather than realistic spectra from uniform point sources shows **no significant changes.**

Similarly, sprs as a function of  $\%dd(10)$  **do not change when flattening filter is removed**

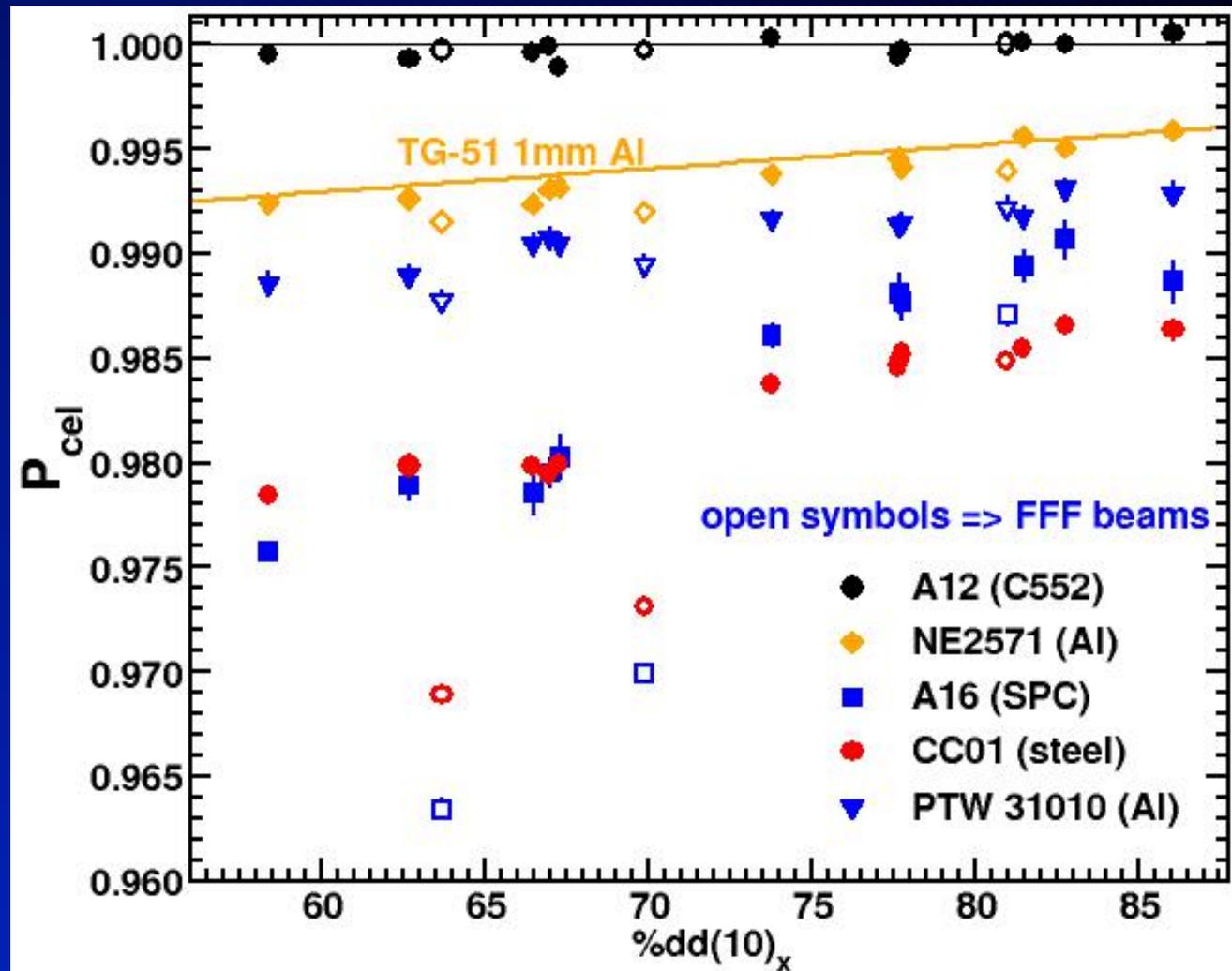
# $P_{cel}$ : electrode correction

- TG-51 corrects for chambers with **Al central electrodes**
- more recent, more precise calculations **agree with values TG-51** used for Al electrodes in filtered beams
- even more recent calculations for **higher-Z electrodes** show major effects
  - $P_{cel}$  effects much larger
  - $P_{cel}$  in FFF (flattening filter free) beams even larger effects

# $P_{cel}$ : state-of-the-art

• high  $Z \Rightarrow$   
much larger  
effect

• FFF beams  $\Rightarrow$   
even larger  
effects

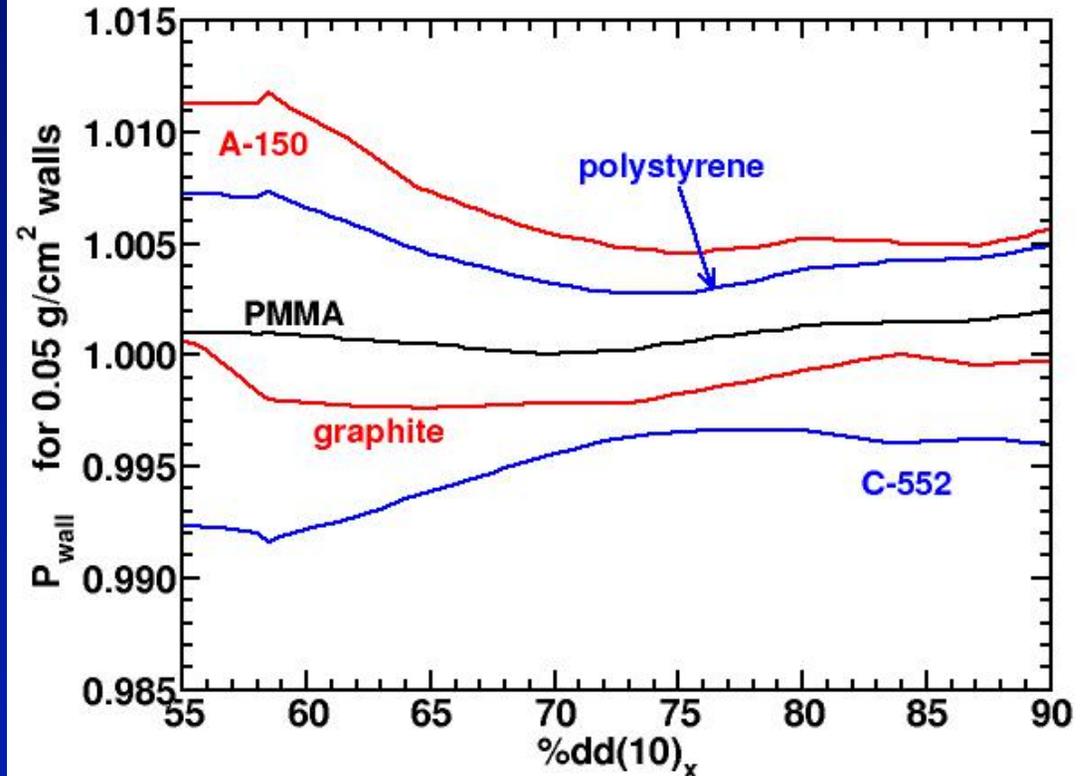


# $P_{wall}$ in TG-51

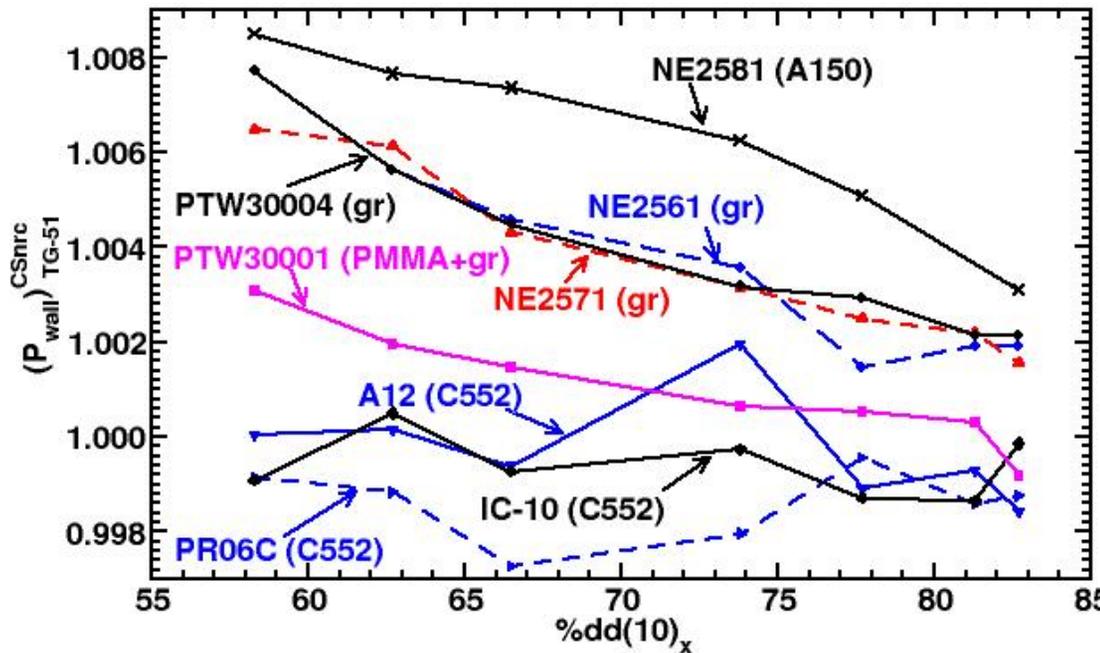
$$P_{wall} = \frac{\alpha \left(\frac{\bar{L}}{\rho}\right)_{air}^{wall} \left(\frac{\overline{\mu_{en}}}{\rho}\right)_{wall}^{med} + \tau \left(\frac{\bar{L}}{\rho}\right)_{air}^{sheath} \left(\frac{\overline{\mu_{en}}}{\rho}\right)_{sheath}^{med} + (1 - \alpha - \tau) \left(\frac{\bar{L}}{\rho}\right)_{air}^{med}}{\left(\frac{\bar{L}}{\rho}\right)_{air}^{med}}$$

- accounts for wall not being water

For walls 0.05g/cm<sup>2</sup>

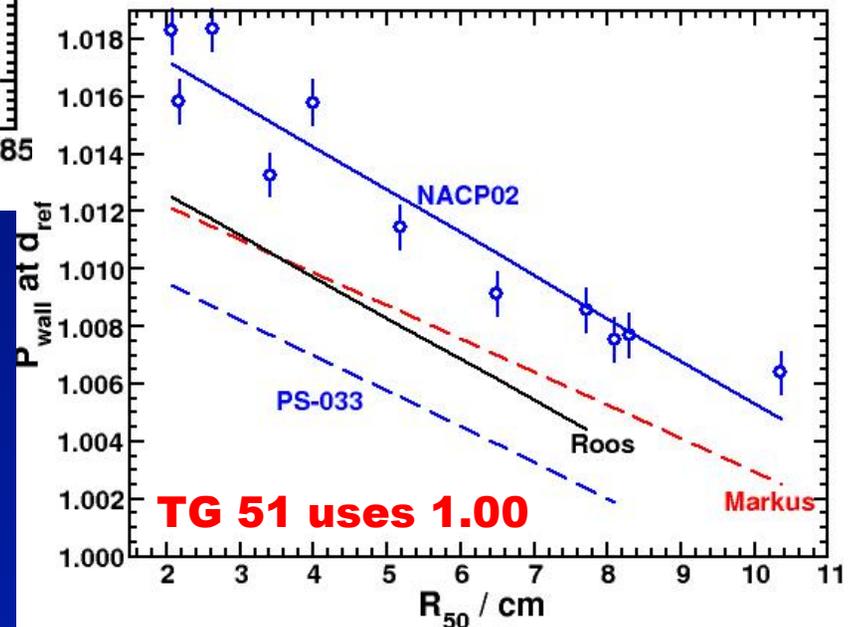


# Monte Carlo values of $P_{wall}$



← photon beams

↓ electron beams



Buckley et al MP 33(2006) 455  
MP 33(2006) 1788

# $P_{gr}$ for cylindrical chambers

TG-51 (& TG-21) use  $P_{gr}$  values from Cunningham & Sontag(1980)  
-values buried in  $k_Q$  values

IAEA uses values from Johansson et al (1977).  
Same paper led to the  $0.75 r_{cav}$  and  $0.6 r_{cav}$   
offsets used in the effective point of  
measurement approach

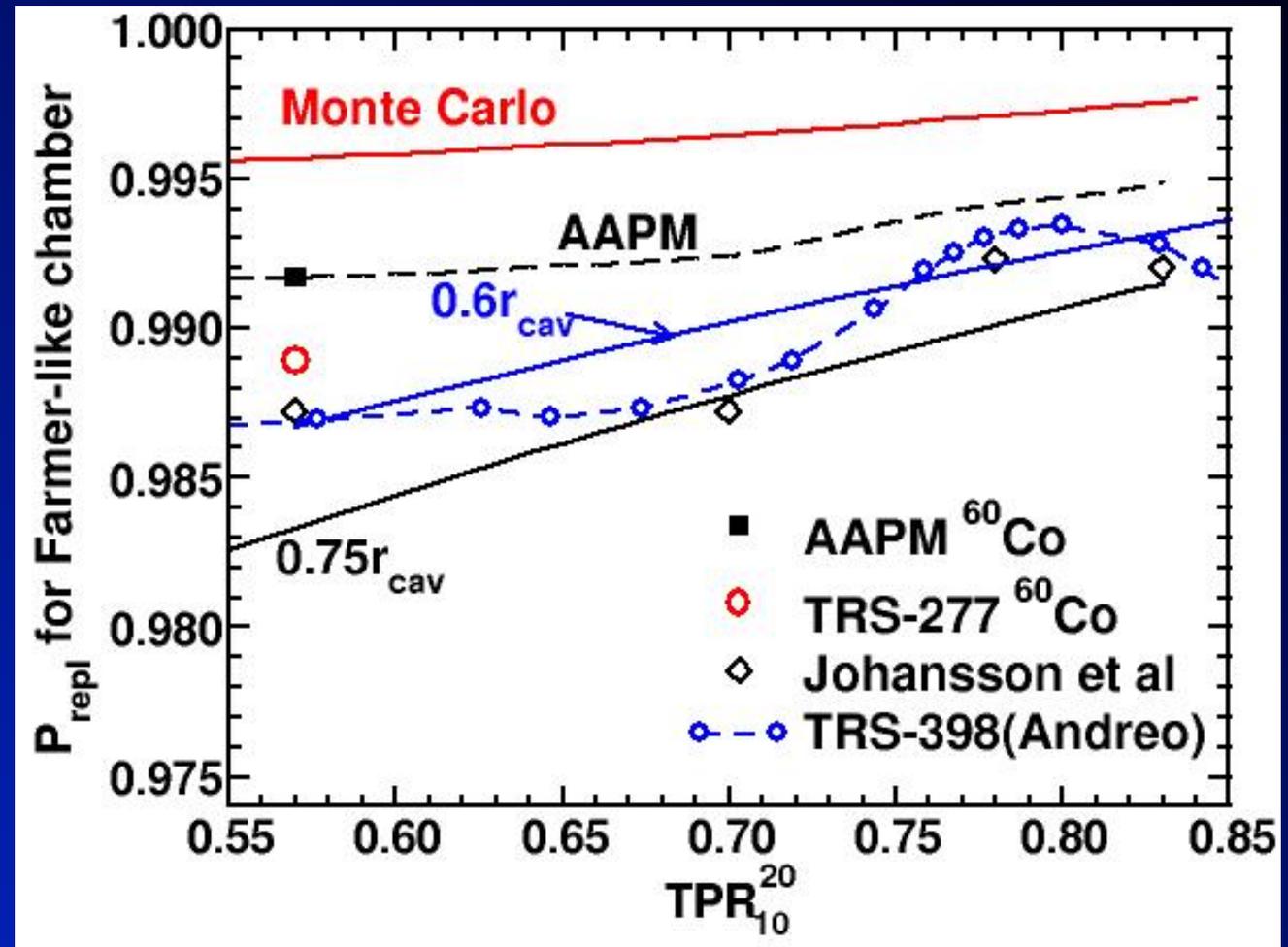
$$P_{gr}^{offset} = 1 + \left( \frac{1}{10} \ln \frac{D_{20}}{D_{10}} \right) \Delta z$$

Relates multiplicative factor to offset

# $P_{gr}$ for cylindrical chambers

$P_{gr}$  is largest  
TG51-TRS398  
difference

Wang's MC  
calns disagree  
with both:  
while explaining  
previous  
measurements  
PMB 54(2009)1609



TG-51's  $P_{gr}$  ratio hardly changes since lines parallel  
15/39

# Status of TG-51 $k_Q$ photon calc'ns

$$k_Q = \frac{\left[ \left( \frac{\bar{L}}{\rho} \right)_{\text{air}}^w P_{\text{wall}} P_{\text{fl}} P_{\text{gr}} P_{\text{cel}} \right]}{\left[ \left( \frac{\bar{L}}{\rho} \right)_{\text{air}}^w P_{\text{wall}} P_{\text{fl}} P_{\text{gr}} P_{\text{cel}} \right]}$$

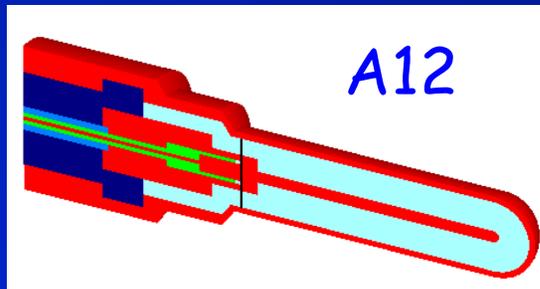
- $\text{spr}$  is OK, even in FFF beams
- $P_{\text{wall}}$  values wrong by up to 0.5%
- $P_{\text{fl}}$  value of unity assumed OK
- $P_{\text{gr}}$  values wrong by up to 0.2%
- $P_{\text{cel}}$  values wrong by up to 3% for high-Z electrodes

# Which path forward?

- rework TG-51 **analytic calculations** accounting for all the new data?
- base  $k_Q$  values on **measured values**
  - McEwen published an extensive set of values in 2010 (Med Phys 37(2010) 2179)
- do **ab initio Monte Carlo** calculations of  $k_Q$

# *ab initio Monte Carlo calculations*

- **EGSnrc** has been shown to calculate doses in an ion chamber within **0.1%** relative to its own cross sections (Fano test)
- **egs\_chamber** code of Wulff et al (Med Phys 35 (2008) 1328)
  - very efficient: **correlated sampling**
  - handles complex **realistic geometries**



# Calculating $k_Q$

- definitions:

$$k_Q = \frac{N_{D,w}^Q}{N_{D,w}^{Co}}$$

$$D_{\text{gas}} = \frac{Q \left( \frac{W}{e} \right)_{\text{air}}}{m_{\text{air}}}$$

$$N_{D,w} = \frac{D_w}{Q} = \frac{D_w}{D_{\text{gas}} \frac{m_{\text{air}}}{\left( \frac{W}{e} \right)_{\text{air}}}}$$

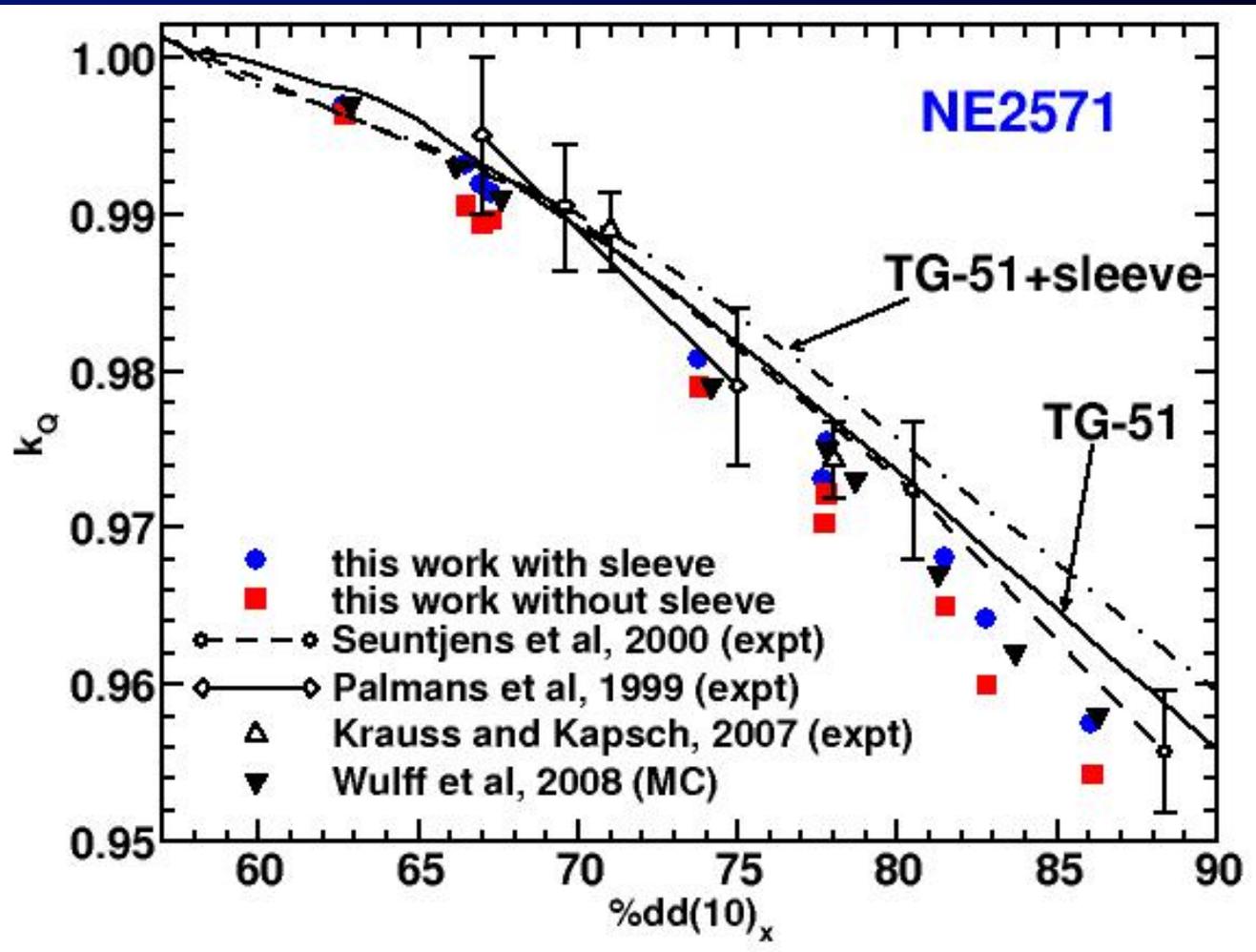
assume  $(W/e)$   
is independent  
of beam  
quality

$$k_Q = \frac{N_{D,w}^Q}{N_{D,w}^{Co}} = \frac{\frac{D_w^Q}{D_{\text{gas}}^Q}}{\frac{D_w^{Co}}{D_{\text{gas}}^{Co}}} = \left( \frac{D_w}{D_{\text{gas}}} \right)^Q \frac{Co}{Co}$$

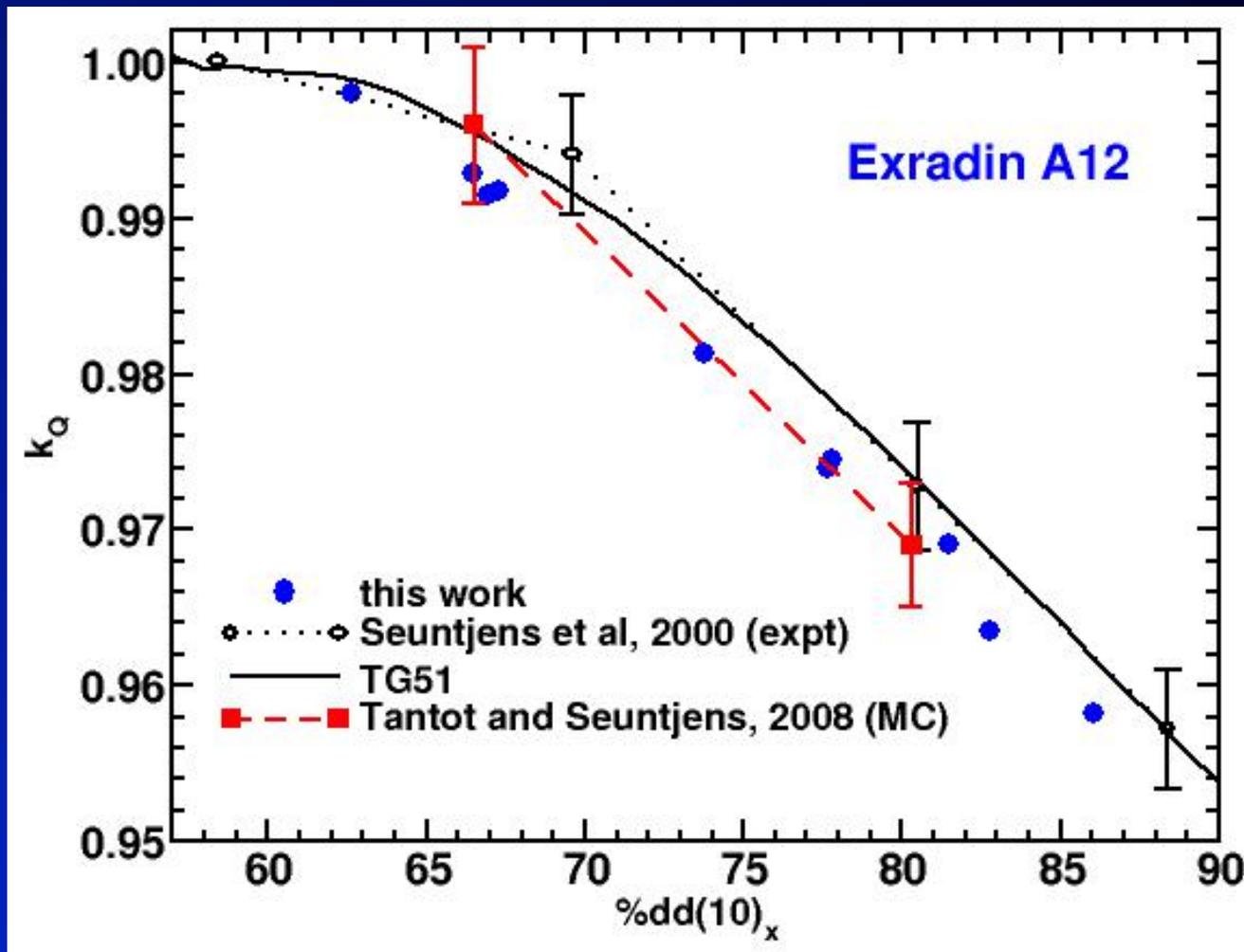
# Some $k_Q$ results: NE2571

Comparison  
to  
McEwen's  
data comes  
later

NE2571  
one of  
furthest  
from  
TG-51



# more $k_Q$ results: A12

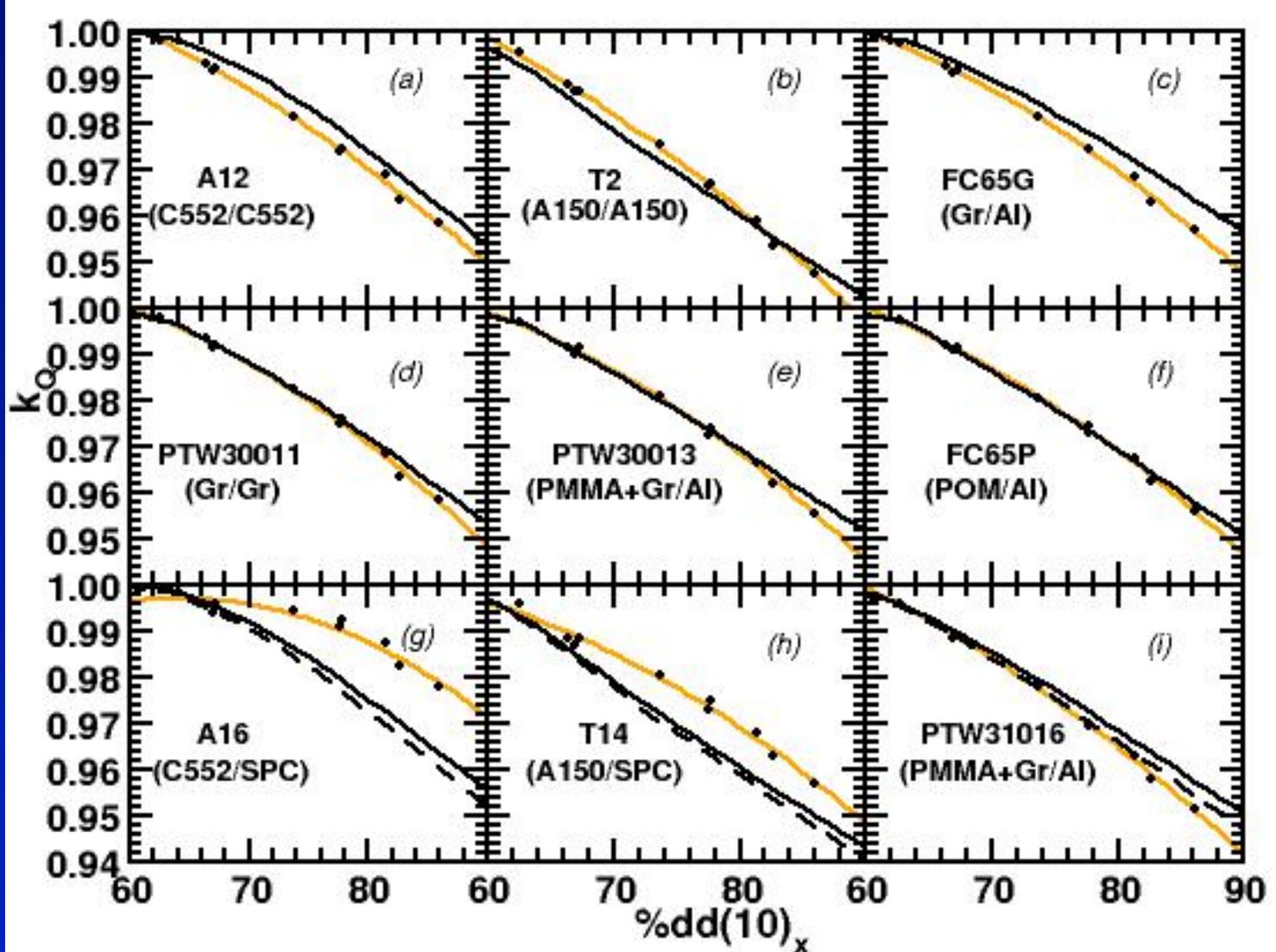


# 9 different "classes" of detectors

black: TG51  
gold: fit

labels:  
(wall/  
electrode)

Note large  
effects of  
high-Z  
electrodes



## *fits to $k_Q$*

$$k_Q = a + b(\%dd(10)_x) + c(\%dd(10)_x)^2$$
$$\%dd(10)_x \geq 62.7\%$$

- **rms deviation:** less than 0.1 % for 10 WFF beams, except for 1 chamber (A14, 0.2%, very small volume, SPC electrode).
- **a,b,c tabulated** in paper and report (also as a function of TPR)

# Uncertainties on calculated $k_Q$

- EGSnrc is accurate to 0.1 % against its own cross sections (Kawrakow, Med Phys 27(2000) 499)
- what is effect of cross section uncertainties?
- what is uncertainty on  $(W/e)_{\text{air}}$  being constant?
  - TRS-398 says 0.5% but evidence for any value is very thin

# Cross section uncertainties on $k_Q$

standard error propagation, assuming uncorrelated

$$u_{k_Q} = \left[ \sum_{i=1}^n \left( \frac{\partial k_Q}{\partial x_i} \right)^2 u^2(x_i) \right]^{\frac{1}{2}}$$

where  $u(x_i)$  is the uncertainty on cross section  $x_i$

Approximate

$$\left( \frac{\partial k_Q}{\partial x_i} \right) = \frac{\Delta k_Q}{\Delta x_i}$$

where  $\Delta k_Q$  is change in  $k_Q$  when cross section  $i$  is changed by  $\Delta x_i$ . Calculate  $\Delta k_Q$  for a  $\Delta x_i$  corresponding to  $u(x_i)$ .

$$u_{k_Q} = \left[ \sum_{i=1}^n (\Delta k_Q)_i^2 \right]^{\frac{1}{2}}$$

# Cross section uncertainties on $k_Q$ (cont)

- Calculating  $\Delta k_Q$  as a % change

$$\Delta k_Q = \left| \frac{R_{ch}(Co)}{R_{ch}(Q)} - 1 \right| \times 100\%$$

where  $R_{ch}$  is the **ratio** dose to gas calculated with a cross section **changed by its stated uncertainty** over that calculated with **standard cross section**.

Can use correlated sampling.

# Cross section uncertainties on $k_Q$ (cont)

Material	Mean Excitation Energy		Photon Cross-sec
	$\Delta I$ (%)	$\Delta k_Q$ (%)	$\Delta k_Q$ (%)
<u>Water (NE2571)</u>	1.5	0.03	0.55
C552 (Exradin A12)	5	0.30	0.53
Graphite Wall (NE2571)	4.5	0.19	0.29
PMMA (PTW 30013)	2	0.09	0.16
Air (NE2571)	2.5	0.03	0.02
Aluminum (NE2571)	0.5	0.00	0.01
POM/Delrin (IBA FC65P)	5	0.23	0.21
A150 (Exradin T2)	5	0.33	0.53
Steel Wire (IBA CC01)	5	0.01	0.05
SPC Electrode (Exradin A16)	5	0.03	0.02

# NE2571 $k_Q$ uncertainty components

Variable, $x_i$	$u(x_i)$ (%)	$\Delta(k_Q)_i$ (%)
<u>Mean Excitation Energy, I</u>		
Water	1.5	0.03
Air	2.5	0.03
Graphite Wall	4.5	0.19
Aluminum Electrode	0.5	0.00
<u>Photon Cross-sections</u>		
Water	1.0	0.55
Air	1.0	0.03
Graphite Wall	1.0	0.29
Aluminum Electrode	1.0	0.01
<u>All (Correlated)</u>	1.0	0.0

# NE2571 $k_Q$ uncertainties (cont)

<u>Other Sources</u>	$u(x_i)$ (%)	$\Delta(k_Q)_i$ (%)
Statistical Uncertainty	-	0.1
EGSnrc <sup>30</sup>	-	0.1
Wall Thickness	5.0	0.1
Cavity Dimensions	5.0	0.00
Source model	-	0.1
$\frac{W}{e}$	-	0.5
<u><math>u_{k_Q}</math></u>		
corr, no W/e	-	0.28
uncorr, no W/e	-	0.68
corr, with W/e	-	0.57
uncorr, with W/e	-	0.85

# Uncertainties on $k_Q$ for all chambers

Group (Wall/Electrode)	$u_{k_Q}$			
	corr no W/e	uncorr no W/e	corr with W/e	uncorr with W/e
a (C552/C552)	0.36	0.85	0.62	0.98
b (A150/A150)	0.39	0.86	0.63	0.99
c (Graphite/Al)	0.28	0.68	0.57	0.85
d (Graphite/Graphite)	0.28	0.68	0.57	0.85
e/i (PMMA+Graphite/Al)	0.31	0.71	0.58	0.86
f (POM/Al)	0.32	0.66	0.59	0.83
g (C552/SPC)	0.36	0.85	0.62	0.98
h (A150/SPC)	0.39	0.86	0.63	0.99

worst case: 0.39% 0.86% 0.63% 0.99%

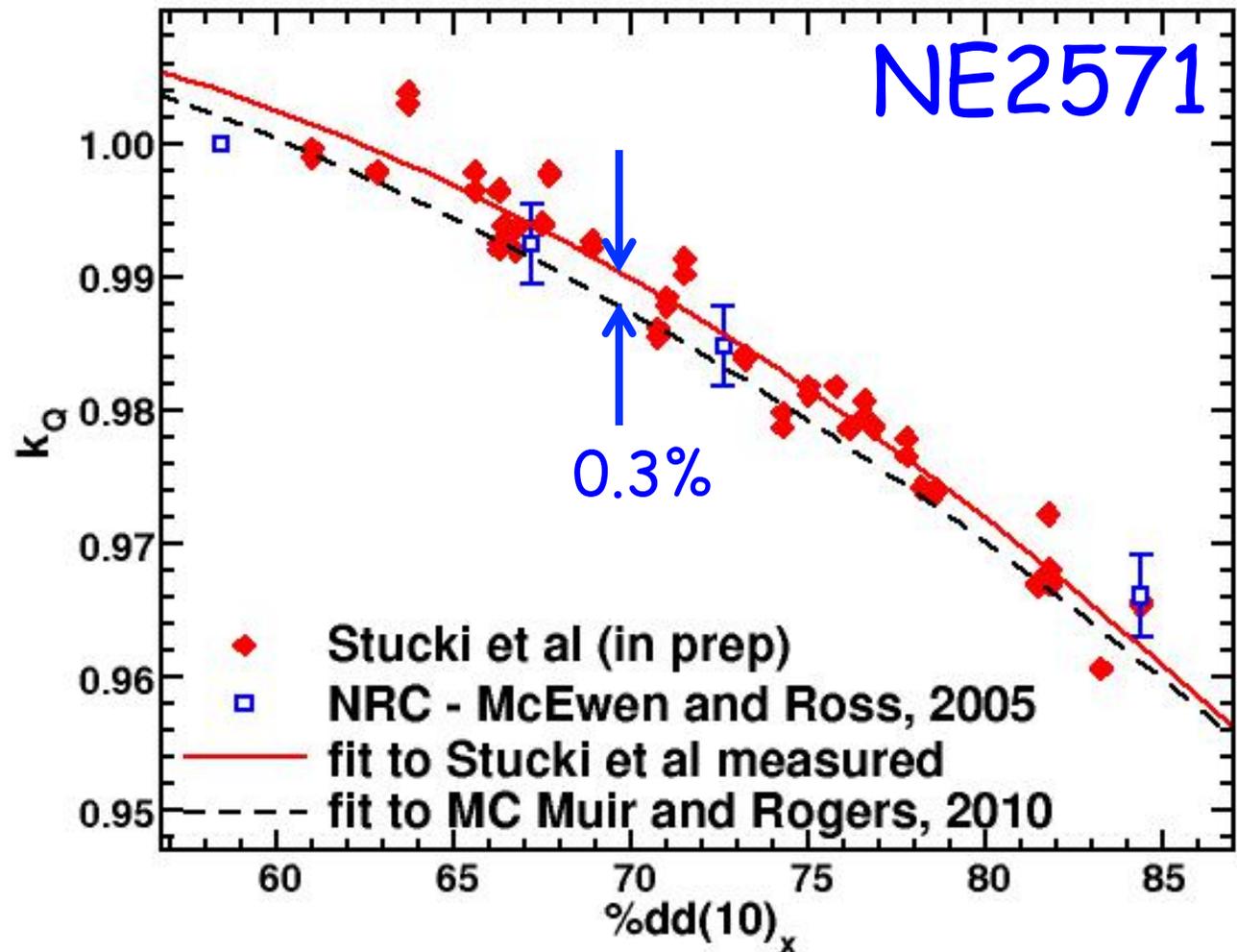
# Experimental measurements of $k_Q$

- many measurements done, but most papers measure one or two types of chambers
- **McEwen** measured  $k_Q$  for **27 different** types against the Canadian primary standards of absorbed dose---->  
(Med. Phys. 37 (2010) 2179)
- for “**well-behaved**” chambers  
measurement uncertainty on  $k_Q$  was **0.30%**
- **agreement with TG–51** values is excellent, typically **0.5%** or better for “**well-behaved**”

$$k_Q = \frac{N_{D,w}^Q}{N_{D,w}^{Co}}$$

# Consistency of measured $k_Q$

diamonds are from standards labs (Stucki et al, to be published)



# How well do calculations and measurements agree?

$$\Delta_i = \frac{k_{Q,i}(\text{calculated}) - k_{Q,i}(\text{measured})}{k_{Q,i}(\text{measured})} \times 100\%$$

$$\chi^2/df = \frac{1}{f} \sum_{i=1}^f \frac{\Delta_i^2}{s_m^2 + s_c^2}$$

For 26 chambers in common,

- $\chi^2/df < 0.65$  for all chambers at 1 energy

- $\chi^2/df < 1$  for all chambers vs energy except 1

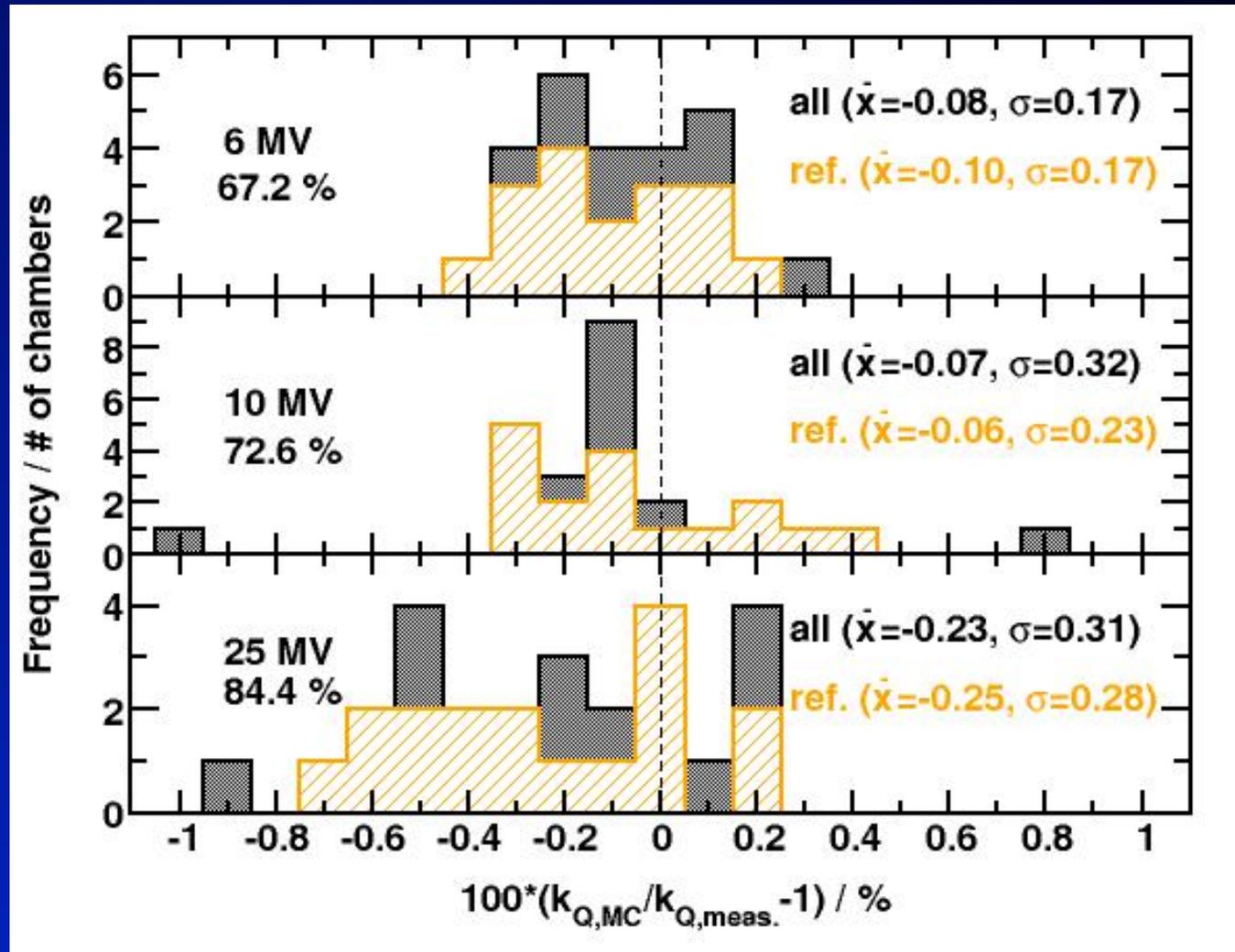
Suggests, if anything, **uncertainties are too large**

# Measured vs calculated $k_Q$

26 chambers  
in common

shaded part  
is less  
precise  
chambers

remarkable  
agreement



# Can we use this agreement to set a limit on the variation of $(W/e)_{air}$ ?

- assume some variation of  $(W/e)_{air}$

$$\alpha = \left( \frac{W}{e} \right)_{Co}^Q$$

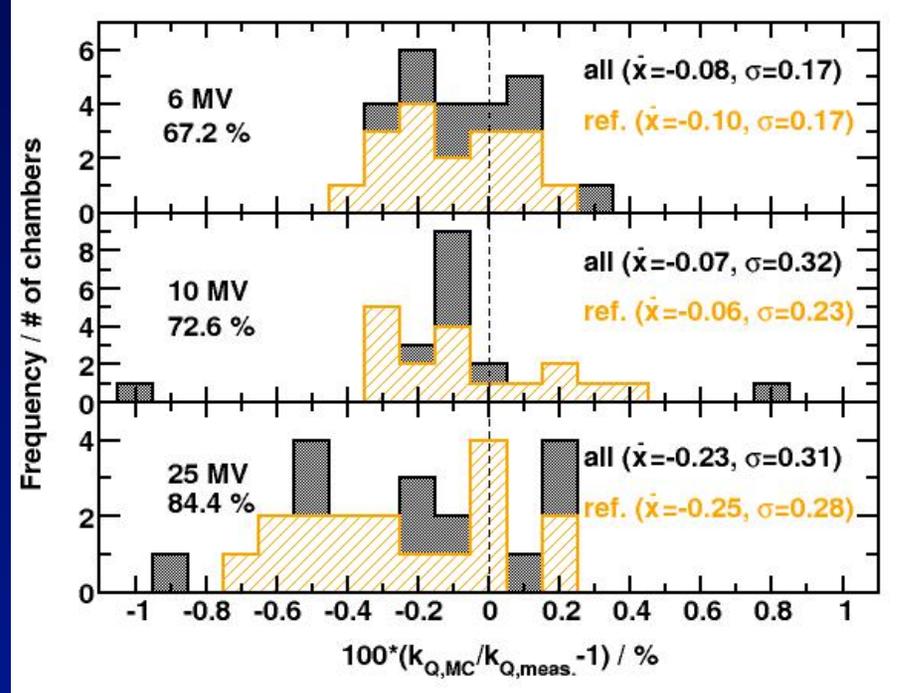
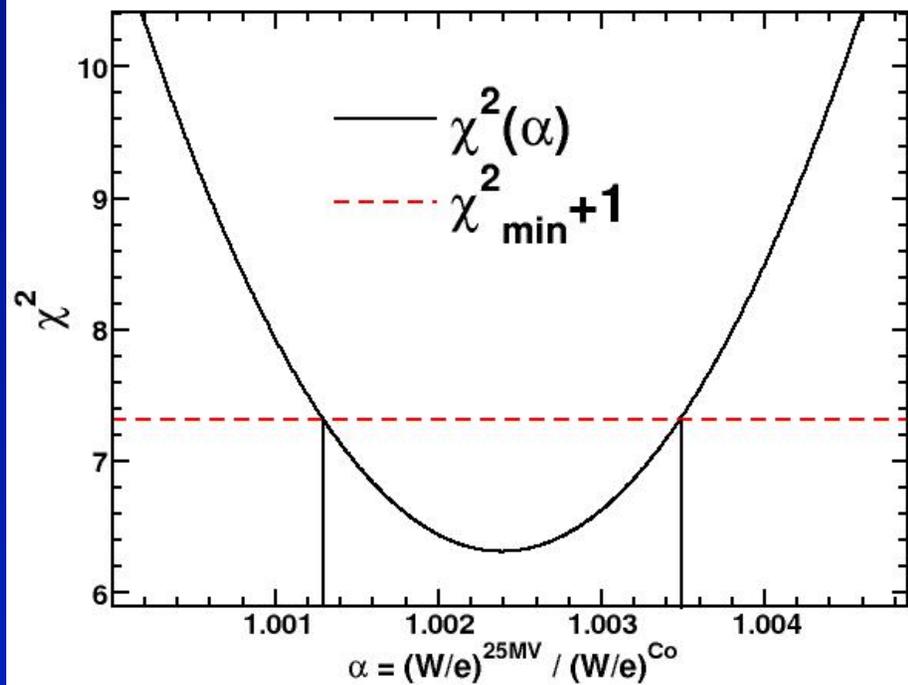
- in this case:  $k_Q = \alpha \left( \frac{D_w}{D_{ch}} \right)_{Co}^Q$

- now we have

$$\Delta_i = \frac{\alpha \times k_{Q,i}(\text{calculated}) - k_{Q,i}(\text{measured})}{k_{Q,i}(\text{measured})}$$

- calculate  $\chi^2$  as before as function of  $\alpha$

# What is the value of $\alpha$ ?



$$\alpha = 1.0024 \pm 0.0011$$

Conservatively one can say

$W/e$  is constant within 0.29% (0.42%) with 68% (95%) confidence

$$\alpha = \left( \frac{W}{e} \right)_{Co}^Q$$

# Conclusions

- Monte Carlo **calculations** of  $k_Q$  are **feasible**
- experimental **agreement** is **exceptional**
  - 0.13 % mean difference for 26 chambers
  - **0.31 % RMS deviation** for 26 chambers
- uncertainty on **variation of  $(W/e)_{\text{air}}$**  from Co to 25 MV is 0.29 % (68 % limit)

## Conclusions (cont)

- uncertainty on calculated  $k_Q$  values is between 0.40 and 0.49 % depending on wall material
- individual chambers appear **representative** else agreement could not be so good
- results apply **only to filtered beams**
  - with low-Z electrodes, results still apply
  - with  $Z > 13$  electrodes, values will not hold in FFF beams (OK in WFF beams if reference quality)

# Acknowledgements

- as mentioned before, much of the work was done by Bryan Muir
- Malcolm McEwen
- work supported by an NSERC CGS, OGSSTs, the CRC program, an NSERC DG, CFI and OIT

## Thanks for your attention

