IMRT Optimization Based on Physical Criteria

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Motivation

3D conformal Rx
Intensity Modulation

Outline

1. Physical optimization criteria
2. The objective function
3. Variables to be optimized
4. Optimization algorithm: An example

1. Physical optimization criteria

1.1 Clinically relevant criteria
1.1.1 Deviation from prescribed dose
1.1.2 Target dose homogeneity issues
1.1.3 Min/Max dose
1.1.4 Mean dose
1.1.5 DVH criteria
1.1.6 EUD

1.2 Physical and technical criteria
1.2.1 Non-negativity of intensities
1.2.2 Deliverability, smoothness of IM maps
1.1.1 Deviation from prescribed dose

Least squares deviation

\[ F(x) = \sum_{i=1}^{N} (d_i - y_i)^2 \]

Clinically this is NOT very useful!

\[ F_x(h) = \sum_{i=1}^{N} (d_i(h) - d_i)^2 \]

"Positivity operator": \[ [a]_+ = \begin{cases} a & \text{for } a \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

1.1.2 Target dose homogeneity issues

- IMRT does not necessarily lead to bigger target dose inhomogeneity
- For complex cases, IMRT is the only way to make target dose uniform
- In practice, IMRT target dose distributions are often less homogeneous than 3DCRT
  - Margins
  - Number of beams
  - Tradeoff in critical structures
  - Objective function

1.1.3 Min/Max doses

- Soft constraint
- Hard constraint
1.1.3 Min/Max doses – Example: Clivus Chordoma

Non-intensity-modulated (4 beams, non-coplanar, MLC) vs. Intensity-modulated (9 beams, coplanar)

Brainstem

Target

1.1.3 Min/Max doses – Example: Clivus Chordoma

Non-intensity-modulated (4 beams, non-coplanar, MLC) vs. Intensity-modulated (9 beams, coplanar)

Brainstem

NTCP = 7%

NTCP = 0.7%

1.1.3 Min/Max doses – Example: Thyroid

Technique: 9 beams, coplanar, intensity-modulated

Lungs

Spinal cord

Target volume

Transversal view

Target

Lung

Lung

Spinal cord

1.1.4 Mean dose

The mean dose is a good descriptor of the dose effect in parallel critical structures such as lung.


1.1.5 Dose-volume criteria
1.1.5 Dose-volume criteria

*No more than 1/3 of the lung should get more than 15 Gy.*

$$DVH_k(D^\text{max}_k) \leq V^\text{max}_k$$

1.1 Physical criteria in clinical protocols:

RTOG-022 oropharyngeal cancer

Target dose prescription (homogeneity):

- No more than 1% of planning target volume (PTV) can receive less than 93% of prescription.
- No more than 20% of the PTV can receive greater than 110% of prescription.
- No more than 1% or 1cc of tissue outside of PTVs will receive greater than 110% of prescription dose to primary PTV.

Critical Structures dose limits:

- Brainstem: 54 Gy
- Cord (+ 5mm): 45 Gy
- Mandible: 70 Gy

Parotids (3 options):

- Mean dose to either parotid < 26 Gy or
- At least 50% of either parotid gland < 30 Gy or
- At least 20 cc of the combined parotid volume will receive < 22 Gy.
1.1.5 EUD

- The equivalent uniform dose (EUD) is the uniform dose that causes the same effect as the given non-uniform dose.

Power-law relationship for tolerance dose (TD):

\[ TD(v) = \frac{TDU}{v^a} \]

Brahme et al., Acta Radiol. Oncol. 23(5), 379-391, 1984
Kwa et al., Radiother. Oncol. 48(1), 61-69, 1998
Niemierko, Med. Phys. 26(6), 1100, 1999

Examples:

- \( a = 1 \): EUD = \( \bar{D} \)
- \( a = \infty \): EUD = \( D_{\text{max}} \)

Use generalized Equivalent Uniform Dose (EUD) with negative \( a \) also for target volumes.


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- 1.1.5 DVH criteria
- 1.1.6 EUD

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- 1.2.1 Non-negativity of intensities
- 1.2.2 Deliverability, smoothness of IM maps
1.2.1 Non-negativity of intensities – The CT analogy

CT Imaging

IMRT Treatment

1.2.2 Deliverability – smoothness of intensity maps

Original

After smoothing

Dose distributions are almost identical

1.2.2 Deliverability – smoothness of intensity maps

- Include smoothness term in objective function
- Apply smoothing filter during optimization

2. The objective function

2.1 Combination of criteria using weights
2.2 Constraints, feasibility search
2.3 Mono-criteria vs. multi-criteria
2.4 A new multi-criteria optimization concept

2.1 Combination of criteria using weights - Target

\[
F_T(\vec{b}) = \sum_{i=1}^{N} \left( a \left( D_{\text{min}} - d_i(\vec{b}) \right)^2 + \right.
\left. w \left[ d_i(\vec{b}) - D_{\text{max}} \right] \right)
\]

"Positivity operator": \([x] = \begin{cases} 
  x & \text{for } x \geq 0 \\
  0 & \text{otherwise}
\end{cases}\)

- This is not the only or the "best" measure of deviations from prescription or tolerance.
- For example, instead of the quadratic deviation, higher powers (4, 6, ...) have been suggested (see the work from Ann Arbor, Fraass et al 2002)
2.1 Combination of criteria using weights - OAR

Max dose

Volume

penalize

Dose

Dmax

Max dose

“Penalty” function:

\[
F_{R,k}(b) = w_k \sum_{i=1}^{N,k} \left[ d_{R,k}(i) - D_{max,i} \right]
\]

Penalty

dose at

voxel \( i \)

in OAR \( k \)
tolerance
dose

“Positivity operator”:

\[
[x] = \begin{cases} 
  x & \text{for } x \geq 0 \\
  0 & \text{otherwise}
\end{cases}
\]

EUD criteria

Volume

EUD(D) > EUDmax

Dose

EUD criteria

C. Thieke et al., Med. Phys, in print

DVH criteria

Volume

penalize

Vmax

Dmax

Dose

Helios, BrainLab, KonRad, ...

Optimize (minimize)

\[
F = w_{Target} F_{Target} + w_{Risk1} F_{Risk1} + w_{Risk2} F_{Risk2} + \ldots
\]
2. The objective function

2.1 Combination of criteria using weights
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2.3 Mono-criteria vs. multi-criteria
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2.2 Constraints, feasibility search

- Feasibility search: Find a plan that fulfills all constraints — no optimization
- See previous presentation by Yair Censor
2. The objective function

2.1 Combination of criteria using weights
2.2 Constraints, feasibility search
2.3 Mono-criteria vs. multi-criteria
2.4 A new multi-criteria optimization concept

Problems with weights (penalties) and constraints

Optimization using weights
- Trial and error to determine weights
- “Sensitivity” of the solution

Constrained optimization
- Non-efficient solutions

\[ F = w_{Target} F_{Target} + w_{Risk1} F_{Risk1} + w_{Risk2} F_{Risk2} + \ldots \]

\( F \) is a single number!

Example: Head&Neck

- Brainstem
- Parotis
- Spinal Cord

Problems with weights (penalties) and constraints

Optimization using weights
- Trial and error to determine weights
- “Sensitivity” of the solution

Constrained optimization
- Non-efficient solutions

![Graph showing dose-volume histograms for different plans and weights.](Image)

Plan 1
- Plan 2

- Target
- Spinal Cord
- Parotis

- \( w=1 \)
- \( w=10000 \)
Problems with weights (penalties) and constraints

Optimization using weights
- Trial and error to determine weights
- “Sensitivity” of the solution

Constrained optimization
- Non-efficient solutions

Change “penalties” or “weight factors”

2.3 Mono-criteria vs. multi-criteria

- Radiotherapy planning (and IMRT planning) is a multi-criterial problem
- It always involves tradeoffs, one can never optimize all criteria at the same time
- A treatment plan cannot be scored with a single “grade”

Optimization for multi-criteria problems “efficiency”, Pareto-optimality

Pareto optimization

Vilfredo Pareto, 1848-1923
Italian-Swiss socio-economist
Pareto-optimality, “efficient”
You cannot make anybody better off without making someone else worse off.
Equilibrium
A Pareto-optimal or "efficient" treatment plan is a plan in which we cannot improve one aspect (e.g., reduce the dose in one OAR) without compromising at least one other aspect (e.g., reduce the target dose).

Towards multi-criteria optimization:
- Head & Neck example
- Hard constraint: target EUD = 60 Gy, homogeneity criteria
- Vary EUD constraints for Parotis and spinal cord from 0 Gy to 30 Gy in steps of 2 Gy
- 17 hours for 256 plans

Outline:
1. Physical optimization criteria
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3. Variables to be optimized
4. Optimization algorithm: An example
3. Variables to be optimized

- Intensity profiles
- MLC shapes, segment weights
- Number of beams
- Beam angles (gantry angle, table angle)
- Number of intensity levels
- Energy (mainly in charged particle therapy)
- Type of radiation (photons, electrons, ...)
- ...
3. Variables to be optimized

- Beam angles, number of beams

  - If many beams (≥ 9) are used, the selection of beam angles is uncritical:
    - Evenly spaced coplanar beams can be used

  - For smaller numbers of beams,
    - Careful selection (optimization) of beam angles can lead to improvements

  - The number is case-dependent
    - Pugachev et al., "Role of beam orientation optimization in IMRT" IJROBP 50(2): 551-60, 2001

Number of intensity levels


Number of intensity levels, Example: Clivus chordoma

<table>
<thead>
<tr>
<th>Beam 1 of 7 (0°)</th>
<th>7 Beams, Target</th>
<th>7 Beams, Brainstem</th>
</tr>
</thead>
<tbody>
<tr>
<td>continuous</td>
<td>5 levels</td>
<td>continuous</td>
</tr>
<tr>
<td>5 levels</td>
<td>3 levels</td>
<td>5 levels</td>
</tr>
<tr>
<td>3 levels</td>
<td>2 levels</td>
<td>3 levels</td>
</tr>
<tr>
<td>2 levels</td>
<td>continuous</td>
<td>2 levels</td>
</tr>
</tbody>
</table>

Energy

- Energy plays a surprisingly small role in IMRT. 6MV is a good choice for many cases.

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Gradient technique

\[ F(b) = \sum_{i=1}^{N} w_i [d_i(b) - P_i]^2 \]

\[ d_i(b) = \sum_j D_{ij} b_j \]

\[ D_{ij} : \text{dose contribution of pencil beam } j \text{ to voxel } i \]

Gradient:

\[ \frac{\partial F}{\partial b_j} = 2 \sum_{i=1}^{N} w_i (d_i(P_i) - P_i) D_{ij} \]

2nd derivative:

\[ \frac{\partial^2 F}{\partial b_j^2} = 2 \sum_{i=1}^{N} w_i D_{ij}^2 \]

Algorithm (Newton-like):

\[
\begin{align*}
b_{j+1} &= b_j' - \alpha \frac{\partial F}{\partial b_j} \\
&= b_j' - \alpha \left[ \sum_{i=1}^{N} w_i (P_i - d_i') D_{ij} \right] \\
&= b_j' + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i') D_{ij}^2}{\sum_{i=1}^{N} w_i D_{ij}^2}
\end{align*}
\]

\[ \alpha = 1/N \]

4. Optimization algorithm: An example

A very simple example:

Dose distribution:

\[ D_{ij} = \begin{cases} 1 & \text{if pencil beam (ray) } j \text{ hits voxel } i \\ 0 & \text{otherwise} \end{cases} \]

Initial values:

\[ b_i^0 = 0 \]

\[ d_i^0 = 0 \]
Optimization
Iteration 1

\[ b_i' = b_i' + \alpha \left( \frac{\sum w_i (P_i \cdot d_i)^2}{\sum w_i D_i^2} \right) \]
Optimization Iteration 2

\[ b^\alpha = \left[ b^\alpha + \alpha \frac{\sum \nu_i (P_i - d_i^2) D_i}{\sum \nu_i D_i^2} \right] \]

Saves for the region at risk. Here the difference is 0.5.

Reading correction matrix.
$b^o = \frac{\sum_i w_i (P_i - d_i) D_i}{\sum_i w_i D_i^2}$

$\alpha = \sum_i D_i$
Optimization
Iteration 3

\[ b' = b + \alpha \left( \sum_{i} n_i (P_i - d_i) \Delta_0 \right) / \sum n_i \phi_i \]

Repeat the steps of evaluation and optimization until an acceptable treatment plan is found.

4. Optimization algorithms

- Projecting back and forth between dose distribution and intensity maps

Summary

1. Physical optimization criteria
2. Objective functions
3. Variables to be optimized
4. Optimization algorithm: An example