Optimization Techniques and their Suitability and Robustness for IMRT Planning

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Overview of Talk
- Optimization approaches for intensity-modulated radiation therapy
  - Modeling approaches (MIP versus nonlinear)
- Management of solution spaces
- Clinical Comparisons
- Discussion

Size of Treatment Models
- Beam Geometry
  - Number of input candidate beams – 0/1 variables
  - Couch angles, collimator sizes, energy, isocenters
  - Beamlet intensity – continuous variables
- Constraints/Objectives
  - Planning target volume (PTV), Organs-at-risks (OARs) in discretized representation
  - Clinical constraints (dose-based, dose-volume, spatial, EUD, biological TCP, NTCP)
- Uncertainties
  - Estimate in dose calculation
  - Uncertainty in voxel positions (Time-Dependent Planning)
  - Delivery parameters (Setup errors)

Treatment Models and Tractability
- Polynomial-time
  - Convex
    - Linear
    - Nonlinear
- NP-Hard
  - Non-linear
  - Discrete/combinatorial Optimization
    - (Linear/nonlinear mixed integer programming)
Treatment Models and Tractability

- Convex
  - Linear
  - Nonlinear
- Non-convex
  - Nonlinear
  - Discrete/combinatorial optimization (linear/nonlinear mixed integer programming)

Some Algorithms
- Exact Algorithms
  - Simplex
  - Gradient-based/interior-point based/exact-type methods
- Heuristic Algorithms
  - Simulated annealing
  - Genetic algorithms
  - Evolutionary-genetic algorithms

Constraint Types
- Dose-based
- Fluence intensity
- DVHs
- Coverage
- Beam orientation
- Couch angles, energy type
- EUD, Biological TCP/NTCP

Issues to Expand in this Talk
- Selection of an optimization method and the associated “steering” objectives for solution procedure
- DVH objectives – discrete vs continuous models
- Multiple-objective IMRT treatment planning

Methods
- In-house modeling (CAMELA, Lee, Cha, Shi ’01–’03) and optimization modules (MIPSOL, Lee, ’97–’03) developed based on (linear and nonlinear) mixed integer programming
- Treatment planning models incorporate multiple objectives, dose constraints, spatial DVHs, PTV coverage, homogeneity (underdose/overdose), and a large collection of candidate beam orientations.
- Compare MIP approach versus nonlinear programming approach with (DVHs) as the steering objective.
- Compare two multiobjective strategies for IMRT planning with 4 steering objectives: PTV Homogeneity, min OAR doses, PTV Conformity, OAR-DVHs
- Analyze plans based on clinical metrics: coverage, conformity, homogeneity, DVHs and isodose curves
Treatment Planning Model Variables

- Continuous variables to represent intensity for each beamlet
- Continuous variables to measure dose discrepancy of each voxel from desired dose bounds, and 0/1 variable to record if voxel meets certain dose bounds (PTV coverage/over/underdose, OARs’ DVHs)
- 0/1 variables to indicate, respectively, the use of beam directions, couch angle, energies, and collimator size

Modeling of DVHs: MIP/discrete approach versus nonlinear programming approach

- Imposed Constraints:
  - Upper/lower/mean dose-based constraints, spatial DVHs, homogeneity constraints (PTV Underdose / Overdose), PTV coverage, maximum number of beams/angles allowed
- Steering objective:
  - Optimize DVHs (maximizing volume versus penalizing total dose deviation)

A Head-and-neck Case

Observations

- Little change in PTV coverage, dose homogeneity, and DVH
- DVHs of OARs change slightly with spatial information
  - Lower dose to brainstem and spinal cord
  - Slightly higher dose to left parotid
  - Max dose remains the same for OARs
  - Mean dose difference is within 0.5% of each other

Nonlinear NLP With spatial information
- Significant lower dose to brainstem and spinal cord, drastic reduction in max/mean/min dose
- Slightly lower dose to right parotid
- Slight higher dose to left parotid

Nonlinear NLP without spatial information
- Little change in PTV coverage, dose homogeneity, and DVH
- DVHs of OARs change more significantly with spatial information
Modeling of Multiple-Objectives
MIP/discrete nonlinear approach
Comparison –
4-stage preemptive versus goal programming

• Imposed Constraints:
  – Upper/lower/mean dose-based constraints, spatial DVHs, homogeneity constraints (PTV Underdose / Overdose), PTV coverage, maximum number of beams/angles allowed

• Steering objectives:
  PTV Homogeneity, min total OAR doses, PTV Conformity, OAR-DVHs

Observations
– Compatible PTV coverage, DVHs and homogeneity
– OARs (some tradeoffs in DVHs)
– Lower dose to brainstem, spinal cord, left parotid and normal tissue in goal programming
Table 1: Dose Statistics from Optimal Plans Obtained by 4-stage preemptive programming vs Goal Programming

<table>
<thead>
<tr>
<th>PTV</th>
<th>4-stage preemptive programming</th>
<th>Goal programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coverage</td>
<td>Homogeneity</td>
<td>Mean Dose</td>
</tr>
<tr>
<td>min (Gy)</td>
<td>max (Gy)</td>
<td>min (Gy)</td>
</tr>
<tr>
<td>0.962</td>
<td>1.21</td>
<td>1.103</td>
</tr>
<tr>
<td>0.964</td>
<td>1.19</td>
<td>1.114</td>
</tr>
</tbody>
</table>

Table 2: Dose Statistics from Optimal Plans Obtained by 4-stage preemptive programming vs Goal Programming

<table>
<thead>
<tr>
<th>Structure</th>
<th>Goal Programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol (cm³)</td>
<td>10.7</td>
</tr>
<tr>
<td>V30</td>
<td>23.6%</td>
</tr>
<tr>
<td>V40</td>
<td>8.9%</td>
</tr>
<tr>
<td>V50</td>
<td>0.7%</td>
</tr>
<tr>
<td>mean dose</td>
<td>12.8</td>
</tr>
<tr>
<td>min dose</td>
<td>0.2</td>
</tr>
</tbody>
</table>

4-stage Preemptive Programming vs Goal Programming

4 Objectives: PTV Homogeneity, min OAR doses, PTV Conformity, OAR-DVHs

Observations

- More homogeneous PTV dose from 4-stage preemptive programming
- OARs
  - Lower dose to rectum, and bladder in goal programming
  - Slightly higher normal tissue max dose for goal programming plan

A Head-and-Neck Case

A Prostate Case

Observations

- Under the same solution space with explicit constraints, in the treatment model:
  - Upper/lower/mean dose-based constraints, spatial DVH constraints, Homogeneity constraints (PTV Underdose/Overdose), PTV Coverage constraints, OAR EUDs, maximum number of beams/angles allowed
- With DVH-driven objective:
  - 1) maximize volume of OARs satisfying DVHs (MIP) versus
  - 2) Quadratic dose difference penalization (NLP)
- Findings:
  - NLP optimal plans are very sensitive to incorporation of spatial information within DVH-driven objective: In both head-and-neck and prostate cases, including spatial information drastically improves the dose-distribution in OARs while PTV maintains homogeneity and good coverage
  - MIP optimal plans are not as sensitive to spatial information
  - MIP plans are superior to NLP plans in both tumor sites
- Tradeoffs – MIP is NP-complete, there is no known polynomial-time algorithm, hence proven-optimal solution time can be very significant.
- The type of models dictate the optimization algorithms used, and vice versa
Observations

- Under the same solution space with explicit constraints in the treatment model:
  Upper/lower/mean dose-based constraints, spatial DVH constraints, Homogeneity constraints (PTV Underdose/Overdose), PTV Coverage constraints, OAR EUDs, maximum number of beams/angles allowed
- With 4 clinical objectives: PTV Homogeneity, min total OAR doses, PTV Conformity, OAR-DVHs, contrast multi-stage preemptive programming versus goal programming
- Findings:
  - Both approaches arrive at Pareto optimal points on the efficient frontier, and result in plans with improvement over single objective plans
  - Tradeoffs occur in PTV homogeneity versus OARs dose improvement
  - More detailed discussion on multiple objective IMRT planning on Tuesday: TU-C-BRA-8 (11:10 am, Ballroom A)

Related Manuscripts

Recent Manuscripts (materials of this talk were extracted from these two papers)


Previous work


Modeling and Computational Engines

E.K. Lee (1997), Computational Experience with a General Purpose Mixed 0/1 Integer Programming Solver (MIPSOL), Software Report, School of Industrial and Systems Engineering, Georgia Institute of Technology.


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