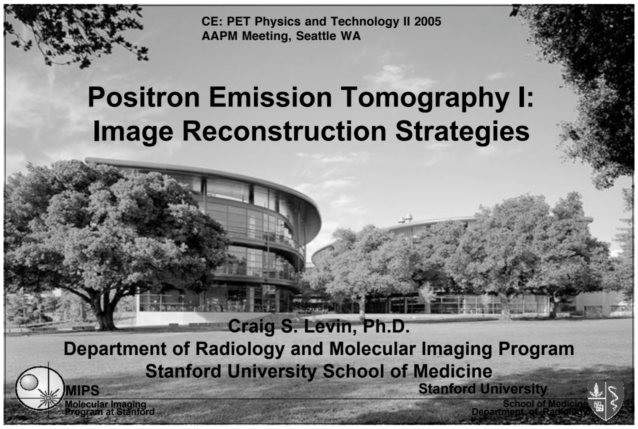


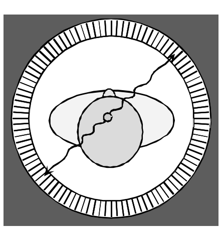
Outline

I. Brief Introduction to PET
II. Organization of PET data
III. Image Reconstruction Methods for PET

<u>NOTE</u>: TOPICS DISCUSSED ARE SUBJECTS OF ACTIVE RESEARCH - HERE WE DESCRIBE SOME OF THE ALGORITHMS CURRENTLY IMPLEMENTED IN COMMERCIAL CLINICAL SYSTEMS.

What is Positron Emission Tomography? (PET)

PET is a Nuclear Medicine tomographic imaging technique that uses a tracer compound labeled with a radionuclide that is a positron emitter. The resulting radio-emissions are imaged.



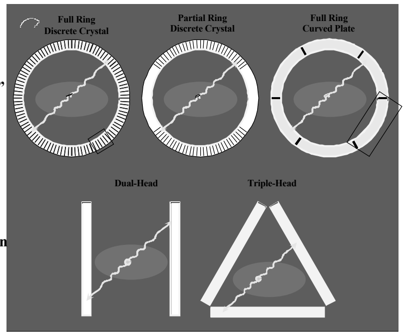
RESULT

- **Cross-sectional image slices representing regional uptake of the radio-chemical**
- **Quantitative information in absolute units of $\mu Ci/cm^3$ or in terms of actual rates of biological processes that utilize or incorporate that chemical**

What Does a PET Scanner Look Like ?

"Dedicated"

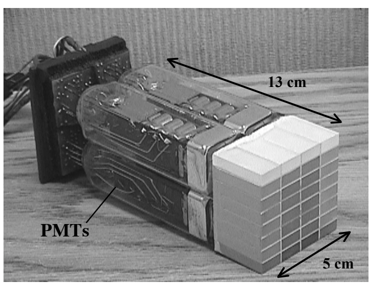
"Scintillation Cameras"



PET Annihilation Photon Detectors

"Block Detector"

Example



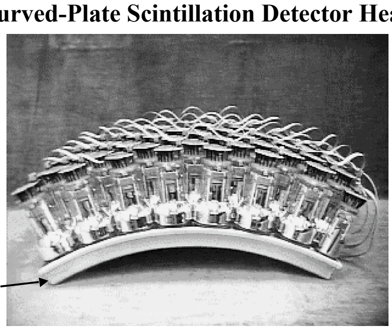
From Siemens/CTI "ECAT 931" (BGO)

PET Detectors - ctd.

"Curved-Plate Scintillation Detector Head"

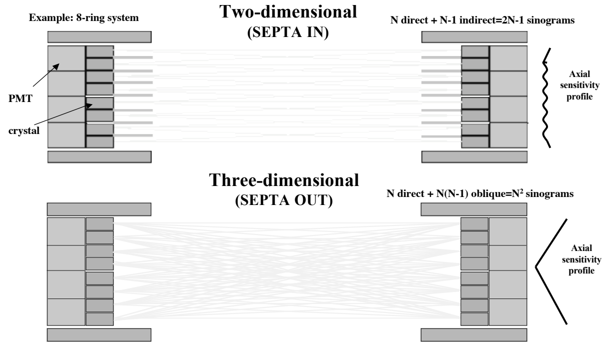
Example

Scintillator: Sodium Iodide NaI(Tl)

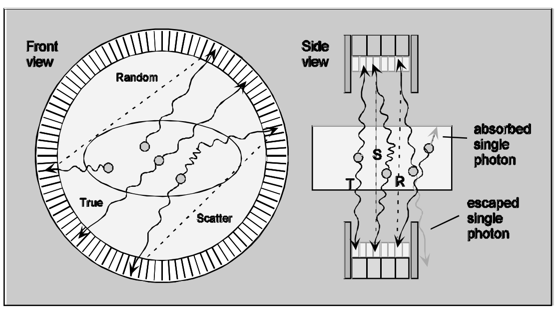


From Philips-ADAC "C-PET" (NaI(Tl)), Courtesy of J. Karp, UPENN

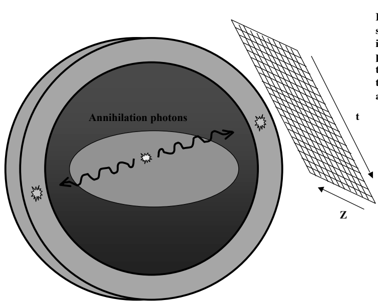
PET Data Collection Modes



Types of Coincident Photon Events



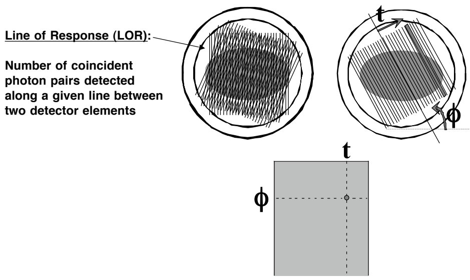
Organization of PET Data



In modern 3-D PET systems data is acquired into a set of planar projections parallel to the z-axis for each transaxial projection angle.

Organization of Tomographic Data

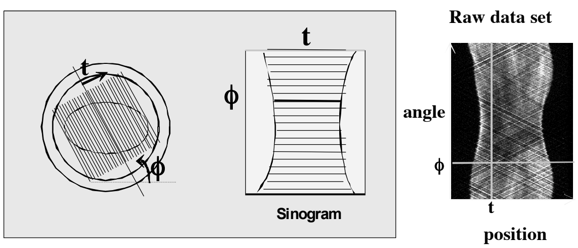
For reconstruction the tomographic projection data is often organized into a “sinogram”



Sinogram

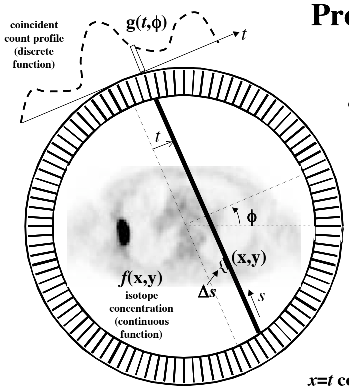
Organization of Tomographic Data

Sinogram = Stacked profiles or “projections” from all angles



PET Image Reconstruction Algorithms

Projection Operator



"Radon Transform"

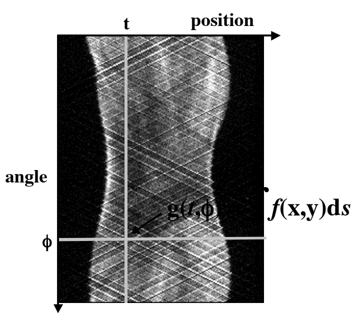
$$g(t,\phi) = \int_{FOV} f(x,y)\,ds$$

"Ray-Sum" **Integrate all coincident photon activity along a given line-of-response:**

$t = x\cos\phi + y\sin\phi$

$x = t\cos\phi - s\sin\phi$, $y = t\sin\phi + s\cos\phi$

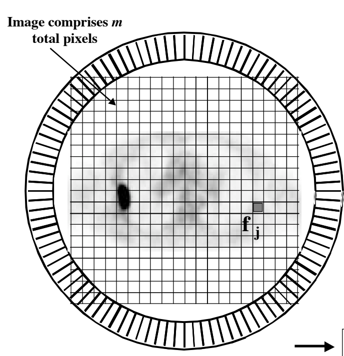
Sinogram



The "sinogram" is the set of all $g(t,\phi)$ for all possible t,ϕ.

Fourier reconstruction methods assume ideal line integrals so all corrections should be applied to the data.

System Matrix (*A*): "The projection operator"



In reality both $f(x,y)$ (the image to reconstruct) and $g(t,\phi)$ (the sinogram) are discrete functions.

$f(x,y) \dashrightarrow f_j$ (image pixel values)

$g(t,\phi) \dashrightarrow g_i$ (sinogram bin values)

$g(t,\phi) = \int f(x,y)\,ds \dashrightarrow$

$g_i = \sum_{j=1}^{m} a_{ij} f_j$ (discrete formula for the projection operation)

$\longrightarrow \boxed{g = A f}$ (a matrix product)

Discrete Version of the Projection Operation

$$g_i = \Sigma\, \mathrm{a}_{ij} f_j \quad or \quad g = A f$$

a_{ij} is the weighting factor representing the contribution of image pixel *j* to the number of counts detected in sinogram bin *i*, or equivalently, the probability that a photon emitted from image pixel *j* is detected in sinogram bin *i*.

$$\mathrm{a}_{ij} \in [0,1]$$

A variable fraction of an image pixel *j* contributes to the counts detected in a given sinogram bin *i*, depending upon the relative positions of *i* and *j* and ϕ and on physical effects such as photon attenuation and scatter.

For later discussion of iterative algorithms

Note: By choosing appropriate values for the a_{ij} 's accurate modeling of physical phenomena such as photon attenuation or scatter can be included during the reconstruction process. The capability to accurately account for physical factors and the system model (perhaps non-ideal) is one of the advantages of the iterative approaches.

Definition: The matrix A is said to be "*ill-conditioned*" when small changes in sinogram data *g*, such as from statistical noise may produce large differences in *f*. In that case, the solution $f = A^{-1} g$ is difficult to determine. (A^{-1} may not exist or may not be unique either, and such "direct inversion" is computationally intensive).

Reconstruction Algorithms for 2-D PET (that we will cover)

- Filtered Back-Projection (FBP) (Analytical)
- Ordered Sets Expectation Maximization (OSEM) (Iterative)

Analytical Reconstruction Methods in 2-D

Back-Projection Operator

Backprojection Operator:

Number of projections

$$bp(x,y) = \int_{\phi=0}^{\pi} g(t,\phi)d\phi \;\text{---->}\; bp(x,y) = \sum_{k=0}^{p} g(t_k, \phi_k)\Delta\phi$$

$t = x \cos \phi + y \sin \phi$

Discrete form of backprojection operator

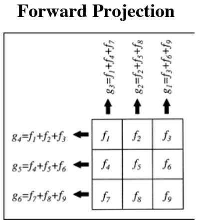
$\Delta\phi = \pi/p$

For a given location (x,y) in the image add the value $g(t_k, \phi_k)$ to the current value. Repeat the process for all projection angles and divide by the number of projection angles, *p*. Represents the accumulation of the ray-sums of all rays that pass through any given point (x,y).

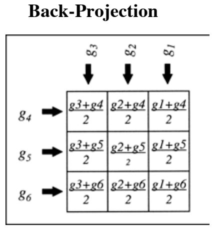
Back-Projection Process

Example: p=2 projection angles

Forward Projection



Back-Projection



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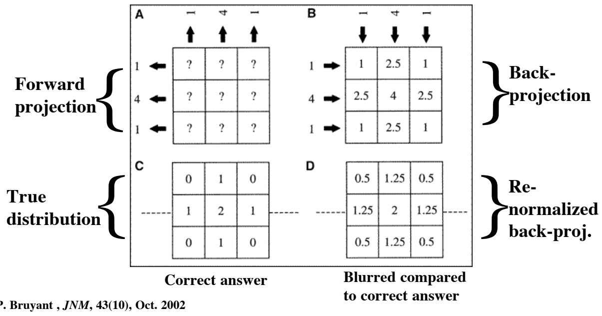
Projection/Back-Projection Process

Example: p=2 projection angles



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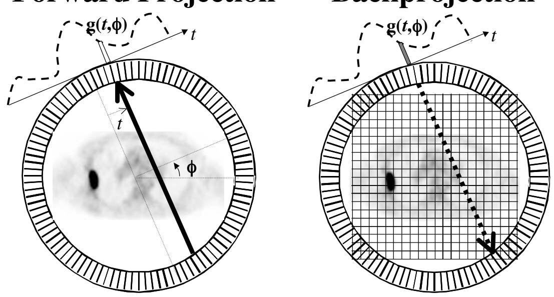
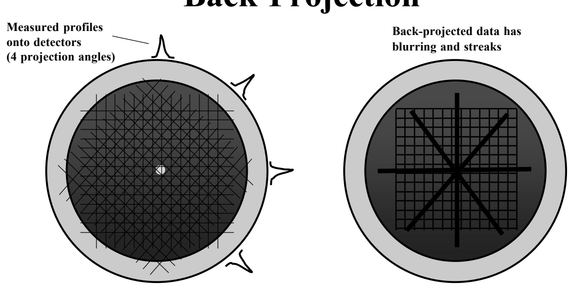
Forward Projection Backprojection



The back-projection operation is not the inverse of the projection operation:

$$bp(x,y) = \sum g(t_k, \phi_k)\Delta\phi \neq f(x,y) \text{ ---> blurred } f(x,y)$$

Back-Projection

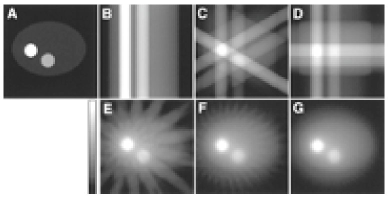


Limited Angular Sampling

When p (the number of angular projections) < m (the matrix size) ----> <u>streak artifact</u>

1 projection angle 3 4

$f(x,y)$



No streaks but $f(x,y)$ is blurred due to backprojection process

16 32 64

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Filtered Back-Projection Algorithm

Instead of:

$$bp(x,y) = \int g(t,\phi)d\phi, \text{ where } bp(x,y) \neq f(x,y)$$

Backproject filtered sinogram:

$$fbp(x,y) = \int F^{-1}\{F[g(t,\phi)]\cdot w\}d\phi ,$$

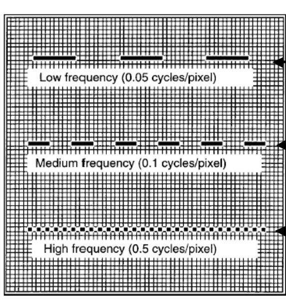
where,
F is the Fourier Transform (FT) operator, tranforms g into the frequency domain;
F^{-1} is the inverse FT operation;
w is the spatial frequency filter weight function

or in discrete form:

$$fbp(x,y) = \sum \mathrm{F}^{-1}\{\mathrm{F}[\mathrm{g}(t_k, \phi_k)]\cdot w\}\Delta\phi \approx f(x,y)$$

The Concept of "Spatial Frequency"

Rectangles that regularly repeat themselves in the horizontal direction



Repeats itself every 20 pixels

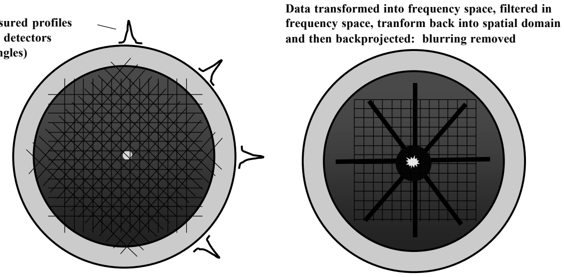
Repeats itself every 10 pixels

Repeats itself every 2 pixels

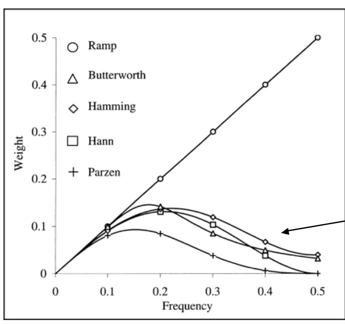
"Nyquist" frequency = 0.5 cycles/pixels
the highest frequency a system can see

Image Reconstruction

Filtered Back-Projection (FBP)



Spatial Frequency Filter Function (*w*)



$$\text{Butterworth}(f) = \frac{1}{1 + \left(\frac{f}{f_c}\right)^{2n}}$$

f_c = cutoff frequency

n= order of filter

De-emphasize high spatial frequency components to suppress image noise: a “low pass filter” in frequency domain

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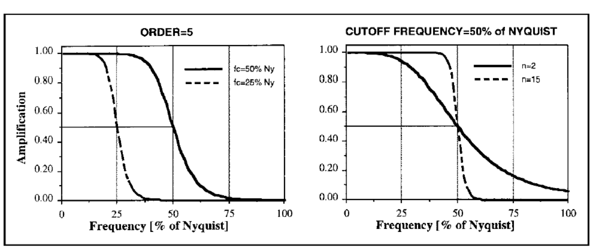
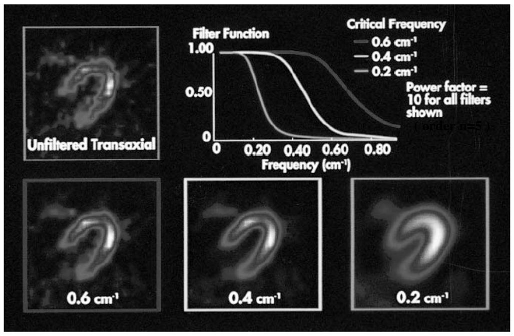
Order and Cutoff Frequency of Filter Function

Same filter order but different cutoff frequencies | **Same cutoff but different orders**



Filter cutoff controls location of midpoint of slope | **Filter order controls its slope to zero**

Effect of Frequency Filtering on Reconstructed Image



Lower critical frequencies correspond to more smoothing

Filter Function in Frequency and Spatial Domains

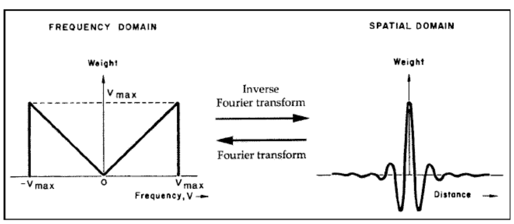
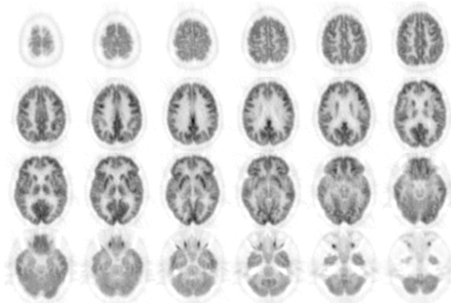


Image Reconstruction

Filtered Back-Projection Steps

1. **Collect projection data and apply corrections (attenuation, scatter, deadtime, detector efficiency, randoms)**
2. **1st row of sinogram (projection angle 0)**
3. **Take Fourier transform**
4. **Multiply by frequency filter**
5. **Take inverse Fourier transform**
6. **Backproject into image matrix (typically 128x128)**
7. **Repeat steps 2-6 for all projection angles**

Normal Brain-FDG
2-D Filtered Back-Projection



Drawbacks with FBP in PET

•A smoothing filter is required to reduce the noise due to low photon counting statistics, resulting in a trade-off between image noise and spatial resolution.

•Streak artifacts may appear if the sampling rate is insufficient.

•Streak artifacts appear when there is a hot region adjacent to a cold one.

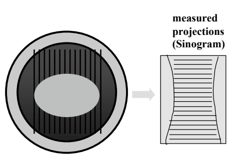
2-D Iterative Reconstruction Methods

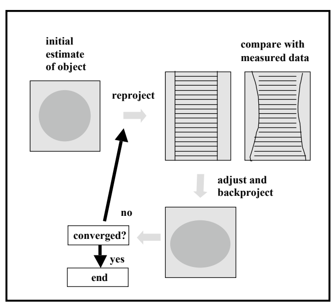
General Problem of Iterative Methods

- **Find the vector f that is the solution of: $g = A f$ using the principle of "successive estimates":**
  - **(1) Make an initial guess for the solution.**
  - **(2) The forward projections for the current estimate are compared with the measured projections, for each iteration.**
  - **(3) The result of the comparison is used to modify the current estimate, creating a new estimate.**
- **Iterative algorithms differ in:**
  - **(1) the algorithm by which the measured and current estimated projections are compared and**
  - **(2) the algorithm for the correction that is applied to modify the current estimate.**

Image Reconstruction

Iterative Reconstruction





Example iterative algorithm names:
-Ordered-set expectation maximization (OSEM)
-Maximization a priori (MAP)

Algebraic Reconstruction Technique (ART)

New estimate **Current estimate** **Comparison method**

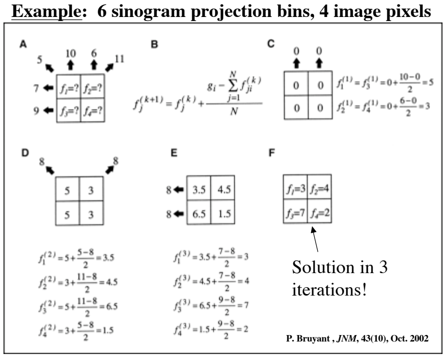
$$f_j^{(k+1)} = f_j^{(k)} + \frac{g_i - \sum_{j=1}^{N} f_{ji}^{(k)}}{N},$$

Additive correction term to current estimate

where $f_j^{(k)}$ and $f_j^{(k+1)}$ are the current and new estimates, respectively;
N the number of image pixels along ray i;
$\sum f_{ji}^{(k)}$ is sum of counts in the N image pixels of the current estimate along ray i for the kth iteration;
g_i the measured number of counts for ray i. Each new projection represents a different iteration. N is the total number of projection bins in the data.

Algebraic Reconstruction Technique (ART)

Example: 6 sinogram projection bins, 4 image pixels



Maximum Likelihood Expectation Maximization (MLEM) Algorithm

- **An optimization method: find the best estimate for the solution of $g = Af$ fitting a given criterion (g is the vector of sinogram values, A is the system matrix and f is the unknown vector of image pixel values).**
- **The criterion is maximization of the *likelihood function* (MLEM) of the reconstructed image.**
- **Measurements of the process of radioactive disintegration are subject to variations that follow a Poisson probability distribution; Any given measured data set g is one particular realization.**
- **Goal of the MLEM algorithm is to find a general solution for the best estimate of f; That is, find the mean number of radioactive disintegrations in the image that can produce the sinogram g with the highest likelihood.**

MLEM Algorithm - Formulation

The mean number of photons $\bar{g}_i$ detected by sinogram bin i is the sum of the mean number of photons emitted by each image pixel j:

$$\bar{g}_i = \sum_{j=1}^{m} a_{ij} \bar{f}_j ,$$

where $\bar{f}_j$ is the mean number of disintegrations in image pixel j;
a_{ij} is the probability sinogram bin i detects a photon from image pixel j;
$a_{ij}\bar{f}_j$ is the mean number of photons emitted from image pixel j and detected at sinogram bin i.
m is the total number of pixels in the image.

MLEM Algorithm (ctd.)

Poisson law allows the prediction of a realized number of detected counts, given the mean number of disintegrations. We assume both the number of emitted and detected disintegrations are Poisson random variables. The probability of detecting g_i photons is given by:

$$P(g_i) = \frac{e^{-\bar{g}_i}\,\bar{g}_i^{\,g_i}}{g_i\,!},$$

The maximum probability is reached when the number of counts g_i detected in the sinogram bin j is equal to the mean number $g_i=\bar{g}_i$.

MLEM Algorithm (ctd.)

Since the g_i Poisson variables are independent, the conditional probability $P(g|\bar{f})$ of observing the vector g when the emission map is $\bar{f}$ is the product of the individual probabilities $p(g_i)$. The likelihood function $L(\bar{f})$ is given by:

Total # projection bins

$$L(\bar{f}) = P(g \mid \bar{f}) = p(g_1)p(g_2).....p(g_n) = \prod_{i=1}^{n} p(g_i) = \prod_{i=1}^{n} \frac{e^{-\bar{g}_i}\,\bar{g}_i^{\,g_i}}{g_i!}.$$

$$\ln[L(\bar{f})] = \sum_{i=1}^{n} [-\bar{g}_i + g_i \ln(\bar{g}_i) - \ln(g_i!)],$$

$$= \sum_{i=1}^{n} \{-\sum_{j=1}^{m} a_{ij}\bar{f}_j + g_i \ln[\sum_{j=1}^{m} (a_{ij}\bar{f}_j) - \ln(g_i!)].$$

MLEM Algorithm (ctd.)

$\ln[L(\bar{f})]$ **Gives the log of the probability to observe a given projection data set for any given mean image.**

The vector $\bar{f}$ for which the likelihood function L attains its maximum value is found by computing its derivative [actually that of $\ln(L)$] and setting the result equal to zero:

$$\frac{\partial \ln[L(\bar{f})]}{\partial \bar{f}_j} = -\sum_{i=1}^{n} a_{ij} + \sum_{i=1}^{n} \frac{g_i}{\sum_{j'=1}^{m} a_{ij}\bar{f}_{j'}}\, a_{ij} = 0.$$

MLEM Algorithm (ctd.)

$$\longrightarrow \quad \bar{f}_j = \frac{\bar{f}_j}{\sum_{i=1}^{n} a_{ij}} \left(\sum_{i=1}^{n} \frac{g_i}{\sum_{j'=1}^{m} a_{ij'} \bar{f}_{j'}} a_{ij} \right),$$

Which leads to the iterative algorithm of the MLEM algorithm (Lange and Carson *J Comp Ass Tomog*, 1984):

$$\boxed{\bar{f}_j^{(k+1)} = \frac{\bar{f}_j^{(k)}}{\sum_{i=1}^{n} a_{ij}} \left(\sum_{i=1}^{n} \frac{g_i}{\sum_{j'=1}^{m} a_{ij'} \bar{f}_{j'}^{(k)}} a_{ij} \right)}$$

Note: Correction term is multiplicative, so must be greater than zero

MLEM Algorithm - Interpretation

$$\bar{f}_j^{(k+1)} = \frac{\bar{f}_j^{(k)}}{\sum_{i=1}^{n} a_{ij}} \left(\sum_{i=1}^{n} \frac{g_i}{\sum_{j'=1}^{m} a_{ij'} \bar{f}_{j'}^{(k)}} a_{ij} \right)$$

Normalization factor

Ratio of the measured counts to the current estimate of the mean counts in bin *i*.

back-projection of that ratio for image pixel *j*.

A set of successive projections/backprojections. For the whole image:

$$\boxed{\text{Image}^{(k+1)} = \text{Image}^{(k)} \times \text{Normalized Backprojection of} \left(\frac{\text{Measured Projections}}{\text{Projections of Image}^{(k)}} \right)}$$

•The MLEM converges slowly and may require 50-200 iterations, especially if an accurate forward projection model is incorporated.

MLEM Algorithm - Incorporation of Accurate System Model

•At each iteration *k*, a current estimate of the image is available.

•Using an accurate system model (including for example attenuation and resolution blurring), it is possible to simulate more accurately what projections of the current estimate should look like.

•The ratio between simulated and measured projections is used to modify the current estimate to produce an updated estimate for iteration *k*+1.

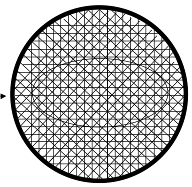
•Since a more accurate forward projection model has been provided, the new estimate $f^{(k+1)}$ should be more accurate.

•This process is repeated.

Ordered Subsets Expectation Maximization (OSEM) Algorithm - "Fast MLEM"

- **Proposed to accelerate the MLEM algorithm.**
- **Divide the set of projections into subsets of projections ordered in a particular sequence around the gantry (ordered sequence of subsets).**
- **To promote convergence each subset should contain projections equally distributed about the FOV.**

<u>Example</u>: 32 subsets, each containing 4 projections 45° apart. An example of the 4 projections from one subset is shown at the right (in red, green, orange, yellow). The next subset would have projections in between those shown.



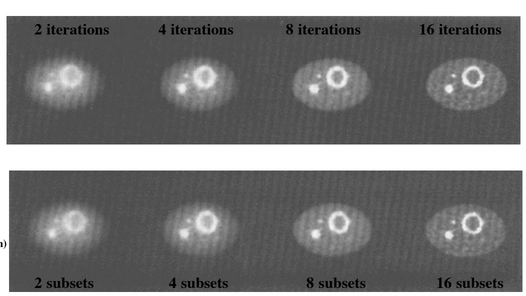
OSEM Algorithm - ctd.

- **The MLEM algorithm is applied to each subset, as a sub-iteration.**
- **The resulting reconstruction after each subset becomes the starting value for the next subset.**
- **The likelihood function for the whole set of projection data is increased within each sub-iteration of an individual subset.**
- **One full iteration comprises a single MLEM pass through all specified subsets.**
- **By processing the data in subsets within each iteration, this procedure accelerates MLEM convergence by a factor $\propto$ the number of subsets.**
- **The number of iterations required for convergence $\propto$ 1/(the number of subsets) up to a critical number, after which noise is amplified.**
- **With noisy data OSEM converges to a non-ML solution.**

Reconstructed Image Examples: 2-D data

EM

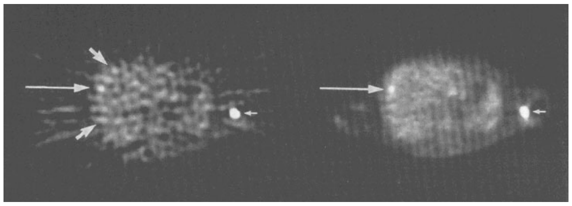
OSEM
(1 iteration)



S Meikle et al, *PMB* 39 (1994)

Reconstructed Image Examples: 2-D data

FBP | OSEM (8 subsets, 1 iteration)

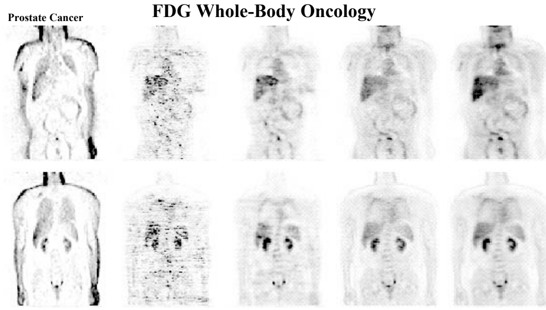


S Meikle et al, *PMB* 39 (1994)

Reconstructed Image Examples - 2D data

Prostate Cancer

FDG Whole-Body Oncology



FBP no AC | OSEM no smooth AC | OSEM 1.2 mm smooth AC | MAP no smooth AC | MAP 3 mm smooth AC

(Not covered in this talk)

R. Leahy, USC

Attenuation-Weighted OSEM Algorithm (AWOSEM)

•Fourier deconvolution methods assume that the emission data g_i corresponding to LOR$_i$ are ideal line integrals. Hence the measured data have to be corrected for randoms, dead-time, detector efficiencies, geometry factors, scatter and attenuation before applying Fourier methods.

•These corrections modify the statistical distribution of the data to be non-Poisson (variance $\not\propto$ mean), which limits applicability of EM approaches.

•For whole-body PET studies, the largest deviation from Poisson statistics is caused by performing attenuation correction since the range of variations of the ACFs in a sinogram can be ~ 100.

(Corrections for randoms, scatter, and normalization also alter the Poisson nature, but these effects are difficult to estimate in a consistent way.)

AWOSEM Algorithm - ctd.

•To overcome this deviation from Poisson statistics, in AWOSEM, the data is de-corrected for atttenuation by multiplying the measured data by the inverse of the 2-D ACFs. So the iteration step for each subset S_n in image slice z becomes:

Attenuation correction factor for LOR_i

$$\bar{f}_{j,z}^{(k+1)} = \frac{\bar{f}_{j,z}^{(k)}}{\sum_{i \in S_n} a_{ij} / A_{i,z}} \left(\sum_{i \in S_n} \frac{g_i / A_{i,z}}{\sum_{j'=1}^{m} a_{ij'} \bar{f}_{j',z}^{(k)}} a_{ij} \right)$$

AWOSEM incorporates the ACFs in the system matrix used to forward project the estimated sinogram based on the current estimate of the image. The estimated sinogram is then compared with the measured (attenuated sinogram), thus approximately preserving the Poisson statistics assumed by the EM alogrithm.

Reconstruction Algorithms for 3-D PET (that we will cover)

Project Missing Data (PROMIS) or 3-D Reprojection (3DRP) [Analytical]

Fourier Rebinning (FORE)+2D(FBP or OSEM) [Analytical or Iterative]

Row Action Maximum Likelihood Algorithm (RAMLA) [Iterative]

2-D acquisition: N direct +(N-1) indirect=2N-1 sinograms

3-D acquisition: N direct + N(N-1) oblique=N^2 sinograms

3DRP Algorithm

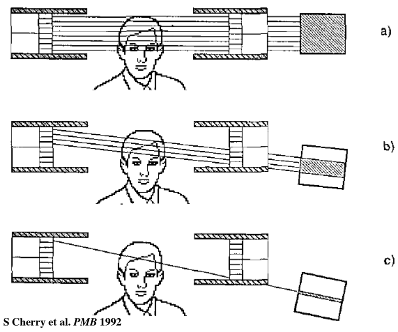
•Multi-slice PET scanners operating in 3-D mode have the geometry of a truncated cylinder.

•Response function varies strongly as a function of axial position across the FOV, which violates the condition of “shift-invariance” required to use filtered backprojection.

•Response function has incomplete projection data, resulting in poor image quantification, spatial distortion or data loss.

3D RP Algorithm - ctd.

Projections are increasingly truncated as the polar angle θ of acceptance increases due to limited axial extent of scanner.



Complete projections for 2-D acquisition

Incomplete projections for 3-D acquisition (incomplete sampling of the 3-D volume)

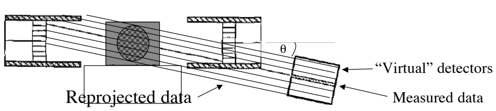
S Cherry et al. *PMB* 1992

3D RP Algorithm - ctd.

•To complete the projections and satisfy the condition of shift-invariance, the direct and cross planes are reconstructed using standard 2-D filtered backprojection methods to give a low statistics estimate of the image.

•This estimate is then used to forward-project the missing line-of-response data and complete the projections, such that all points within the FOV contribute all the projections. The amount of forward-projected data increases rapidly for larger θ.

•Once all missing projections are estimated, standard 3-D FBP methods may be applied: The projections are scaled, convolved with a 2-D filter function, e.g. the Colsher (1980) or Defrise filter (1989), smoothed, and then backprojected into the image array.

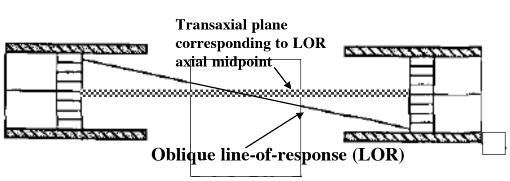


3D-->2D Rebinning Algorithms

•Rebinning is a procedure for sorting 3D projection data set into a stack of ordinary 2D-equivalent data sets that can be reconstructed by ordinary 2-D methods.

•Estimate the contribution to the direct LORs from the measured oblique LORs, which traverse several transaxial slices.

•Simplest rebinning algorithm: Single-slice Rebinning (SSRB): Assign axially tilted LOR data to transaxial slices midway axially between the two detectors (see below).



Fourier Rebinning (FORE) Algorithm

- **SSRB is accurate only for activity close to the system axis. For data off axis, very rough approximations result producing significant axial blurring.**
- **Fourier Rebinning (FORE) method allows a more accurate estimate of the source axial location, thereby strongly reducing off-axis distortion.**
- **FORE rebins the data in the frequency domain, rather than the spatial domain; Can be interpreted geometrically by exploiting the "frequency-distance relation" for oblique sinograms.**

FORE Algorithm - ctd.

Idea: Given the 4-parameter/3D Radon Transform of a 3-D distribution $f(x,y,z)$:

$$g(t,\phi,z,\delta) = \int f(x,y,z)\,ds\ ,$$

where $x=t\cos\phi - s\sin\phi$, $y=t\sin\phi + s\cos\phi$, $z=z+s\delta$, with z the axial coordinate of the midpoint of the oblique LOR, and $\delta=\tan\theta$, where θ is the angle between oblique LOR's and the transaxial plane.

For an exact rebinning formula, calculate the 3-D Fourier Transform Γ by (1) Fourier transform of g with respect to t, (2) Fourier series with respect to ϕ, and (3) Fourier transform of the result with respect to z:

$$\Gamma(\omega,k,\omega_z,\delta)=\int dz\, e^{-i\omega_z z}\int d\phi \int dt\, e^{-i(k\phi+\omega t)}\, g(t,\phi,z,\delta),$$

where ω and ω_z are the transaxial and axial spatial frequencies, respectively.

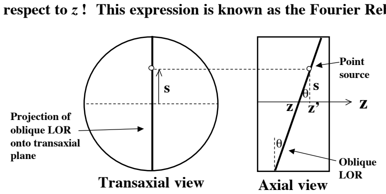
$$\rightarrow \underbrace{\Gamma(\omega,k,\omega_z,\delta)}_{\text{3D FT of oblique sinograms}} = \exp\{-ik\tan^{-1}(\frac{\delta\omega_z}{\omega})\}\cdot \underbrace{\Gamma(\omega\sqrt{1+\frac{\delta^2\omega_z^{\,2}}{\omega^2}},k,\omega_z,0)}_{\text{3D FT of direct sinograms}}$$

FORE Algorithm - ctd.

Expanding Γ to first order in $(\delta\omega_z/\omega)$ yields a result for the 2-D Fourier transform G:

$$\boxed{G(\omega,k,z,\delta) \approx G(\omega,k,z-(k\delta/\omega),0),}$$

which relates the 2-D Fourier transform G (w.r.t. t and ϕ only) of an oblique sinogram (z,δ) to the 2-D Fourier transform of the direct plane sinogram of a slice shifted axially by a frequency-dependent offset $\Delta z = -k\delta/\omega$. Note: do not need to take an axial FT with respect to z ! This expression is known as the Fourier Rebinning Approximation.



Geometric Interpretation

The "frequency distance relation": the value of $G(\omega,k,z,\delta)$ at the frequency (ω,k) receives contributions mainly from sources located at a fixed distance $s=-k/\omega$ along the lines of integration.

The contribution of the source at (z',δ) is rebinned into a direct sinogram $(z',0)$ where: $z' = z + s\tan\theta = z+\delta s = z-k\delta/\omega$

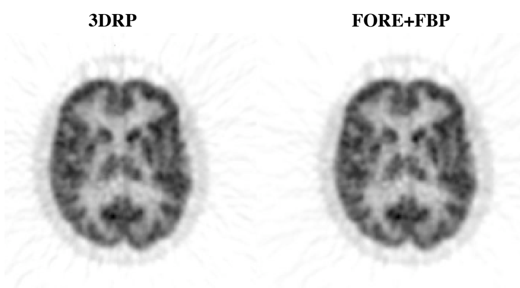
FORE Algorithm - ctd.

1. Initialize a stack of re-binned 2-D Fourier transformed sinograms $G_{2D}(\omega,k,z)$.
2. For each oblique sinogram (z,δ), calculate the 2-D FFT with respect to t and θ.
3. For each sample (ω,k), calculate $z' = z - \delta k/\omega$, for $|z'|\leq L/2$, L the scanner length.
4. a. High frequency region: Using linear interpolation increment the two closest sampled slices $z_1 < z' < z_2$: add $(z_2-z')G(\omega,k,z,\delta)$ to $G_{2D}(\omega,k,z_2)$ and add $(z'-z_1)G(\omega,k,z,\delta)$ to $G_{2D}(\omega,k,z_1)$.

   b. Low frequency region: add $G(\omega,k,z,\delta)$ to $G_{2D}(\omega,k,z)$.
5. Normalize the rebinned data $G_{2D}(\omega,k,z)$.
6. Take the inverse 2-D FFT of $G_{2D}(\omega,k,z)$. To obtain the stack of rebinned sinograms $g(t,\phi,z)$
7. Reconstruct each slice with any 2-D reconstruction algorithm.

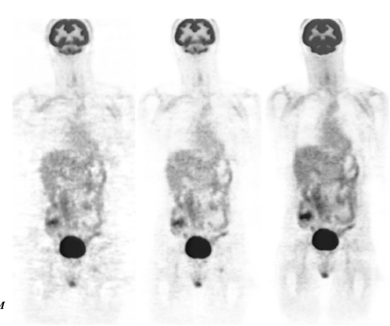
The algorithm essentially factors a 3-D backprojection into a 1-D axial backprojection and a 2-D transaxial backprojection of rebinned sinograms $g_{2D}(t,\phi,z)$.

FORE Algorithm - Image Examples 3-D data sets



M Defrise et al, *IEEE TMI* 16(2) April 1997

FORE Algorithm - Image Examples Reconstructed 3D data sets



C Lartizien et al, *JNM* 44(2) Feb. 2003

FORE Algorithm - Image Examples
Reconstructed 3-D data sets

FORE+FBP FORE+OSEM FORE+AWOSEM

C Lartizien et al, *JNM* 44(2) Feb. 2003

Row Action Maximum Likelihood Algorithm (RAMLA)

•RAMLA was developed as a faster alternative to the EM algorithm for maximizing the Poisson likelihood.

•Reconstructed image is updated after each projection in a controlled way using a relaxation parameter.

•The relaxation parameter prevents the algorithm from giving excessive weight to imperfect data or too large corrections for certain projection lines in the early iterations. It allows gradual and fast convergence.

•Deals with only one row of the projection matrix *A* at a time. In processing the projection lines a special ordering is chosen to ensure that sequential projection lines are as independent as possible to obtain a fast Poisson likelihood increase.

RAMLA iterative form:

$$f_j^{(k+1)} = f_j^{(k)} + \lambda_k f_j^{(k)} \left(\frac{g_i}{\sum_{j=1}^{m} a_{ij} f_j^{(k)}} - 1 \right) a_{ij}$$

"Relaxation" parameter

Additive correction term

(Similar to that of ART)

RAMLA + "blob"

•Can use alternative volume elements to voxels that are spherically-symmetric, bell-shaped (a.ka. "blobs") basis functions for the image model.

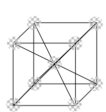
•Using certain definitions of the the blob's amplitude and shape the resolution of the measured data may be preserved, while at the same time suppressing the noise in the RAMLA reconstructed images.

•Forward projection and backprojection are "smeared out" over defined footprint of blob.

•To obtain uniform volume sampling, blobs are partially overlapped with neighbors and counts in common regions are averaged.

$$f(x,y,z) = \sum_{j=1}^{m} f_j \, \Phi(x-x_j, y-y_j, z-z_j),$$

$$\Phi(x,y,z) = b(r) = (b\sqrt{x^2+y^2+z^2})$$

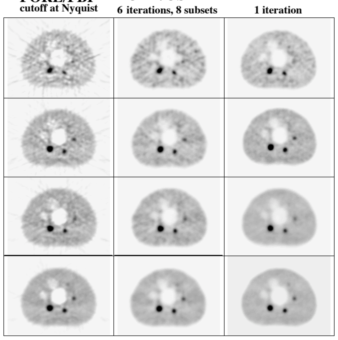


Spherically symmetric volume elements

Reconstructed Image Examples - 3D data sets

FORE/FBP (cutoff at Nyquist) | FORE/OSEM (6 iterations, 8 subsets) | FORE/RAMLA (1 iteration)

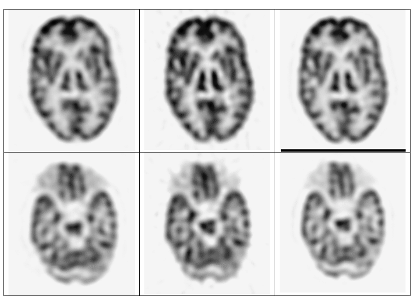
Acquisition times: 1 min. | 3 min. | 6 min. | 26 min.



M Daube-Witherspoon *et al. IEEE TNS* 48(1), Feb. 2001

Reconstructed Image Examples - 3D data sets

FORE/OSEM | 3DRP | 3D RAMLA

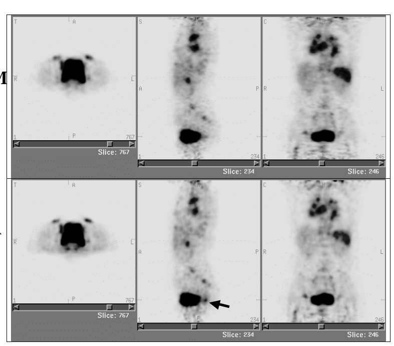


M Daube-Witherspoon *et al. IEEE TNS* 48(1), Feb. 2001

Reconstructed Image Examples - 3D data sets

FORE/OSEM

FORE/RAMLA

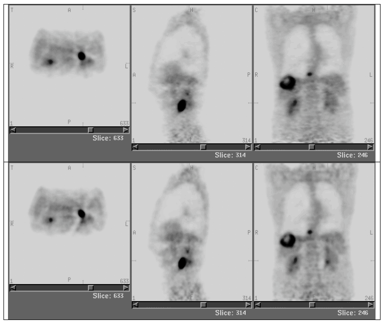


M Daube-Witherspoon *et al. IEEE TNS* 48(1), Feb. 2001

Reconstructed Image Examples - 3D data sets

FORE/OSEM

FORE/RAMLA



M Daube-Witherspoon *et al.* *IEEE TNS* 48(1), Feb. 2001

Summary

- **Both analytical and interative methods may be used to reconstruct the corrected PET data.**
- **Analytical methods are linear but due to noise require frequency filtering that results in a compromise in spatial resolution.**
- **Iterative methods allow an improved tradeoff between spatial resolution and noise, but are more computationally intensive.**