Evolution of Standards for Ionizing Radiation

Bert Coursey, NIST
Stephen Seltzer, NIST
Peter Almond, M.D. Anderson
Larry DeWerd, University of Wisconsin
1895  Roentgen discovers x rays

1896  Diagnostic applications

1898  Curies separate radium

1900  Therapeutic applications of radium

1901  National Bureau of Standards founded
Evolution of Standards for Ionizing Radiation

Standards for Brachytherapy, Coursey

Standards for Exposure and Air kerma, Seltzer

Standards for Absorbed Dose, Almond

Standards for Beam Quality Specification, DeWerd
Mass radium (mg)
Activity (Ci, Bq)

Exposure (R)
Air kerma (Gy/s)
Raw materials for initial Ra separations
Marie Curie 1898 - 1902

0.1 kg  pitchblende

0.5 kg  carnotite

100 kg  pitchblende

8000 kg  pitchblende
Commission internationale des étalons de radium

<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
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<tbody>
<tr>
<td>Marie Curie</td>
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<tr>
<td>André Debierne</td>
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<td>Ernest Rutherford</td>
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<td>Stefan Meyer</td>
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<td>Egon von Schweidler</td>
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<tr>
<td>Arthur Eve</td>
<td>McGill</td>
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<tr>
<td>Bertram Boltwood</td>
<td>Yale</td>
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</table>
International Standards of 1912

The Paris Standard

21.99 mg RaCl₂

The Vienna Secondary Standard

10.11 mg RaCl₂

31.17

40.43
Marie Curie placing weights on piezoelectric electrometer to achieve null balance with gold-leaf electroscope
COMMISSION INTERNATIONALE DES ÉTALONS DE RADIUM.

CERTIFICAT.

La Préparation de Chlorure de Radium contenus dans l'ampoule Nr. 6 provient de la pitchblende de St. Joachimsthal. Elle est donc pratiquement exempte de Meso-thorium.

Elle contient 21.50 Milligrammes de sel.

Le sel a été enfermé le 1/8/1913 dans un tube de verre (Verrc de Thüringa.) d'un diamètre extérieur de 27 mm; longeur 22 mm. Un fil de platine fin a été soudé à l'extrémité du tube.

En qualité d'Étalon secondaire l'ampoule a été comparée à l'Étalon de Vienne et à l'Étalon International de Paris, au moyen de méthodes de mesures basées sur le rayonnement β. La comparaison a été faite indépendamment à Vienne et à Paris.

D'après son rayonnement β, la Préparation équivalent en l'année 1913 à 20.22 mg. RaCl₂. (La diminution par année est de 0.4 pour mille.)

En adoptant les poids atomiques suivants:

- Radium 226
- Chlore 35.457
- Brom 79.916

on déduit la teneur correspondante en Radium élément et en Bromure de Radium:

\[
\begin{align*}
Ra & : 15.97 \text{ mg.} \\
RaCl₂ & : 20.22 \text{ mg.} \\
RaBr₂ & : 26.16 \text{ mg.}
\end{align*}
\]

La précision de ces résultats est considérée comme assurée à une approximation de 0.5%.

Specimen No. 6 of Radium is prepared as chloride from pitchblende of St. Joachimsthal and is consequently practically free from Meso-thorium.

It contains 21.50 Milligrammes of salt.

It was enclosed the 1/8/1913 in a glass tube (Thuringian glass) of 0.27 mm thickness, exterior diameter 22 mm, length 22 mm, a thin platinum wire being fused into the end of the tube.

It is calibrated as Secondary Standard by comparison with the Vienna Standard and with the International Standard at Paris, several independent β-ray methods being used.

Measured by the β-radiation, it is in the year 1913 equivalent to 20.22 mg. RaCl₂. (The yearly decay is about 0.4 per mille.)

Taking the atomic weights:

- Radium 226 for Radium
- 35.457 for Chlore
- 79.916 for Bromine

this corresponds to

\[
\begin{align*}
Ra & \text{ : 15.97 mg.} \\
RaCl₂ & \text{ : 20.22 mg.} \\
RaBr₂ & \text{ : 26.16 mg.}
\end{align*}
\]

These statements are considered correct to 0.5%.

Pour les mêmes faits à Paris:

[Signature]

[Signature]

Président de la Commission

[Signature]
Marie Curie visits Harding White House, May 1921
## 1934 Höningschmid Standards

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<td>PTB</td>
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FIG. 5. Geometrical relationships used in calculation of exposure at point P, from a linear γ-ray source. The source extends from $l_1$ to $l_2$. Filter thickness $T = 0$ in the case of Eq. (2); $T > 0$ for Eq. (3), the Sievert integral.

\[
\left( \frac{d X}{d t} \right)_P = A'\Gamma \int_{l_1}^{l_2} \frac{e^{-\mu_{en} T / \cos \theta}}{r^2} \, dl
\]
Energy (keV)

Relative Counting Rate

Relative Count Rate / 5

x-rays

186.2 $^{226}$Ra
242.0 $^{214}$Pb
295.4 $^{214}$Pb
352 $^{214}$Pb
609.3 $^{214}$Bi

$^{214}$Bi
Exposure Standard for Radium

Mass 1 mg Ra

Distance 1 cm

Filtration 0.5 mm Pt

Units $R \, \text{cm}^2$

----------- mg hr
NBS Exposure Rate Constant for Radium

Attix and Ritz (1957)
Cavity chamber measurement of exposure

Calorimetric measurement of activity (mg radium)

\[ 8.25 \text{ R cm}^2 \text{ mg hr} \]
Present NIST air kerma standards for brachytherapy

Cesium-137, Cobalt-60
B-G Cavity chambers
Iridium-192
B-G Cavity chamber
Iodine-125, Palladium-103
WAFAC
Beta Emitters
Extrapolation chamber
Evolution of Physical Measurement Standards for Exposure / Air Kerma

Stephen M. Seltzer
Ionizing Radiation Division
National Institute of Standards and Technology

1895: W.C. Röntgen discovers x rays

1896: research on the ionization of gases by x rays

- They showed that the ionization produced in gases could be related to the intensity of the x rays.
- Röntgen - first to demonstrate the role of air in the discharge of electrified bodies
- Perrin and Thomson & Rutherford proved the existence of saturation in the ionization of a gas.

March 1896: Röntgen, in his second communication

“Electrified bodies in air, charged either positively or negatively, are discharged if X rays fall upon them; and this process goes more rapidly the more intense the rays are. The intensity of the rays was estimated by their action on a fluorescent screen or a photographic plate… The observations … indicate that air through which X rays have passed possesses the power of discharging electrified bodies with which it comes in contact.”

August 1896: Perrin
(Comptes-Rendus de l’Academie des Sciences de Paris)

Perrin used an experimental set-up very similar to what we now call a free-air ionization chamber. He found a maximum rate of discharge independent of the collecting field.

“It is this maximum rate which … gives the quantity of neutral electricity dissociated with the rays … The quantity of X rays radiated into a cone centered on the source is proportional to the quantity of electricity dissociated in this cone per unit length, in a given gas, at a given pressure and temperature … This definition is only acceptable if the rays are not much reduced over their pathway.”
1897: Perrin
(Annales de Chimie et de Physique)

“One sees easily how, in considering the experimental limiting rate, it is possible to define and to measure the neutral quantity of electricity dissociated in a volume of any shape, under the influence of a given source, during a given time. This quantity measures the ionization I inside this volume. Let dI be the ionization produced in an infinitesimal volume dv containing point A; the quotient dI/dv is the ionization at the point A.”

1897: Perrin
(Annales de Chimie et de Physique)

“The ionization at a point ... varies as the inverse square of the distance between the point and the source.”

1897: Perrin
(Annales de Chimie et de Physique)

“The ionization thus plays a similar role as the illuminance in photometry and one conceives that it possible to define, if not the intensity of the source of Röntgen rays, as these sources are discontinuous like the discharges that produce them, at least the quantity of Röntgen rays. The unit of quantity of rays will be, for example, the quantity which produces, in air, at a pressure of 76 cm, in a spherical layer centered on the source having a thickness of 1 cm, an ionization equal to 1, that is to say an ionization corresponding to the liberation of 1 electrostatic CGS unit of positive electricity.”

1902

Holzknecht observed that some chemicals changed color in a systematic manner with exposure to x rays in varying quantities.

1905

W. Duane, while measuring radon, noted scattering from the walls of his ionization chamber that caused additional ionization, which led to his introduction of the large parallel-plate ionization chamber in which the beam would not strike any walls.

1907

P. Villard designed one of the first practical measuring instruments based on a thimble ionization chamber connected to an electrometer. He presented an instrument to determine the quality of x rays by measuring the ratio of charges liberated in two successive thimble chambers separated by a filter.

We had to wait more than a quarter of a century for these ideas to be put into practical use and the free-air chamber re-invented.
**1908**
Villard modified his chamber: the chamber is discharged by the x rays, and - when the electrometer reaches a given reading - an electric contact recharges the chamber. The number of discharges is counted, thus realizing one of the first digital instruments.

“The instrument is temporarily calibrated in units of H (Holzknecht) but this opportunity should be taken to define a more rational unit. It seems quite logical to adopt the following fundamental unit or its multiples: The unit of quantity of x rays is that which liberates by ionization one electrostatic unit per cubic centimeter of air at standard temperature and pressure.”

**1914**
Duane defines the unit of intensity as

“that intensity of radiation which produces under saturation conditions one electrostatic unit of current per cubic centimeter of air under normal temperature and pressure.”

**1921**
I. Solomon was probably the first to attach the name Röntgen to a unit for x rays, calling his unit R, defined as

“the intensity of roentgen radiation producing the same ionization per second as one gram of radium placed two centimeters from the ionization chamber and filtered with 0.5 mm platinum.”

*Poor standard, as it was entirely dependent on a specific instrument and electroscope located in a hospital laboratory in Paris.*

**1925 The First International Congress of Radiology (London)**
Behnken of the PTR proposed to change the name of Villard’ electrostatic unit to the “Roentgen” unit (1R), with the following definition.

“The absolute unit of the roentgen-ray dose is obtained from the roentgen-ray energy, which, by fully utilizing the secondary electrons produced, and by avoiding secondary radiations from the wall of the ionization chamber, produces in one c.c. of atmospheric air of 18 °C (64.4 °F) and 760 mm atmospheric pressure, such a degree of conductivity that the quantity of electricity measured by saturation current equals one electrostatic unit.”

**1925 The First International Congress of Radiology**

- No national laboratory in any country was then directly involved in radiation measurement standards.
- The International X-Ray Unit Committee was appointed to consider the establishment of a “uniform X-ray standard of intensity and an X-ray unit.”
- Committee became the International Commission on Radiation Units and Measurements (ICRU)
1927
- The Radiological Society of North America (RSNA) urged the National Bureau of Standards (NBS) to participate in the standardization of x radiation.
- The NBS agreed, and hired Lauriston S. Taylor

1928 The Second International Congress of Radiology (Stockholm)
The ICRU, essentially adopting Behnken’s definition, recommended:
“That the international unit be the quantity of x radiation which, when the secondary electrons are fully utilized and the wall effect of the chamber is avoided, produces in one cubic centimeter of atmospheric air at 0 °C and 76 cm mercury pressure, such a degree of conductivity that one electrostatic unit of charge is measured at saturation current.”
“That the international unit of X-radiation be called the Röntgen and that it be designated by the letter small r.”

1929
Taylor develops free-air chamber to serve as US standard for x-ray beam measurements.

1931
- Large FACs were in place in England, Germany, and US
- Comparisons: NBS-PTR ~1%, NBS-NPL ~3%

1936 Bragg-Gray cavity theory
Provided the cavity is small and the surrounding solid medium thick enough to exclude secondary electrons from any other material,

\[ E_{m} = J W S_{m} \]

- \( E_{m} \): energy absorbed per unit mass in the solid
- \( J \): ionization per unit mass of the gas
- \( W \): average energy per ion pair
- \( S_{m} \): ratio of mean mass stopping powers of the solid to that of the gas
The ICRU refined the definition of the Röntgen:

“That the international unit be the quantity or dose of x rays shall be called the ‘roentgen’ and shall be designated by the symbol ‘r’ … The roentgen shall be the quantity of x or γ radiation such that the associated corpuscular emission per 0.001293 gram of air produces, in air, ions carrying 1 e.s.u. of quantity of electricity of either sign.”

Gamma rays are now included. No mention of wall effects recognizes the use of cavity chambers with air-equivalent walls, particularly for high-energy photons. Volume of air was replaced with mass of air at 0 °C and 76 cm Hg pressure (implying dry air). “Associated corpuscular emissions” can be interpreted to imply a measurement of the x-ray field at a point, rather than averaged over the whole region within the maximum range of secondary electrons.

NBS develops pressurized FAC for higher energy x-ray beams.

Taylor/NBS constructs new building for x-ray beam generation and measurements.

Ionization measurements should be made under conditions normally referred to as infinitesimal cavity conditions. The ICRU should promote the compilation and distribution of the data needed to convert ionization to absorbed dose.

Gladys White, in Fano’s Radiation Theory Section, publishes one of the first critically compiled set of cross sections for photon interactions.
1953

ICRU (1954) defines absorbed dose:

“Absorbed dose of any ionizing radiation is the amount of energy imparted to matter by ionizing particles per unit mass of irradiated material at the place of interest. It shall be expressed in rads. The rad is the unit of absorbed dose and is 100 ergs per gram.”

The ICRU recognized the importance of preparing tables of best available data for calculating absorbed dose from measurements of ionization, and retained the definition of the roentgen.

~1954

NBS develops new FAC that serves still today as US standard for medium-energy x-ray beams (50-300 kV).

Wyckoff-Attix chamber

1955

Spencer-Attix cavity theory

Extends Bragg-Gray cavity theory by considering the secondary electrons produced in the course of an electron slowing down. The mean mass stopping-power ratio is now a more complicated calculation and depends somewhat on an energy cut-off related to the size of the cavity.

1957

ICRU (1957) recommends that the quantity expressed in roentgens be called “exposure dose” rather than “dose” to avoid confusion with “absorbed dose.”

~1957-59

F.H. Attix develops cylindrical graphite-wall cavity-ionization chamber to serve as US standard for γ-ray beam exposure dose.

1958

The ICRU approaches the General Conference of Weights and Measures (established under the Treaty of the Meter) and its laboratory, the BIPM, who agree to expand to include the measurement of ionizing radiation.

The Consultative Committee on Ionizing Radiation is established to compare and harmonize national standards through the BIPM.
1958

W.C. Roesch points out a quantity of fundamental importance that had neither been defined nor given a name: The Kinetic Energy Released per unit Mass to secondary charged particles at a point in a medium by γ rays or neutrons. This quantity he called KERM.

1959

V.H. Ritz develops new FAC that still serves today as US standard for low-energy x-ray beams (20-100 kV).

1962

ICRU (1954) adopts rigorous definitions, among which:

- The quantity for which the roentgen is the unit is changed to exposure, and the the roentgen is denoted by the symbol R, where 1 R = 2.58 × 10⁻⁴ C/kg.
- The quantity kerma is defined similar to Roesch’s KERM, to be expressed in ergs/g, but not in rads.
- The quantities fluence and energy fluence are defined.
- The mass energy-transfer coefficient and the mass energy-absorption coefficient are established.

1963

M.J. Berger publishes seminal paper on Monte Carlo calculations of electron (and other charged particle) transport.

1964

From work organized by Fano for the National Academy of Science, Berger & Seltzer compile critically evaluated electron stopping powers, both due to collisions with orbital electrons (including the density effect) and due to the emission of bremsstrahlung. These data were for use also in their Monte Carlo calculations.

1965

P.J. Lamperti develops FAC that still serves today as US standard for very-low-energy x-ray beams (10-20 kV).
1969
J.H. Hubbell publishes classic compilation of critically evaluated cross sections for photon interaction with atoms.

1969
T.P. Loftus uses three new spherical graphite-wall cavity-ionization chambers to establish US standard for exposure from $^{137}$Cs sources.

1972-74
T.P. Loftus and J.T. Weaver use now a set of eight spherical graphite-wall cavity-ionization chambers to establish US standard for exposure from $^{137}$Cs and $^{60}$Co $\gamma$-ray beams.

1979
T.P. Loftus uses three of the spherical graphite-wall chambers to establish US standard for exposure from $^{192}$Ir sources.

1980
ICRU adopts SI units:
- The roentgen R is retained for exposure (for temporary use).
- The preferred quantity kerma is expressed in gray, 1 Gy = 1 J/kg.
- Absorbed dose is expressed also in gray, 1 Gy = 100 rads.

1984
Berger & Seltzer develop new evaluation of electron stopping powers.
ICRU publishes Report 37.

1984
T.P. Loftus establishes the US exposure standard for $^{125}$I brachytherapy seeds using the Ritz FAC to measure the output from an array of from 4 to 6 sources.
1986
Due to implementation by the Consultative Committee on Ionizing Radiation of new ratios of electron stopping-powers from Berger & Seltzer (1984), new ratios of photon mass energy-absorption coefficients from Hubbell (1982), and humidity corrections, NBS lowers its exposure standard for $^{60}$Co $\gamma$-ray beams by 1.1%.

\[ X = \frac{Q_{e}}{2.58 \times 10^{-4} \rho_{\text{air}} V_{\text{eff}}} \prod k_{i}, \quad V_{\text{eff}} = (\text{Aperture Area}) \times (\text{Collecting Length}) \qquad \text{regardless of beam divergence} \]

\[ k_{i} = \frac{1}{2} - \frac{R_{\text{air}}}{V_{\text{eff}}} \]

1989
In accordance with recommendations of the Consultative Committee on Ionizing Radiation, NIST changes standards/calibrations from exposure to air kerma.

\[ K_{\text{air}} = 2.58 \times 10^{-4} X \cdot \frac{(W/e)}{(1 - \bar{g})}. \]

\( W \) mean energy expended in air per ion pair formed
\( g \) fraction of energy of liberated charged particles that is lost in radiative processes

1995
NIST adopts the Attix FAC, designed and built at the University of Wisconsin, as the US standard for air kerma from mammography x-ray beams.

1995
Reunion of Radiation Theory Section at NIST celebrating the award of the Fermi Prize to Ugo Fano

1993-97
R. Loevinger develops new Wide-Angle Free-Air Chamber (WAFAc) for measurement of air kerma from low-energy photon-emitting brachytherapy seeds.
1999
Loevinger’s WAFAC adopted as US air-kerma strength standard for $^{125}$I and $^{103}$Pd seeds.

WAFAC

Loevinger’s WAFAC Automated WAFAC

$^{60}$Co

International comparison of air kerma standards in $^{60}$Co γ radiation

$^{137}$Cs

Medium-Energy X Rays

AAPM, July 26, 2001
Low-Energy X Rays

2001

- Bielajew and Rogers: wall corrections for cavity-ionization chambers have been in error
- Independent Monte Carlo calculations confirm NIST carrying out Monte Carlo calculations for spherical standard chambers, for $^{60}\text{Co}$, $^{137}\text{Cs}$ and $^{192}\text{Ir}$ γ rays.
- Air-kerma standards expected to be increased by ~1% for $^{60}\text{Co}$ and $^{137}\text{Cs}$

- Monte Carlo calculations by Burns at the BIPM: consistent set of correction factors for FACs
- NIST implementation is expected to change x-ray standards typically by ~0.1-0.2 %, approaching ~1% for a few beam qualities
- Spectroscopy/calculations for prostate seeds suggests a change in standards of nearly 1% for some models
Standards and Traceability for Absorbed Dose

by

Peter R. Almond
“In the past, the unwritten principle was followed that within the calibration chain from primary standard to field instrument the same quantity (and unit) should be used throughout.”
“It would seem natural to choose as a standard the exposure at a designated point in a reference beam. But the definition of such a beam cannot be envisaged.”
“By reason of the difficulties the unit of exposure is not maintained by the use of a radiation beam. The role of standard is assigned to the instrument for measuring exposure: An ionization chamber can constitute a standard of exposure rate...”

“The advantages of a uniform calibration chain maybe outweighed by advantages connected with the following two principles:

1) Quantities, in terms of which primary standards deliver their readings should be close to the physical effect on which the standard is based.

2) Quantities, in terms of which field instruments are calibrated should be closest to the purpose of the measurement.”
“Changing the quantity between primary standards and field instruments means that conversion factors have to be introduced at a certain point in the calibration chain.”

“...eventually, for all instruments, the certificates should give both the calibration factor in terms of the quantity of the primary standard and the derived calibration factor so that the indicated values can be traced.”

The representation of the calibration chain by Loevinger and Loftus can be modified to represent the present approach as follows:

Steps
1. \( N_1 = X_{11} X_{12} X_{13} \ldots \)
2. \( N_2 = N_1 X_{21} X_{22} X_{23} \ldots \)
3. \( N_3 = N_2 X_{31} X_{32} X_{33} \ldots \)
4. \( N_4 = N_3 X_{41} X_{42} X_{43} \ldots \)
5. \( N_5 = N_4 X_{51} X_{52} X_{53} \ldots \)

or

\[
N_5 = \pi X_{ij}
\]

\(ij\)
The $X_{ij}$ are the results of measurements, or are correction factors, conversion factors or physical constants.

$N_1$ is the exposure or air-kerma rate determined by the standard instrument.

$N_2$ and $N_3$ are the calibration factors of the transfer and field instruments respectively.

$N_4$ changes the quantity from exposure or air-kerma to absorbed dose.

$N_5$ is the dose rate of the treatment beam.
For the United States of America, steps 1 and 2 are done at the National Institute of Science and Technology (NIST). Step 3 is done at an Accredited Dosimetry Calibration Laboratory (ADCL) and steps 4 and 5 are done at the user’s beam.
\[ X_c = J_{gas} \left[ \left( \frac{L}{\rho} \right)_{gas} \left( \frac{\mu_{en}}{\rho} \right)_{wall} \right]_c \left( \beta_{wall} \right)^{-1} \prod_i K_i \]
\[ N_{x,c}^{tr} = X M_{tr,c}^{-1} \]
\[ N_{x,c}^u = N_{x,c}^{tr} M_{tr,c} M_{u,c}^{-1} \]
\[ N_{gas}^u = N_{x,c}^u \left[ \frac{(W / e) A_{ion} A_{wall} \beta_{wall}}{(L / \rho)_{wall} (\mu_{en} / \rho)_{air}} \right]_c \]
\[ D_{med} = M_Q N_{gas}^u \left[ (L/\rho)_{gas}^{med} P_{ion} P_{repl} P_{wall} \right]_Q \]
\[ D_{med} = M_Q \left[ J_{gas} \left( \frac{L}{P} \right)_{gas} \left( \mu_{en} / P \right)_{wall} \left( \beta_{wall} \right)^{-1} \prod_i K_i \right] \]

\[
\left( M_{tr,c}^{-1} \right) \left( M_{tr,c} M_{u,c}^{-1} \right) \left[ \frac{(w / e) A_{air} A_{wall} \beta_{wall}}{(L / \rho)_{air} \left( \mu_{en} / P \right)_{wall}} \right]_c
\]

\[
\left[ \left( \frac{L}{\rho} \right)^{med} \left( \frac{L}{\rho} \right)_{gas} P_{ion} P_{repl} P_{wall} \right]_Q
\]

The calibration factors do not explicitly appear in this equation. They are in fact a convenient short hand method of rewriting the equation.
The beam calibration in terms of absorbed dose rate depends upon all the individual measurements, correction factors, etc. but does not explicitly involve the intermediate results, $N_1$, $N_2$, $N_3$, etc.
The fundamental measurement is the determination of $J_{\text{air}}$ (C/kg) and is the basis of the primary measurement standard.
All other measurements are relative and their units (coulombs, scale divisions, etc.) cancel out, but it is these measurements and the precision with which they are done that determines the traceability of the final result.
Step 4 is unique in that it does not involve a measurement and hence can be put at various places in the chain. It could have been put before step 3 instead of after step 3. Any step between 1 and 4 however must be done with the same modality and beam quality as step 1 because the results of the intercomparison do not depend upon describing the instrument making the measurement. Step 4 depends upon knowledge of certain parameters of the chamber. After step 4 any appropriate modality and energy may be used as long as the various perturbation and correction factors are used.
Assuming that the initial measurements at the standards laboratory are with standard cavity ionization chambers for a single photon energy (i.e. Co-60 gamma rays) then it makes no difference whether the initial quantity calculated from those measurements is exposure, air-kerma in air, water kerma in air, or even absorbed dose in air (where “in air” means the appropriate quantity is calculated for a small sphere of water of equilibrium thickness centered at the point of measurement).
Traceability is the ability to relate the response of the field instrument per unit of the desired quantity in any stated radiation beam, through the calibration chain by intercomparison (measurement) or calculation, to the determination of a specified quantity (in appropriate units) for a specified radiation beam.