Strategies for Learning to Solve Physics Problems

“I understand the concepts, I just can’t solve the problems.”

Ken Heller
School of Physics and Astronomy
University of Minnesota

20 year continuing project to improve undergraduate education with contributions by:
Many faculty and graduate students of U of M Physics Department
In collaboration with U of M Physics Education Group

Details at http://groups.physics.umn.edu/physed/

Supported in part by Department of Education (FIPSE), NSF, and the University of Minnesota
**TASK**
Discuss why you assign problems in your courses.

List the common goals of the problems.

Report the single most important goal

**TIME ALLOCATED**
5 minutes

**PROCEDURES**
Form a group of 3 people
Choose one person as a recorder

Formulate a response individually.
Discuss your response with your partners.
Listen to your partners' responses.
Create a new group response through discussion.
Learning Problem Solving
Using Cooperative Group Problem Solving
A Guide for Discussion

1. What is Problem Solving?
2. What is Learning?
3. Why Learn Problem Solving?
4. Strategies for Teaching Problem Solving
   1. Goals
   2. Guidance from learning theory
   3. Logical framework
   4. Useful problems
   5. Cooperative groups
   6. Grading and assessment
Some Goals of Problem Solving

• Students can make both qualitative and quantitative predictions about the real world from basic, well-understood principles.

• Students will know the difference between fundamental principles, special cases, and specific applications.

• Students can make decisions, know the assumptions that underlie them, and be able to evaluate them.

• Students can construct and communicate a long chain of logic (including mathematics) to themselves and others.
What Is Problem Solving?

“Process of Moving Toward a Goal When Path is Uncertain”

• If you know how to do it, its not a problem.

Problems are solved using general purpose tools

Heuristics

Not specific algorithms

“Problem Solving Involves Error and Uncertainty”

A problem for your student is not a problem for you

Exercise vs Problem

M. Martinez, Phi Delta Kappan, April, 1998
Metacognition – Reflecting on Your Own Thought Process

- Managing time and direction
- Determining next step
- Monitoring understanding
- Asking skeptical questions

Some General Tools (Heuristics)

- Means - Ends Analysis (identifying goals and subgoals)
- Working Backwards (step by step planning from desired result)
- Successive Approximations (idealization, approximation, evaluation)
- External Representations (pictures, diagrams, mathematics)
- General Principles of Physics

M. Martinez, Phi Delta Kappan, April, 1998
Problem Solving Is an Organized Framework for Making Decisions

- Visualize situation
- Determine goal
- Choose applicable principles
- Choose relevant information
- Make necessary simplifications
- Construct a plan
- Arrive at an answer
- Evaluate the solution

This is a process not a linear sequence. It requires students to reflect on their work.

Not natural for most students – must be explicitly taught in every new academic environment.

Problems that facilitate learning should

- Explicitly connect important concepts
- Explicitly connect to reality
Learning is a Biological Process

Neural Science Gives the Constraints

Knowing is an individual’s neural interconnections

Knowing something means a student can use it in novel (for them) situations and communicate that usage.

Learning is expanding the network of neural connections by linking and changing existing ones and establishing new ones.

Teaching is putting the student in a situation that stimulates neural activity that renovates the relevant network of neural connections.

Teaching requires Interactive Engagement (Active Learning, Activities)

Cognitive Apprenticeship

Neurons that fire together, wire together

Simplification of Hebbian theory:
Hebb, D (1949). *The organization of behavior.*
New York: Wiley.

Brain MRI from Yale Medical School
Neuron image from Ecole Polytechnique Lausanne
Strategies for Learning Problem Solving (or anything else)

- **Watch experts solve problems** – Ask yourself (and them) what are they doing and why. Don’t be surprised if they don’t know.

- **Develop expert problem solving skills by repeated practice.** – Always use an organized framework for your problem solving. “Practice does not make perfect, only perfect practice makes perfect” – Vince Lombardy.

- **Practice problem solving in different contexts that are meaningful to you** – Solving the same problem over and over until it becomes automatic will not help in learning how to solve problems.

- **Practice isolated skills by doing exercises** – However you can’t learn to solve problems by doing exercises.

- **Get a coach that will make sure you engage in perfect practice.** - A good practitioner is not necessarily a good coach.

- **Work with others solving problems.** - Learn from their successes and struggles as you solve problems together.

- **Work on your own to solve problems.** - Get coaching only after you have tried your best and failed. The help should never be directed at how to solve that specific problem.

- **Don’t get discouraged.** - Applying newly learned skills will lead to slower and more error prone practice. Get through that in your practice.
Problem-solving Framework
Used by experts in all fields

G. Polya, 1945

STEP 1
Recognize the Problem
What's going on and what do I want?

STEP 2
Describe the problem in terms of the field
What does this have to do with ...... ?

STEP 3
Plan a solution
How do I get what I want?

STEP 4
Execute the plan
Let's get it.

STEP 5
Evaluate the solution
Can this be true?
Learning is Too Complex to Predetermine

Phenomenological Learning Theory
Apprenticeship Works
Cognitive Apprenticeship

Learning in the environment of expert practice

- Why it is important
- How it is used
- How is it related to a student’s existing knowledge

Brain MRI from Yale Medical School
Neuron image from Ecole Polytechnique Lausanne

Learning Requires Scaffolding

Additional structure used to support the construction of a complex structure.
   Removed as the structure is built

Examples of Scaffolding in teaching Introductory Physics using problem solving

• Problems that discourage novice problem solving
• An explicit problem solving framework
• Cooperative group structure that facilitates peer coaching by encouraging productive group interactions
   Grouping rules
   Group roles
• A worksheet that structures the framework
• Limit use of formulas by giving an equation sheet (only allowed equations)
• Explicit grading rubric to encourage expert-like behavior
Cooperative Group Problem Solving is an Implementation of Cognitive Apprenticeship

**Essential Elements**

1. Organized Framework for Problem Solving
2. Problems that Require Using an Organized Framework
3. Cooperative Groups to provide coaching to students while solving problems

Peer coaching  
Instructor coaching
Appropriate Problems for Practicing Problem Solving

The problems must be challenging enough so there is a *real* advantage to using a *problem solving framework*.

1. The problem must be **complex** enough so the best student in the class is not certain how to solve it.

   The problem must be **simple** enough so that the solution, once arrived at, can be understood and appreciated by everyone.
2. The problems must be designed so that

- the major problem solving **heuristics** are **required** (e.g. physics understood, a situation requiring an external representation);

- there are **decisions** to make in order to do the problem (e.g. several different quantities that could be calculated to answer the question; several ways to approach the problem);

- the problem **cannot be resolved in one or two steps** by copying a pattern.
3. The task problem must connect to each student’s mental processes

- the situation is **real** to the student so other information is connected;

- there is a **reasonable goal** on which to base decision making.

This is not what is called Problem Based Learning (PBL). These are closed ended problems with a definite answer (or a few possible answers) appropriate for novice problem solvers and directed toward a specific learning goal.
Context Rich Problem

You are investigating using MRI to identify cancer cells. To do this, you have constructed a 3.0 cm diameter solenoid into which you can place a tissue sample. You will then change the magnetic field at your sample by changing the current through the solenoid. You need to monitor the magnetic field inside the solenoid but its size makes inserting your Hall probe impractical. Instead you put the solenoid through the center of a 5 cm diameter, 10 turn coil of wire and measure the voltage across that coil. To decide if this gives enough precision, you calculate the change in the coil voltage as a function of time as you change the solenoid current. The solenoid is 20 cm long and consists of 2000 turns of wire. Your signal generator varies the current through the solenoid as a sine function at a frequency of 500 Hz with a maximum of 15 A.

Gives a motivation – allows some students to access their mental connections. Gives a realistic situation – allows some students to visualize the situation. Does not give a picture – students must practice visualization. Uses the character “you” – more easily connects to student’s mental framework.

Decisions must be made
Context Rich Problem

You are investigating using MRI to identify cancer cells. To do this, you have constructed a 3.0 cm diameter solenoid into which you can place a tissue sample. You will then change the magnetic field at your sample by changing the current through the solenoid. You need to monitor the magnetic field inside the solenoid but its size makes inserting your Hall probe impractical. Instead you put the solenoid through the center of a 5 cm diameter, 10 turn coil of wire and measure the voltage across that coil. To decide if this gives enough precision, you calculate the change in the coil voltage as a function of time as you change the solenoid current. The solenoid is 20 cm long and consists of 2000 turns of wire. Your signal generator varies the current through the solenoid as a sine function at a frequency of 500 Hz with a maximum of 15 A.

What is happening? – you need a picture.
What is the question? – it is not in the last line.
What quantities are important and what should I name them? – choose symbols.
What physics is important and what is not? – Faraday's Law, definition of flux
What assumptions are necessary? – Can you ignore the field outside the solenoid?
Is all the information necessary? – There is a lot of information.
The Dilemma

Start with complex problems so novice framework fails

Difficulty using strange new framework with challenging problems.

Why change?

Start with simple problems to learn expert-like framework.

Success using novice framework.

Why change?

Coaching is the necessary ingredient that allows students to work complex problems that require an expert-like framework.
Cooperative Groups

Provide peer coaching and facilitates expert coaching. Allow success solving complex problems with an organized framework from the beginning.

- Positive Interdependence
- Face-to-Face Interaction
- Individual Accountability
- Explicit Collaborative Skills
- Group Functioning Assessment

Johnson & Johnson, 1978
Scaffolding

Structure and Management of Groups

1. What is the "optimal" group size?
   - three (or occasionally four) for novices

2. What should be the gender and performance composition of cooperative groups?
   - heterogeneous groups:
     - one from top third
     - one from middle third
     - one from bottom third
     - based on past test performance.
   - two women with one man, or same-gender groups
3. How often should the groups be changed?

For most groups:

- stay together long enough to be successful
- enough change so students know that success is due to them, not to a "magic" group.
- about four times per semester
Structure and Management of Groups

4. How can problems of dominance by one student and conflict avoidance within a group be addressed?

- Group problems are part of each test. One common solution for all members.

- Assign and rotate roles:
  - Manager
  - Skeptic
  - Checker/Recorder
  - Summarizer

- Most of grade is based on individual problem solving.

- Students discuss how they worked together and how they could be more effective.
5. How can individual accountability be addressed?

- assign and rotate roles, group functioning;
- seat arrangement -- eye-to-eye, knee-to-knee;
- individual students randomly called on to present group results;
- a group problem counts as a test question -- if group member was absent the week before, he or she cannot take group test;
- most of the test is taken individually. The final exam is all individual. All lab reports are individual.
Identify Critical Failure Points

Fail Gracefully
Non-optimal implementation gives some success

1. Inappropriate Tasks
   Engage all group members (not just one who knows how to do it)

2. Inappropriate Grading
   Don’t penalize those who help others (no grading on the curve)
   Reward for individual learning

3. Poor structure and management of Groups
Scaffolding

Control of Equations that are Allowed

Useful Mathematical Relationships:

\[ \sin \theta = \frac{a}{c}, \quad \cos \theta = \frac{b}{c}, \quad \tan \theta = \frac{a}{b}, \]
\[ a^2 + b^2 = c^2, \quad \sin^2 \theta + \cos^2 \theta = 1 \]

For a circle: \( C = 2\pi R, \quad A = \pi R^2 \)

If \( Ax^2 + Bx + C = 0 \), then \( x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \)

\[ \frac{d(x^n)}{dx} = nx^{n-1}, \quad \frac{d(\cos x)}{dx} = -\sin x, \quad \frac{d(\sin x)}{dx} = \cos x, \quad \frac{d(e^x)}{dx} = e^x, \quad \frac{d(\ln x)}{dx} = \frac{1}{x}, \]
\[ \frac{df(x)}{dt} = \frac{dx}{dt} \frac{df(x)}{dx} \]

Fundamental Concepts, Principles, and Definitions:

<table>
<thead>
<tr>
<th>( \sum F = m\ddot{a} )</th>
<th>( \rho = \frac{m}{V} )</th>
<th>( E_f - E_i - E_{in} - E_{out} )</th>
<th>( \text{KE} = \frac{1}{2} mv^2 )</th>
<th>( P = \frac{F}{A} )</th>
<th>( \varepsilon = \frac{E_{\text{desired}}}{E_{\text{input}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau = rF_{\perp} )</td>
<td>( \frac{dW}{d\ell} = F_{\ell} )</td>
<td>( f = \frac{1}{T} )</td>
<td>( \frac{dU}{dV} = -F_{\text{internal}} )</td>
<td>( S = k\ln \Omega )</td>
<td>( \theta = \frac{\delta C}{r} )</td>
</tr>
<tr>
<td>( F = U - TS )</td>
<td>( \frac{dx}{dt} = \dot{x} )</td>
<td>( \frac{dv_x}{dt} = a_x )</td>
<td>( s_{ev} = \frac{\text{distance}}{\Delta t} )</td>
<td>( v_{x,av} = \frac{\Delta x}{\Delta t} )</td>
<td>( a_{x,av} = \frac{\Delta v_x}{\Delta t} )</td>
</tr>
</tbody>
</table>

Under Certain Conditions:

<table>
<thead>
<tr>
<th>( F = mg )</th>
<th>( F - kx )</th>
<th>( F = \mu_k n )</th>
<th>( F &lt; \mu_s n )</th>
<th>( F = G \frac{m_1 m_2}{r^2} )</th>
<th>( F = k_c \frac{Q_1 Q_2}{r^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum \tau = 0 )</td>
<td>( \Delta E_{\text{internal}} = mL )</td>
<td>( \Delta E_{\text{internal}} = Cm\Delta T )</td>
<td>( PV = NkT )</td>
<td>( PV = nRT )</td>
<td>( \frac{dW}{dV} = P )</td>
</tr>
<tr>
<td>( U = mgh )</td>
<td>( U = \frac{1}{2} kx^2 )</td>
<td>( W = -T\Delta S )</td>
<td>( \Delta S = \frac{Q}{T} )</td>
<td>( F = bv )</td>
<td>( a = \frac{v^2}{r} )</td>
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</table>

Useful constants: 1 mile = 5280 ft, \( g = 9.8 \text{ m/s}^2 \) = 32 \text{ ft/s}^2, \( k_B = 1.4 \times 10^{-23} \text{ J/K} \), \( N_{av} = 6 \times 10^{23} \), \( R = 8.3 \text{ J/(mol K)} \), \( \rho_{\text{water}} = 1 \text{ g/cm}^3 \)
Scaffolding

Grading Guidance

This is a closed book, closed notes quiz. Calculators are permitted. The ONLY formulas that may be used are those given below. Define all symbols and justify all mathematical expressions used. Make sure to state all of the assumptions used to solve a problem. Credit will be given only for a logical and complete solution that is clearly communicated with correct units. Partial credit will be given for a well communicated problem solving strategy based on correct physics. MAKE SURE YOUR NAME, ID #, SECTION #, and TAs NAME ARE ON EACH PAGE!! START EACH PROBLEM ON A NEW PAGE. Each problem is worth 25 points: In the context of a unified solution, partial credit will be awarded as follows:

- a useful picture, defining the question, and giving your approach is worth 6 points;
- a complete physics diagram defining the relevant quantities, identifying the target quantity, and specifying the relevant equations with reasons is worth 6 points;
- planning the solution by constructing the mathematics leading to an algebraic answer and checking the units of that answer is worth 7 points;
- calculating a numerical value with correct units is worth 3 points; and
- evaluating the validity of the answer is worth 3 points.

The multiple choice questions are each worth 1.5 points.
Your task is to design an artificial joint to replace arthritic elbow joints. After healing, the patient should be able to hold at least a gallon of milk while the lower arm is horizontal. The biceps muscle is attached to the bone at the distance 1/6 of the bone length from the elbow joint, and makes an angle of 80° with the horizontal bone. How strong should you design the artificial joint if you can assume the weight of the bone is negligible.
**Student 1**

- **Knowns**
  - \( \theta = 80^\circ \)
  - \( V_m = 3.76 \text{ L} = 3760 \text{ ml} \)

- **Target**: \( F_j = \) Force of joint

- **Approach**: Use forces
  - \( \Sigma F_x = 0 \)
  - \( \Sigma F_y = 0 \)
  - \( \Sigma T = 0 \)

- Assume density of milk is similar to water, \( 1 \text{ g/cm}^3 = 1 \text{ g/ml} = 0.001 \text{ kg/ml} \)

- \( \Sigma F_x = 0 \)
  - \( F_{jx} - F_{bx} = 0 \)

- \( \Sigma F_y = 0 \)
  - \( F_{by} - mg - F_{jy} = 0 \)

- \( \Sigma T = 0 \) (joint is pivot point)

---

**Equation**

\[
F_j^2 = F_{jx}^2 + F_{jy}^2
\]

\[
F_{jx} - F_{bx} = 0
\]

\[
F_{by} - mg - F_{jy} = 0
\]

\[
F_{bx} = \frac{F_{by}}{\sin \theta}
\]

\[
F_{by} = F_{bx} \cos \theta
\]

\[
F_{jx} = \frac{bm g \cos \theta}{\sin \theta}
\]

\[
F_{jy} = F_{by} - mg = bm g - mg
\]

\[
F_j = \sqrt{\left(\frac{bm g \cos \theta}{\sin \theta}\right)^2 + (bm g - mg)^2}
\]

- **Plug in**
  - \( \theta = 80^\circ \)
  - \( V_m = 3760 \text{ ml} \)

\[
F_j = \sqrt{\left(\frac{3760 \text{ kg/m} \cdot \text{m}^3 \cdot \text{m} \cdot \text{m}}{\sin 80^\circ} \right)^2 + (3760 \text{ kg/m} \cdot \text{m}^3 \cdot \text{m} \cdot \text{m})^2}
\]

\[
= 43.3 \text{ N}
\]

**Units**

\[
\left(\frac{\text{kg/m} \cdot \text{m}^3 \cdot \text{m} \cdot \text{m}}{\text{s}^2}\right)^2 = \left(\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}\right) = \text{kg/m} \cdot \text{s}^2 = \text{m/s}^2
\]

\[
= \text{kg/m} \cdot \text{s}^2 = 1
\]

\[
\checkmark
\]
Question: What is the minimum force for the artificial joint?

\[ L = \text{length of bone} \]
\[ \theta = 80^\circ \]

Let \( L \) be the pivot point (where elbow bends)

\[ 2F_x \]
\[ 2F_y \]
\[ \gamma \]

\[ 2\gamma = TM \sin \theta \left( \frac{L}{2} \right) - NL = 0 \]
\[ TM(\sin \theta) \left( \frac{L}{2} \right) = Wx \]
\[ W = \frac{1}{2} TM \sin \theta \]

\[ F_y = TM \sin \theta - W \]
\[ F_y = TM \sin \theta - \frac{L}{2} TM \sin \theta \]
\[ = \frac{5}{6} TM \sin \theta \]

\[ F_x = + TM \cos \theta \]

\[ \frac{F_y}{F_x} = \frac{\frac{5}{6} TM \sin \theta}{\sqrt{TM \cos \theta}} \]
\[ = \frac{5}{6} \tan \theta \]

Force of the bone should be \( \frac{5}{6} \tan \theta \) (in this case, \( \frac{5}{6} \tan 80^\circ = 4.7 \text{ N} \))

It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.
Problem Solving Assessment – Not Grading
Almost Independent Dimensions

• **Useful Description**
  – organize information from the problem statement symbolically, visually, and/or in writing.

• **Physics Approach**
  – select appropriate physics concepts and principles

• **Specific Application of Physics**
  – apply physics approach to the specific conditions in problem

• **Mathematical Procedures**
  – follow appropriate & correct math rules/procedures

• **Logical Progression**
  – overall the solution progresses logically; it is coherent, focused toward a goal, and consistent (not necessarily linear)

Based on previous work at Minnesota by:
J. Blue (1997); T. Foster (2000); T. Thaden-Koch (2005);
P. Heller, R. Keith, S. Anderson (1992)
## Problem solving rubric at a glance

<table>
<thead>
<tr>
<th>CATEGORY: (based on literature)</th>
<th>SCORE</th>
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<tr>
<td></td>
<td>5</td>
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<th></th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>NA (P)</th>
<th>NA (S)</th>
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<tbody>
<tr>
<td>Useful Description</td>
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<td>Physics Approach</td>
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<td>Specific Application</td>
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<td>Math Procedures</td>
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<td>Logical Progression</td>
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</table>

**Want**
- **Minimum** number of categories that include relevant aspects of problem solving
- **Minimum** number of scores that give enough information to improve instruction
Rubric Scores (in general)

<table>
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<tr>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete &amp; appropriate</td>
<td>Minor omission or errors</td>
<td>Parts missing and/or contain errors</td>
<td>Most missing and/or contain errors</td>
<td>All inappropriate</td>
<td>No evidence of category</td>
</tr>
</tbody>
</table>

NOT APPLICABLE (NA):

<table>
<thead>
<tr>
<th>NA - Problem</th>
<th>NA - Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not necessary for this problem <em>(i.e. visualization or physics principles given)</em></td>
<td>Not necessary for this solver <em>(i.e. able to solve without explicit statement)</em></td>
</tr>
</tbody>
</table>
**Useful Description** assesses a solver’s skill at organizing information from the problem statement into an appropriate and useful representation that summarizes essential information symbolically and visually. The description is considered “useful” if it guides further steps in the solution process. A *problem description* could include restating known and unknown information, assigning appropriate symbols for quantities, stating a goal or target quantity, a visualization (sketch or picture), stating qualitative expectations, an abstracted physics diagram (force, energy, motion, momentum, ray, etc.), drawing a graph, stating a coordinate system, and choosing a system.

5  The description is useful, appropriate, and complete
4  The description is useful but contains minor omissions or errors.
3  Parts of the description are not useful, missing, and/or contain errors.
2  Most of the description is not useful, missing, and/or contains errors.
1  The entire description is not useful and/or contains errors.
0  The solution does not include a description and it is necessary for this problem /solver.

NA (P) A description is not necessary for this problem. (i.e., it is given in the problem statement)
NA (S) A description is not necessary for this solver.
Physics Approach assesses a solver’s skill at selecting appropriate physics concepts and principle(s) to use in solving the problem. Here the term concept is defined to be a general physics idea, such as the basic concept of “vector” or specific concepts of “momentum” and “average velocity”. The term principle is defined to be a fundamental physics rule or law used to describe objects and their interactions, such as the law of conservation of energy, Newton’s second law, or Ohm’s law.

5  The physics approach is appropriate, and complete
4  Some concepts and principles of the physics approach are missing and/or inappropriate.
3  Most of the physics approach is missing and/or inappropriate.
2  All of the chosen concepts and principles are inappropriate.
1  The entire description is not useful and/or contains errors.
0  The solution does not indicate an approach, and it is necessary for this problem/ solver.
NA (P) A physics approach is not necessary for this problem. (i.e., it is given in the problem statement)
NA (S) An explicit physics approach is not necessary for this solver.
Specific Application of Physics assesses a solver’s skill at applying the physics concepts and principles from their selected approach to the specific conditions in the problem. If necessary, the solver has set up specific equations for the problem that are consistent with the chosen approach. A specific application of physics could include a statement of definitions, relationships between the defined quantities, initial conditions, and assumptions or constraints in the problem (i.e., friction negligible, massless spring, massless pulley, inextensible string, etc.)

5  The specific application of physics is appropriate and complete.
4  The specific application of physics contains minor omissions or errors.
3  Parts of the specific application of physics are missing and/or contain errors.
2  Most of the specific application of physics is missing and/or contains errors.
1  All of the application of physics is inappropriate and/or contains errors.
0  The solution does not indicate an application of physics and it is necessary.
NA (P) A specific application of physics is not necessary for this problem.
NA (S) A specific application of physics is not necessary for this solver.
*Mathematical Procedures* assesses a solver’s skill at following appropriate and correct mathematical rules and procedures during the solution execution. The term *mathematical procedures* refers to techniques that are employed to solve for target quantities from specific equations of physics, such as isolate and reduce strategies from algebra, substitution, use of the quadratic formula, or matrix operations. The term *mathematical rules* refers to conventions from mathematics, such as appropriate use of parentheses, square roots, and trigonometric identities. If the course instructor or researcher using the rubric expects a symbolic answer prior to numerical calculations, this could be considered an appropriate mathematical procedure.

5  The mathematical procedures are appropriate and complete.
4  Appropriate mathematical procedures are used with minor omissions or errors.
3  Parts of the mathematical procedures are missing and/or contain errors.
2  Most of the mathematical procedures are missing and/or contain errors.
1  All mathematical procedures are inappropriate and/or contain errors.
0  There is no evidence of mathematical procedures, and they are necessary.
NA (P) Mathematical procedures are not necessary for this problem or are very simple.
NA (S) Mathematical procedures are not necessary for this solver.
*Logical Progression* assesses the solver’s skills at communicating reasoning, staying focused toward a goal, and evaluating the solution for consistency (implicitly or explicitly). It checks whether the entire problem solution is clear, focused, and organized logically. The term *logical* means that the solution is coherent (the solution order and solver’s reasoning can be understood from what is written), internally consistent (parts do not contradict), and externally consistent (agrees with physics expectations).

5  The entire problem solution is clear, focused, and logically connected.
4  The solution is clear and focused with minor inconsistencies.
3  Parts of the solution are unclear, unfocused, and/or inconsistent.
2  Most of the solution parts are unclear, unfocused, and/or inconsistent.
1  The entire solution unclear, unfocused, and/or inconsistent.
0  There is no evidence of logical progression, and it is necessary.

NA (P) Logical progression is not necessary for this problem. (i.e., one-step)
NA (S) Logical progression is not necessary for this solver.
Some References


The End

Please visit our website for more information:

http://groups.physics.umn.edu/physed/

The best is the enemy of the good.
"le mieux est l'ennemi du bien"

Voltaire
Assessment

• Problem Solving Skill
• Drop out rate
• Failure rate
• FCI – some mechanics concepts
• BEMA – some E&M concepts
• CLASS – attitudes toward learning physics
• Math Skills
• What students value in the course
• Engineering student longitudinal study
• Faculty use
• Adoption by other institutions and other disciplines
Question: How far away from the tree does the fruit and acorn accelerate?  

Approach: Use conservation of momentum and kinematics. Assume constant acceleration due to gravity. Assume no wind is lost in the collision. Assume wind resistance.

We know initially from the tree the acorn leaves the tree with just before it is hit by the wind and just after it hits the fruit until the hit by the wind.

Given is the height and gravity for the first part and the wind and gravity for the second part.

Plan the Solution: 

\[ d = v_{ox} t \]
\[ v_{xf} = \frac{m_v}{m_v + m_p} v_{xo} \]
\[ v_{xo} = v_0 \cos \theta \]
\[ t = \sqrt{\frac{2h}{g}} \]
\[ d = \frac{m_v}{m_v + m_p} v_0 \cos \theta \sqrt{\frac{2h}{g}} \]

Check units: 

\[ m = \frac{m_v}{m_v + m_p} \frac{v_0}{\sqrt{g}} \]
\[ m_1 = \frac{m_v}{m_v + m_p} \frac{v_0}{\sqrt{g}} \]
\[ m = m \rightarrow \text{OK} \]

Is the answer complete? 
Yes, the distance was found in terms of the requested units.

Is the answer reasonable? 
Yes, the units check out ok and \( d \) will be smaller than \( h \) due to conservation of moment.

Is the answer correctly stated? 
Yes, it is in units of distance, meters.
Improvement in Problem Solving

Logical Progression

General Approach - does the student understand the physics
Specific Application of the Physics - starting from the physics they used, how did the student apply this knowledge?
Logical Progression - is the solution logically presented?
Appropriate Mathematics - is the math correct and useful?

Algebra based physics 1991
Each letter represents a different professor (39 different ones)

- Incoming student scores are slowly rising (better high school preparation)
- Our standard course (CGPS) achieves average FCI \( \sim 70\% \)
- Our “best practices” course achieves average FCI \( \sim 80\% \)
- Not executing any cooperative group procedures achieves average FCI \( \sim 50\% \)
FCI by discussion/lab section

Same symbol (color and shape) is the same TA
Retention

Drop % Physics 1301

Change from quarters to semesters

Dropout rate ~ 6%, F/D rate ~ 3% in all classes
Males and females do about as well in the course.
CLASS LEARNING ATTITUDES SURVEY BY CATEGORY (PRE-POST)
1202 PHYSICS BIOLOGY & PRE-MEDICINE SPRING 2009

CLASS LEARNING ATTITUDES SURVEY BY CATEGORY
Experienced TAs FALL 2009
**Student Opinion Data: Algebra-based Physics 1998**

Rate the usefulness of the following components of the course. Use a scale from 1 to 10 with 10 being extremely useful and 1 being completely useless in helping you learn physics in this course.

<table>
<thead>
<tr>
<th>Component</th>
<th>Ave.</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ave. All Sections (N = 393)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>108. Textbook</td>
<td>6.6 ± 0.13</td>
<td>1</td>
</tr>
<tr>
<td>106. Discussion Sessions (CGPS)</td>
<td>6.5 ± 0.13</td>
<td>2</td>
</tr>
<tr>
<td>101. Homework (not graded)</td>
<td>6.4 ± 0.14</td>
<td>3</td>
</tr>
<tr>
<td>105. Quizzes and Exams</td>
<td>6.1 ± 0.12</td>
<td>4</td>
</tr>
<tr>
<td>103. Lectures</td>
<td>6.1 ± 0.13</td>
<td>5</td>
</tr>
<tr>
<td>102. Laboratory</td>
<td>5.5 ± 0.12</td>
<td>6</td>
</tr>
<tr>
<td>109. Material on Class Web Pages</td>
<td>5.3 ± 0.14</td>
<td>7</td>
</tr>
<tr>
<td>107. TA’s in tutoring room</td>
<td>4.6 ± 0.14</td>
<td>8</td>
</tr>
<tr>
<td>110. University tutors in Lind Hall</td>
<td>4.2 ± 0.14</td>
<td>9</td>
</tr>
<tr>
<td>104. Lecturer Office Hours</td>
<td>3.9 ± 0.12</td>
<td>10</td>
</tr>
</tbody>
</table>
## CGPS Propagates Through the Department

### Algebra-based Course (24 different majors) 1987

**Goals:** Calculus-based Course (88% engineering majors) 1993

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td><strong>Basic principles behind all physics</strong></td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td><strong>General qualitative problem solving skills</strong></td>
<td></td>
</tr>
<tr>
<td>4.4</td>
<td><strong>General quantitative problem solving skills</strong></td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td><strong>Apply physics topics covered to new situations</strong></td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td><strong>Use with confidence</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Biology Majors Course 2003

**Goals:**  
- 4.9 **Basic principles behind all physics**  
- 4.4 **General qualitative problem solving skills**  
- 4.3 **Use biological examples of physical principles**  
- 4.2 **Overcome misconceptions about physical world**  
- 4.1 **General quantitative problem solving skills**  
- 4.0 **Real world application of mathematical concepts and techniques**

### Upper Division Physics Major Courses 2002  
- Analytic Mechanics  
- Electricity & Magnetism  
- Quantum Mechanics

### Graduate Courses 2007  
- Quantum Mechanics
Scaffolding

Problem Solving Worksheet used at the beginning of the course

Page 1

Page 2
Physics 1001 (Energy & Environment) Conceptual course

Small increase in force and motion concepts

Students have reasonably high math skills

Students do not significantly change attitude toward science

Not using cooperative group problem solving
Significant gain in force and motion concepts

Students have reasonably high math skills

Students can decrease or increase their attitude toward science
Physics 1202 (Biology & Pre-Meds)
Calculus Based

Significant gain in E&M concepts

Students have reasonably high math skills

Students can decrease or increase their attitude toward science
Physics 1101
Algebra Based

Significant gain in force and motion concepts

Students have reasonably high math skills

Students perhaps decrease their attitude toward science
Physics 1301 (Engineer & Physical Sci) Calculus Based

Significant gain in force & motion concepts

Students have reasonably high math skills

Students decrease their attitude toward science
Physics 1302 (Engineer & Physical Sci) Calculus Based

Significant gain in E&M concepts

Students have reasonably high math skills

Students decrease their attitude toward science
The Advantages of Using Cooperative Group Problem Solving

1. Using a problem solving framework seems too long and complex for most students.

   The cooperative-group provides the motivation and knowledge to practice the parts until the framework becomes more natural.

2. Complex problems that need organization are initially difficult.

   Groups can successfully solve them so students see the advantage of a logical problem-solving framework early in the course.
3. The group interaction allows individuals to observe the planning and monitoring skills needed to solve problems. (Metacognition)

4. Students practice the language of physics -- "talking physics."

5. Students must deal with and resolve their misconceptions.

6. Coaching by instructors is more effective – student groups are not sufficient, a more knowledgeable coach for the groups is required.

   External clues of group difficulties
   Group processing of instructor input
Competent Problem Solving

1. **Focus on the Problem**
   Translate the words into an image of the situation.

   Know $T_{\text{max}}$, $\theta$, $\mu$, $W$
   What is $a_{\text{max}}$?

2. **Describe the Physics**
   Translate the mental image into a physics representation of the problem (e.g., idealized diagram, symbols for important quantities).

3. **Plan a Solution**

Identify an approach to the problem.
Relate forces on car to acceleration using Newton's Second Law.

Assemble mathematical tools (equations).

\[
\sum F = ma \\
f_k = \mu N \\
W = mg
\]
3. **Plan a Solution**

*Translate the physics description into a mathematical representation of the problem.*

**Find a:**

\[ \Sigma F_x = ma_x \]

Find \( \Sigma F_x \):

\[ \Sigma F_x = T_x - f_k \]

Find \( T_x \):

\[ T_x = T \cos \theta \]

4. **Execute the Plan**

*Translate the plan into a series of appropriate mathematical actions.*

\[ \Sigma F_x = T \cos \theta - \mu(W - T \sin \theta) \]

\( (W/g)a_x = T \cos \theta - \mu(W - T \sin \theta) \)

\( a_x = (T \cos \theta - \mu(W - T \sin \theta)) \frac{g}{W} \)

5. **Evaluate the Solution**

**Outline the mathematical solution steps.**

Solve [3] for \( T_x \) and put into [2].

Solve [2] for \( \Sigma F_x \) and put into [1].

Solve [1] for \( a_x \).
Context-rich Problems

- Each problem is a short story in which the major character is the student. The problem statement uses the personal pronoun "you."
- Some decisions are necessary to proceed.
- The problem statement includes a plausible motivation or reason for "you" to calculate something.
- The objects in the problems are real (or can be imagined) – students must practice idealization.
- No pictures or diagrams are given with the problems. Students must visualize the situation by using their own experiences.
- The problem can not be solved in one step by plugging numbers into a formula.
Context-rich Problems

In addition, more difficult context-rich problems can have one or more of the following characteristics:

- The **unknown quantity is not explicitly specified in the problem statement** (e.g., Will this design work?).

- **More information** may be given in the problem statement than is required to solve the problems, or relevant **information may be missing**.

- **Assumptions** may need to be made to solve the problem.

- The problem may require more than one **fundamental principle** for a solution (e.g., Newton's 2\textsuperscript{nd} Law and the Conservation of Energy).

- The context can be very unfamiliar (i.e., involve the interactions in the nucleus of atoms, quarks, quasars, etc.)
Solving This

An infinitely long cylinder of radius $R$ carries a uniform (volume) charge density $\lambda$. Use Gauss’ Law to calculate the field everywhere inside the cylinder.

is NOT Problem Solving?
You are investigating the possibility of producing power from fusion. The device being designed confines a hot gas of positively charged ions in a very long cylinder with a radius of 2.0 cm. The charge density of the ions in the cylinder is $6.0 \times 10^{-5} \text{ C/m}^3$. Positively charged Tritium ions are to be injected perpendicular to the axis of the cylinder in a direction toward the center of the cylinder. Your job is to determine the speed that a Tritium ion should have when it enters the cylinder so that its velocity is zero when it reaches the axis of the cylinder. Tritium is an isotope of Hydrogen with one proton and two neutrons. You look up the charge of a proton and mass of the tritium in your Physics text and find them to be $1.6 \times 10^{-19} \text{ C}$ and $5.0 \times 10^{-27} \text{ Kg}$. 
Physics 1301

Amount Learned

FCI Gain

\[ y = -0.0028x + 4.653 \]

\[ R^2 = 0.0005 \]
Student C

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
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- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interaction
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus
Student C

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\begin{align*}
F_j &= \sqrt{(\text{mass})^2 \cdot \text{sin}^2 \phi + (5 \cdot 3)^2} \\
&= \sqrt{45 \cdot 3^2 + 15 \cdot 3^2} \\
&= 9 \cdot 3 = 27 \text{ N}
\end{align*}
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Forces as interactions
- Newton’s 2nd Law
- \( F_j = \sqrt{(\frac{b_2}{\sin \theta} \cdot \cos \theta)^2 + (5 \cdot v_f)^2} \)
- \( F_j = \sqrt{(\frac{b_2}{\sin \theta} \cdot \cos \theta)^2 + (5 \cdot v_f)^2} \)
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Student C

Organizational Framework
- Visualization
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- System selected
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- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interaction
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus

Math
- Trig
- Algebra
- Calculus
Coaching this student

Organizational Framework
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Math
• Trig
• Algebra
• Calculus

Problem #1 - Page 1 of 1

Question: What is the minimum force for the artificial joint? Let be the pivot point where elbow bands

\[ L = \text{length of bone} \]
\[ \theta = 80^\circ \]

Let be the pivot point where elbow bands

\[ \tan \theta \]

Student A

\[ \begin{align*}
F_y &= T \sin \theta - N \\
F_x &= + T \cos \theta \\
F_x &= \frac{F_y}{F_x} = \frac{T \sin \theta}{T \cos \theta} \\
&= \frac{5}{2} \tan \theta \\
\text{Force by the bone should be } &\frac{5}{2} \tan \theta \text{ (in this case, } \frac{5}{2} \tan 80^\circ = 4.7 \text{ N})
\end{align*} \]
Good visualization except incomplete interaction of elbow with the upper arm.

Organizational Framework
- **Visualization**
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- Calculus
Good question statement. Should have alerted the student to the need for forces at the joint.

Organizational Framework
- Visualization
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Math
- Trig
- Algebra
- Calculus

\[
\begin{align*}
\text{Problem: What is the minimum force for the artificial joint?} \\
\text{Let } b = \text{the pivot point where elbow bends} \\
\text{Use } 2F_x, 2F_y \text{ & } T_m \sin \theta \\
\Rightarrow 2F_x = T_m \sin \theta - nL = 0 \\
\Rightarrow T_m \sin \theta = nL \\
\Rightarrow n = \frac{T_m \sin \theta}{L} \\
F_y = T_m \sin \theta - n \\
\Rightarrow F_y = T_m \sin \theta - \frac{n}{2}T_m \sin \theta \\
F_x = \frac{T_m \cos \theta}{L} \\
F_y = \frac{T_m \sin \theta}{L} \\
\frac{F_y}{F_x} = \frac{\frac{n}{2}T_m \sin \theta}{T_m \cos \theta} \\
\Rightarrow \frac{T_m \sin \theta}{T_m \cos \theta} = \frac{n}{2} \\
\text{Force of the bone should be } \frac{n}{2} \tan \theta \\
\text{(in this case, } \frac{n}{2} \tan \theta = 4.7 \text{ N)}
\end{align*}
\]
Recognized the need to sum forces and torque. No statement about relation to acceleration (Newton’s 2nd Law) or about equilibrium.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
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Physics
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Math
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- Calculus
Not clearly stated. From the drawing, the joint might be part of the system. Need to ask the student. Could account for the missing force.

Organizational Framework
- Visualization
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- System selected
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Physics
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Math
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Student A
Could be a free-body diagram but not used to see missing elbow interaction.
Coordinate system defined but force not put on it to make missing force observable.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
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- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2\textsuperscript{nd} Law

Math
- Trig
- Algebra
- Calculus

\begin{align*}
\text{PROBLEM} \#1 - \text{Page 1 of 1} \\
\text{Question: What is the minimum force for the artificial joint?} \\
\text{Let } \theta \text{ be the pivot point where elbow bends} \\
\text{\textbf{Coordinate system defined but force not put on it to make missing force observable.}} \\
\text{\textbf{Organizational Framework}} \\
\text{\textbf{Visualization}} \\
\text{\textbf{Question statement}} \\
\text{\textbf{Appropriate physics approach}} \\
\text{\textbf{System selected}} \\
\text{\textbf{Useful diagram}} \\
\text{\textbf{Useful symbols}} \\
\text{\textbf{Useful equations}} \\
\text{\textbf{Goal oriented plan}} \\
\text{\textbf{Logical math}} \\
\text{\textbf{A conclusion}} \\
\text{\textbf{Evaluation}} \\
\text{\textbf{Physics}} \\
\text{\textbf{Forces as interactions}} \\
\text{\textbf{Newton’s 2\textsuperscript{nd} Law}} \\
\text{\textbf{Math}} \\
\text{\textbf{Trig}} \\
\text{\textbf{Algebra}} \\
\text{\textbf{Calculus}} \\
\end{align*}
Good use of symbols. Defining $L$ even though it is not given in the problem so it can cancel out.

Organizational Framework

- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics

- Forces as interactions
- Newton’s 2nd Law

Math

- Trig
- Algebra
- Calculus

Student A

Question: What is the minimum force for the artificial joint?

\[ L = \text{length of bone} \]
\[ \theta = 80^\circ \]

Let $L$ be the pivot point where elbow bends.

\[ \begin{align*}
T_x &= T_m \sin \theta + T_y - F_x \\
T_y &= T_m \cos \theta - F_y
\end{align*} \]

\[ F_y = T_m \sin \theta + W \]

\[ F_x = T_m \cos \theta \]

\[ F_{net} = F_y/F_x = \frac{T_m \sin \theta}{T_m \cos \theta} = \frac{\tan \theta}{\cos \theta} \]

\[ F_{net} = \frac{5}{6} \tan \theta \]

In this case, $\frac{5}{6} \tan 80^\circ = 4.7 N$

It makes sense that the force of a joint should depend on the angle of the muscle it is connected to for it affects not only movement, but strength of bone.
Does not write equations for the sum of the forces or torques. This might have helped.

Recovers for torques but not forces.

Strange idea of sum = ratio.

Organizational Framework

• Visualization
• Question statement
• Appropriate physics approach
• System selected
• Useful diagram
• Useful symbols
• Useful equations
• Goal oriented plan
• Logical math
• A conclusion
• Evaluation

Physics

• Forces as interactions
• Newton’s 2nd Law

Math

• Trig
• Algebra
• Calculus
No goal oriented plan. Calculates $F_{net}$, described as force of the bone (on what?) Seems to have lost track of the question. Needs an early definition of the target.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus
The math is a logical progression but without a goal.

Organizational Framework
• Visualization
• Question statement
• Appropriate physics approach
• System selected
• Useful diagram
• Useful symbols
• Useful equations
• Goal oriented plan
• Logical math
• A conclusion
• Evaluation

Physics
• Forces as interactions
• Newton’s 2nd Law

Math
• Trig
• Algebra
• Calculus

Forces as interactions

\[ F_y = T_m \sin \theta - W \]

\[ \frac{F_y}{F_x} = \frac{\frac{5}{6} T_m \sin \theta}{T_m \cos \theta} = \frac{5}{6} \tan \theta \]

Force of the bone should be \( \frac{5}{6} \tan \theta \)

In this case, \( \frac{5}{6} \tan 80^\circ = 4.7 \text{ N} \)

It makes sense that the force on a joint should depend on the angle of the muscle it's connected to for it effects not only movement, but strength of bone.
There is a conclusion but not an answer to the question.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2\textsuperscript{nd} Law

Math
- Trig
- Algebra
- Calculus

Student A

\[ F_x = F_x = \frac{5}{6} T \sin \theta \\ \text{Force of the bone should be } \frac{5}{6} \tan \theta \\
\text{(in this case, } \frac{5}{6} \tan 80^\circ \approx 4.7 \text{ N} \)
The evaluation is confused representing the lack of connection of the calculation to a goal.
There is no checking of the units which would have revealed a difficulty.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus
Joint does not interact with upper arm if it is part of the system. Bone does not interact with joint if it is not.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus

Student A
No use of 2nd Law for forces although it is used for torques.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus

Student A
Trigonometry is fine.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus

---

Problem #1

*Question*: What is the minimum force for the artificial joint?

\[ F_y = T_M \sin \theta - W \]

\[ F_y = \frac{5}{6} T_M \sin \theta \]

\[ \frac{F_y}{F_x} = \frac{5/6 T_M \sin \theta}{T_M \cos \theta} \]

\[ \frac{5}{6} \tan \theta \]

---

Force of the bone should be \( \frac{5}{6} \tan \theta \)

In this case, \( \frac{5}{6} \tan 80^\circ = 4.7 \text{ N} \)

It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.
Algebra is fine except when dealing with units.

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton’s 2nd Law

Math
- Trig
- Algebra
- Calculus

Student A

Student A
Some problems given on tests that do not help most students from learn either problem solving or physics concepts.

A block of mass \( m = 3 \text{ kg} \) and a block of unknown mass \( M \) are connected by a massless rope over a frictionless pulley, as shown below. The kinetic frictional coefficient between the block \( m \) and the inclined plane is \( \mu_k = 0.17 \). The plane makes an angle 30\(^{\circ} \) with horizontal. The acceleration, \( a \), of the block \( M \) is 1 m/s\(^2 \) downward.

(a) Draw free-body diagrams for both masses. [5 points]
(b) Find the tension in the rope. [5 points]
(c) If the block \( M \) drops by 0.5 m, how much work, \( W \), is done on the block \( m \) by the tension in the rope? [15 points]
The system of three blocks shown is released from rest. The connecting strings are massless, the pulleys ideal and massless, and there is no friction between the 3kg block and the table.

a) At the instant $M_3$ is moving at speed $v$, how far $d$ has it moved from the point where it was released from rest? (answer in terms of $M_1$, $M_2$, $M_3$, $g$ and $v$.) [10 pts]

b) At the instant the 3 kg block is moving with a speed of 0.8 m/s, how far, $d$, has it moved from the point where it was released from rest? [5 pts]

c) From the instant when the system was released from rest, to the instant when the 1 kg block has risen a height $h$, which statement (1, 2 or 3) is true for the three-block system? (1) The total mechanical energy of the system increases. (2) The total potential energy of the system increases. (3) The net work done on the system by the tension forces is 0. [5pts]

d) Now suppose the table is rough and has a coefficient of kinetic friction $\mu_k = 0.1$. What is the speed, $v$, of the 3 kg block after the 2 kg block drops by 0.5 m? (Assume again that the system is released from rest.) [5pts]
Highest Rated Goals

Goals: Biology Majors Course 2003
4.9 Basic principles behind all physics
4.4 General qualitative problem solving skills
4.3 Use biological examples of physical principles
4.2 Overcome misconceptions about physical world
4.1 General quantitative problem solving skills
4.0 Real world application of mathematical concepts and techniques

Goals: Calculus-based Course (88% engineering majors) 1993
4.5 Basic principles behind all physics
4.5 General qualitative problem solving skills
4.4 General quantitative problem solving skills
4.2 Apply physics topics covered to new situations
4.2 Use with confidence

Goals: Algebra-based Course (24 different majors) 1987
4.7 Basic principles behind all physics
4.2 General qualitative problem solving skills
4.2 Overcome misconceptions about physical world
4.0 General quantitative problem solving skills
4.0 Apply physics topics covered to new situations
The concept test is correlated with the math skills test.
Can a Math Skills Test be used as a placement test?

The Math Skills Test is not a good predictor of performance.
Can the FCI be used as a placement test?

The FCI is not a good predictor of performance.
FINAL EXAM GRADES BY GENDER
CALCULUS-BASED PHYSICS FOR SCIENTISTS & ENGINEERS, FALL TERMS 1997-2007

MALES AVERAGE
61.0±0.3%

FEMALES AVERAGE
57.1±0.5%

FINAL EXAM GRADE (%)
Math Diagnostic Test

**Powers of ten**

\[
\frac{4 \times 10^{-3}}{10^{-4}} = ?
\]

(a) 4 x 10^{-7} [10-20%]  
(b) 4 \times 10^{-3/4}  
(c) 4 [20-28%]  
(d) 40 [51-63%]  
(e) 4 \times 10^7

**Triangles**

For this right triangle, \( \cos \theta = ? \)

- (a) \( \frac{2b}{3c} \)  
- (b) \( \frac{a}{3c} \)  
- (c) \( \frac{2b}{a} \) [7-16%]  
- (d) \( \frac{3c}{a} \) [69-89%]  
- (e) \( \frac{a}{2b} \)

**Graphs**

The slope of the curve pictured is equal to:

(a) 0 m/s  
(b) 1/3 m/s [85-96%]  
(c) 2 m/s  
(d) 3 m/s [4-12%]  
(e) 6 m/s
Algebra

Solve for \( a \) in the equation \( a^2x + cy = t \)

(a) \( \pm \sqrt{t - cy - x} \)  
(b) \( \pm \sqrt{\frac{t - cy}{x}} \)  
(c) \( \pm \frac{1}{a} \sqrt{t - cy} \)

(d) \( \frac{t - cy}{2x} \)  
(e) \((cy - t)(cy + t)\)

Solve for \( y \) in the equation \( \frac{ax + b}{cy + d} = f \)

(a) \( \frac{ax + b - df}{cf} = y \)  
(b) \( \frac{ax + b}{f + d} \)  
(c) \( \frac{ax + b}{d} \left( \frac{1}{cf} \right) \)

(d) \( \frac{ax + b}{cf + d} \)  
(e) \( \frac{1}{c} \left( \frac{f}{ax + b} - d \right) \)
Simultaneous Equations  

If you know \( at = b \) and \( cx + dt = f \) and the values of \( a, b, c, d \) and \( f \), but you don't know the value of \( t \), solve for the value of \( x \).

\[
\begin{align*}
(a) & \quad \frac{f + dt}{c} \\
(b) & \quad \frac{b + f}{c(a + d)} \\
(c) & \quad \frac{f}{c} - \frac{db}{ac} \quad [65-88\%]
\end{align*}
\]

\[
\begin{align*}
(d) & \quad \frac{f}{c} - \frac{db}{a} \\
(e) & \quad \frac{b}{a}
\end{align*}
\]

If you know \( \frac{b}{2}y^2 - cd^2 = 0 \), \( ax + y = d \) and the values of \( a, b, c \) and \( d \) but you don't know the value of \( y \), solve for the value of \( x \).

\[
\begin{align*}
(a) & \quad \frac{y - d}{a} \\
(b) & \quad \frac{d}{a} \left(1 \pm \sqrt{\frac{2c}{b}}\right) \quad [22-40\%]
\end{align*}
\]

\[
\begin{align*}
(c) & \quad \frac{d}{a} \pm \frac{1}{a} \sqrt{\frac{2cd}{b}} \quad [31-45\%]
\end{align*}
\]

\[
\begin{align*}
(d) & \quad \frac{b}{2} (d - ax)^2 - cd^2 \quad [9-28\%]
\end{align*}
\]

\[
\begin{align*}
(e) & \quad \frac{d}{a} - \frac{2cd^2}{ab}
\end{align*}
\]
Derivatives

If \( z = ax^3 + bx + c \), then \( \frac{dz}{dx} = ? \)

(a) \( ax^2 + b \)  
(b) \( a + b + c \)  
(c) \( 3ax^2 + 2b \)

(d) \( 3ax^2 + b + c \)  
(e) \( 3ax^2 + b \) [73-93%]

If \( z = ae^{bt} \), where \( a \) and \( b \) are not functions of \( t \), then \( \frac{dz}{dt} = ? \)

(a) \( bz \) [4-15%]  
(b) \( ae^b \) [7-27%]  
(c) \( az \)

(d) \( abe^t \) [39-58%]  
(e) \( abe^b \) [6-21%]
Anti-Derivatives

If \( \frac{dx}{dt} = 5at^3 + b \), where \( a \) and \( b \) are constants, then \( x = ? \)

(a) \( 15at^2 \) [7-19%]  
(b) \( \frac{5}{4}at^4 + bt + c \) [60-88%]  
(c) \( \frac{5}{4}at^4 + b \)  
(d) \( 5at^2 \)  
(e) \( \frac{5}{4}at^4 \)

If \( \frac{dz}{dt} = -ab^2 \sin(b^2 t) \), where \( a \) and \( b \) are constants, then \( z = ? \)

(a) \( 2abcos(t) + k \)  
(b) \( -2absin(b^2 t) + k \)  
(c) \( -2absin(bt) + k \)  
(d) \( acos(b^2 t) + k \) [33-63%]  
(e) \( -2abcos(bt) + k \) [17-30%]
RUBRIC SCORE VS. PROBLEM GRADE
TEST 1 PROBLEM 2 (SECTION 2, N=110)

y = 0.7656x + 0.1811

$R^2 = 0.84$

R=0.92
The Teaching Process – A Physicist View

Transformation Process

Initial State of Learner

Content
Course Structure
Pedagogy

Instructor

Desired Final State of Learner

<final | T | initial>

F. Reif (1986)
Phys. Today 39
The Teaching Process
The Clear Explanation Misconception

Common Source of Frustration of Faculty, TAs, Students, & Administrators

Instructor pours knowledge into students by explaining things clearly.

Little knowledge is retained.  
Student’s Fault

Impedance mismatch between student and instructor
Instructor’s Fault

Learning is much more complicated

Leonard et. al. (1999). Concept-Based Problem Solving.
An Appropriate Problem

Your task is to design an artificial joint to replace arthritic elbow joints. After healing, the patient should be able to hold at least a gallon of milk while the lower arm is horizontal. The biceps muscle is attached to the bone at the distance 1/6 of the bone length from the elbow joint, and makes an angle of 80° with the horizontal bone. How strong should you design the artificial joint if you can assume the weight of the bone is negligible.

Gives a motivation – allows some students to access their mental connections. Gives a realistic situation – allows some students to visualize the situation. Does not give a picture – students must practice visualization. Uses the character “you” – allows some students to visualize the situation. Requires decisions – students practice decision making.
The result of students “natural” problem solving inclinations

Circled work by evaluators
Desired Student Solution

Plan the Solution:

\[ d = V_{xf} \cdot t \]
\[ V_{xf} = \frac{m}{m + m_i} V_{xo} \]
\[ V_{xo} = V_0 \cos \theta \]
\[ t = \sqrt{\frac{2h}{g}} \]
\[ d = \frac{m}{m + m_i} V_0 \cos \theta \sqrt{\frac{2h}{g}} \]

Check Units:
\[ m = \frac{m_0}{k} \]
\[ m_i = \frac{m_0}{k} \]
\[ m = m_i = 0 \text{ kg} \]

Is the answer complete?
Yes, the distance was found in terms of the requested values.

Is the answer reasonable?
Yes, the units check out ok and \( d \) will be smaller than \( h \) due to conservation of momentum.

Is the answer correctly stated?
Yes, it is in units of distance, meters.
A block of mass \( m = 2.5 \text{ kg} \) starts from rest and slides down a frictionless ramp that makes an angle of \( \theta = 25^\circ \) with respect to the horizontal floor. The block slides a distance \( d \) down the ramp to reach the bottom. At the bottom of the ramp, the speed of the block is measured to be \( v = 12 \text{ m/s} \).

(a) Draw a diagram, labeling \( \theta \) and \( d \). [5 points]
(b) What is the acceleration of the block, in terms of \( g \)? [5 points]
(c) What is the distance \( d \), in meters? [15 points]

**Better**

A 2.5 kg block starts from rest and slides down a frictionless ramp at 25° to the horizontal floor. At the bottom of the ramp, the speed of the block is measured to be 12 m/s.

(a) Draw a diagram, with appropriate labeling. [5 points]
(b) What is the acceleration of the block, in terms of \( g \)? [5 points]
(c) What is the distance the block slides, in meters? [15 points]

**Better**

A 2.5 kg block starts from rest and slides down a frictionless ramp at 25° to the horizontal floor. At the bottom of the ramp, the speed of the block is measured to be 12 m/s. How far did the block slide?

**Better**

A 2.5 kg block starts from the top and slides down a slippery ramp reaching 12 m/s at the bottom. How long is the ramp? The ramp is at 25° to the horizontal floor.

Allow students to practice making decisions about structuring their solution and connecting physics concepts.
A block of mass $m = 2.5$ kg starts from rest and slides down a frictionless ramp that makes an angle of $\theta = 25^\circ$ with respect to the horizontal floor. The block slides a distance $d$ down the ramp to reach the bottom. At the bottom of the ramp, the speed of the block is measured to be $v = 12$ m/s.
(a) Draw a diagram, labeling $\theta$ and $d$. [5 points]
(b) What is the acceleration of the block, in terms of $g$? [5 points]
(c) What is the distance $d$, in meters? [15 points]

You have been asked to design a simple system to transport boxes from one part of a warehouse to another. The design has boxes placed on the top of the ramp so that they slide to their destination. A box slides easily because the ramp is covered with rollers. Your job is to calculate the maximum length of the ramp if the heaviest box is 25 kg and the ramp is at $5.0^\circ$ to the horizontal. To be safe, no box should go faster than 3.0 m/s when it reaches the end of the ramp.

**Context Rich Problem**
- Allows student decisions.
- Practice making assumptions.
- Connects to student reality.
- Has a motivation (why should I care?).
Beginning Context-Rich Problem

You are working with an insurance company to help investigate a tragic accident. At the scene, you see a road running straight down a hill at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Gives a motivation – allows some students to access their mental connections.  
Gives a realistic situation – allows some students to visualize the situation.  
Does not give a picture – students must practice visualization.  
Uses the character “you” – more easily connects to student’s mental framework.

Decisions must be made
Decisions

You are working with an insurance company to help investigate a tragic accident. At the scene, you see a road running straight down a hill at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

What is happening? – you need a picture.
What is the question? – it is not in the last line.
What quantities are important and what should I name them? – choose symbols.
What physics is important? – difference between average and instantaneous.
What assumptions are necessary? – should friction be ignored?
Is all the information necessary? – the angle? The vertical drop? The time?
You are working with an insurance company to help investigate a tragic accident. At the scene, you see a road running straight down a hill at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

**Not an inclined plane problem**

**Not a projectile motion problem**

Same as this textbook question except students must engage with the content

A block starts from rest and accelerates for 3.0 seconds. It then goes 30 ft. in 5.0 seconds at a constant velocity.

a. What was the final velocity of the block?

b. What was the acceleration of the block?
Competent Problem Solving

Step Bridge

1. **Focus on the Problem**

*Translate the words into a useful image of the situation. Decide on what you know and what you don’t. Decide on the question.***

Know $T_{\text{max}}$, $\theta$, $\mu$, $W$

What is $a_{\text{max}}$?

2. **Describe the Physics**

*Translate the image into a physics representation of the problem (e.g., idealized diagram, symbols for important quantities).*

Assemble mathematical tools (equations).

3. **Plan a Solution**

Identify an approach to the problem.

Relate forces on car to acceleration using Newton's Second Law.

$\sum F = ma$

$f_k = \mu N$

$W = mg$
3. **Plan a Solution**

*Decide on the order of using a mathematical representation of the problem.*

**Find a:**

1. \[ \sum F_x = ma \]
2. \[ \sum F_x = T_x - f_k \]
3. \[ T_x = T \cos \theta \]

**Outline the mathematical solution steps.**

- Solve [3] for \( T_x \) and put into [2].
- Solve [2] for \( \sum F_x \) and put into [1].
- Solve [1] for \( a_x \).

4. **Execute the Plan**

*Translate the plan into a series of appropriate mathematical actions.*

\[
\sum F_x = T \cos \theta - \mu(W - T \sin \theta)
\]

\[
(W/g) a_x = T \cos \theta - \mu(W - T \sin \theta)
\]

\[
a_x = (T \cos \theta - \mu(W - T \sin \theta)) \frac{g}{W}
\]

**Check units of algebraic solution.**

\[
\left[ \frac{m}{s^2} \right] \left[ N \right] - \left[ \frac{m}{s^2} \right] = \left[ \frac{m}{s^2} \right] \quad \text{OK}
\]

5. **Evaluate the Solution**
What is the distance magnitude of the force on the elbow joint?

Approach:

The arm is in equilibrium
- Sum of the horizontal forces = 0.
- Sum of the vertical forces = 0.
- Sum of the torques = 0.

Choose the pivot point at the elbow joint.
Assume density of milk = density of water = 1g/cm³

Force Diagram of the arm bone

\[ \sum F_x = B \cos \theta - E_x = 0 \]
\[ \sum F_y = E_y + B \sin \theta - W = 0 \]

\[ E_x^2 + E_y^2 = E^2 \]

Torques: \( \sum \tau = WL - BL \frac{L}{6} = 0 \)
\[ B_L = B \sin \theta \]

Target: \( E \)

Find \( E_x \)
\[ B \cos \theta = E_x = 0 \]  \[ [2] \]

Find \( B \)
\[ WL - B \sin \theta \frac{L}{6} = 0 \]

Find \( E \)
\[ E_x^2 + E_y^2 = E^2 \]

\[ \frac{6 \rho Vg}{\sin \theta} \cos \theta = 0 \]

\[ 6 \rho Vg \cos \theta - E_x = 0 \]

\[ \frac{6 \rho Vg}{\sin \theta} \cos \theta = E_x \] put into [1]  \[ [1] \]

Solve [1] for \( E_x \)
\[ \left( \frac{6 \rho Vg}{\sin \theta} \right)^2 + \left( \rho Vg - \frac{6 \rho Vg}{\sin \theta} \sin \theta \right)^2 = E^2 \]

\[ \left( \frac{6 \rho Vg}{\sin \theta} \cdot \frac{6}{\tan \theta} \right)^2 + \left( \rho Vg - 6 \rho Vg \sin \theta \right)^2 = E^2 \]

\[ \rho Vg \left( \frac{6}{\tan \theta} \right)^2 + (-5)^2 = E \]  \[ W = mg \]
\[ m = \rho V \]
\[ V = 3.76 \text{ liters} \]  \[ W = \rho Vg \]  \[ [4] \]

Find \( E_y \)
\[ E_y + B \sin \theta - W = 0 \]  \[ [5] \]

5 unknown, 5 equations OK

Solve [5] for \( E_y \)
\[ E_y = W - B \sin \theta \] put into [1]
\[ E_x^2 + E_y^2 = E^2 \]
\[ E_x^2 + (W - B \sin \theta)^2 = E^2 \]  \[ [1] \]

\[ E_x^2 + (\rho Vg - B \sin \theta)^2 = E^2 \]  \[ [1] \]

Solve [3] for \( B \)
\[ \rho Vg - B \sin \theta \frac{1}{6} = 0 \]
\[ \rho Vg = B \sin \theta \frac{1}{6} \]
\[ \frac{6 \rho Vg}{\sin \theta} = B \] put into [1] and [2]
\[ \rho V g \sqrt{\frac{36}{\tan^2 \theta}} + 25 = E \]

Check units
\[ \left[ \frac{\text{kg}}{\text{m}^3} \right] \times \left[ \text{m}^3 \right] = \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right] = \left[ \text{N} \right] = \left[ E \right] \]

Units are correct since newtons is a unit of force and E is the force of the elbow joint on the arm.

Put in numbers
\[ \left( \frac{1 \text{ g}}{\text{cm}^3} \right) \times (3.67 \text{ L}) \times \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) \times \sqrt{\frac{36}{\tan^2 80^\circ}} + 25 = E \]
\[ \left( \frac{1 \text{ g}}{\text{cm}^3} \right) \times (3.67 \text{ L}) \times \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) \times \left( \frac{36}{\tan^2 80^\circ} \right) + 25 = E \]
\[ 184 \left( \frac{\text{g}}{\text{cm}^3} \right) \times \left( \frac{\text{m}}{\text{s}^2} \right) = E \]
\[ 184 \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) \times \left( \frac{\text{g}}{\text{cm}^3} \right) \times \left( \frac{1000 \text{ cm}^3}{\text{L}} \right) \times \left( \frac{\text{L}}{\text{m}^2} \right) \times \left( \frac{\text{m}}{\text{s}^2} \right) = E \]
\[ 184 \left( \frac{\text{kg}}{\text{m}^2} \right) = E \]
\[ 184 \text{N} = E \]

By the 3rd law this is equal to the force of the arm on the elbow joint.

The milk has a weight of 38 N. This means the force on the elbow joint is 5 times greater than the weight of the milk. This seems large but not impossible.

Evaluating the answer equation:

The force on the elbow is greater if the volume of the milk is greater. This is reasonable since then the milk would exert a larger force on the arm.
The force on the elbow is greater if the density of the milk is greater. This is reasonable since then the milk would exert a larger force on the arm.
Student Difficulties Solving Problems

• Lack of an Organizational Framework
  – Random walk (knowledge fragments + math)
  – Situation specific (memorized pattern)

• Physics Misknowledge
  – Incomplete (lack of a concept)
  – Misunderstanding (weak misknowledge)
  – Misconceptions (strong misknowledge)

• No Understanding of Range of Applicability – Mathematics & Physics
  – Always true
  – True under a broad range of well-defined circumstances
  – True in very special cases
  – Never true

• Lack of internal monitoring skills (reflection on what they did and why, asking skeptical questions about their actions)

  Students must be taught a problem solving framework that addresses these explicitly