Monitor Unit (MU) Calculation

Timothy C. Zhu[1], Haibo Lin[1], and JiaJian Shen[2]
[1]University of Pennsylvania, Philadelphia, PA
[2]Mayo Clinic, Phoenix, AZ

Introduction

Pencil-beam based dose/MU algorithms

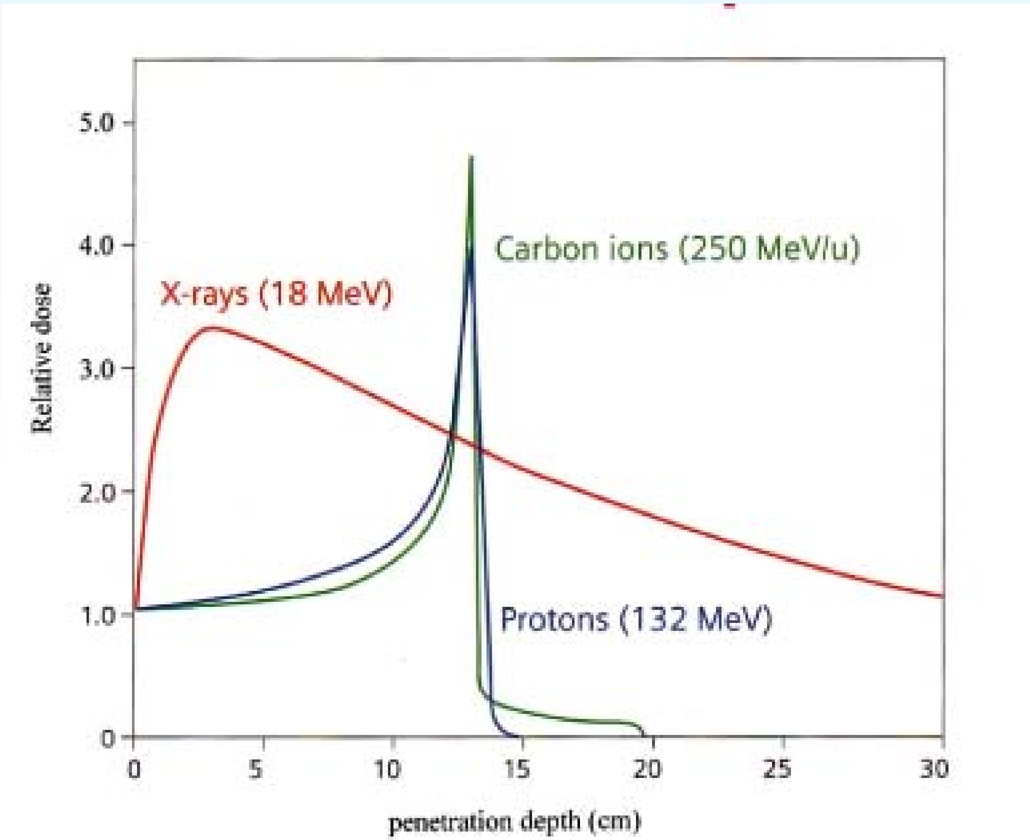
Factor-based dose/MU algorithms

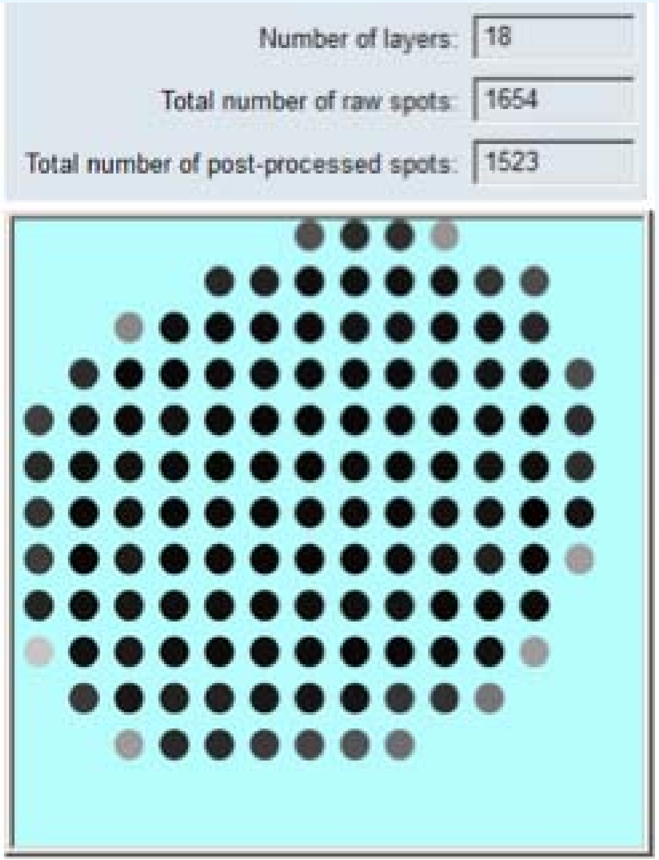
Beam data

Dose and MU verification

Introduction

The fundamental difference between proton and photon beams is that while proton beams provide depth modulation, the photon beams do not





Multiple-plane verification are needed for proton IMPT

Introduction – definition of Monitor Unit

- A monitor unit (MU) is a a measure of machine output by a built-in monitor ionization chamber of a proton machine
  - 1MU equal to dose delivered by a proton beam in a reference condition (e.g., 10x10 cm^2 field, SAD 230 cm, R15M10).
  - Dose per proton passing monitor chamber is used for some centers:

    ion pair/proton = S_{air} x L x ρ_{air} /W_{air} = 37.6 L S_{air}

    where W_{air} = 34.3 eV, ρ_{air} = 0.00129g/cm^3, L is the monitor chamber separation (in cm) and S_{air} is the mass stopping power (in MeV/g/cm^2).

Introduction – categories of dose-to-MU formalism

- Factor-based Dose-to-MU formalism
  - Using product of successive dose ratio factors for a chain of geometries.
- Model-based (Pencil-beam) dose-to MU formalism
  - Calculation of the dose per MU based on predetermined pencil-beam kernels (e.g., MC generated or analytical) because dose is proportional to the incident proton particle fluence.
- Monte Carlo simulation
  - Direct calculation of the absorbed dose per monitor unit using MC simulation (e.g., GEANT 4 or MCNPX)

Introduction – Factor-based D/MU formalism

- Identity Equation
  - Using product of successive dose ratio factors for a chain of geometries.

$$\frac{D}{MU}(\text{caseA}) = \frac{(D/MU)_{\text{caseA}}}{(D/MU)_{\text{caseB}}} \cdot \frac{(D/MU)_{\text{caseB}}}{\ldots} \cdot \frac{\ldots}{(D/MU)_{\text{ref}}} \cdot \frac{D}{MU}(\text{ref})$$

  - In reality, it is expressed as dose ratio or dose-to-fluence ratio

TG74, Med Phys 36, 5261-5291 (2009)

Introduction – Factor-based D/MU formalism

- Identity Equation using dose ratio

$$\frac{D(c,s;z,d)}{D(c_{ref},s_{ref};z_{ref},d_{ref})} = \frac{D(c,s;z,d)}{D(c,s;z,d_{ref})} \cdot \frac{D(c,s;z,d_{ref})}{D(c_{ref},s_{ref};z,d_{ref})} \cdot \frac{D(c_{ref},s_{ref};z,d_{ref})}{D(c_{ref},s_{ref};z_{ref},d_{ref})}$$
$$= TPR(c,s;d) \cdot OF_W(c,s;z,d_{ref}) \cdot ISF(z)$$

- TPR = PDD/ISF, PDD is more commonly used.
- D_{ref}=MU*$(D/MU)_{ref}$, Dose in reference condition is proportional to MU, with $(D/MU)_{ref}$ set by the user, e.g., $(D/MU)_{ref}$ = 1 cGy/MU.

Introduction – Factor-based D/MU formalism

- Identity Equation using dose-to-fluence ratio

$$\frac{D(c,s;z,d)}{D(c_{ref},s_{ref};z_{ref},d_{ref})} = \frac{D(c,s;z,d)}{X(c,s;z)} \cdot \frac{X(c,s;z)}{X(c_{ref},s_{ref};z)} \cdot \frac{X(c_{ref},s_{ref};z)}{X(c_{ref},s_{ref};z_{ref})} \cdot \frac{X(c_{ref},s_{ref};z_{ref})}{D(c_{ref},s_{ref};z_{ref},d_{ref})}$$

$$= TOR(c,s;d) \cdot H_p(c,s;z) \cdot ISF_{air}(z) \cdot \frac{1}{TOR(c_{ref},s_{ref};d_{ref})}$$

$$= \frac{TOR(c,s;d)}{TOR(c_{ref},s_{ref};d_{ref})} \cdot H_p(c,s;z) \cdot ISF_{air}(z)$$

$$= TPR(c,s;d) \cdot PSF(s;d_{ref}) \cdot H_p(c,s;z) \cdot ISF_{air}(z)$$

- OF_w = PSF*Hp.

Introduction – Pencil-beam formalism

- Convolution of pencil beam kernel and proton incident fluence can be used to calculate dose

$$D = k \iint_{x',y'} p(x - x', y - y', z)\Phi(x', y')\,dx'\,dy'$$

- *k* is a constant proportional to MU.

Introduction – Different use of proton beam types

- Broad Proton beam is used as SOBP only, even though fundamentally it is made of a summation of pristine peaks
  - Passive scanning broad beam is produced by inserting scatterers (either double scattering or single scattering)
  - Uniform scanning broad beam is produced by raster-scanning of PBS
- Pencil-beam scanning (PBS) is used by a summation of pristine peaks.

Introduction

Pencil-beam based dose/MU algorithms

Factor-based dose/MU algorithms

Beam data

Dose and MU verification

Pencil-beam algorithm

- Dose/MU calculation algorithm

$$D(x,y,z) = k \cdot \left(\frac{SSD_v + z_n}{SSD_v + z}\right)^2 PDD(z) \cdot \iint f(x-x', y-y', z)\Phi(x', y')dx'dy'$$

where $\Phi(x,y)$ is the lateral proton primary fluence, usually treated as a constant, k is a constant proportional to MU including other factors, PDD(z) is the PDD for infinitely large field size and infinite SSD, otherwise PDD/ISF(z).

Pencil-beam algorithm

- Pencil-beam kernel, f(x,y,z)

$$f(x,y,z)=\frac{\sum_{i=1}^{2} B_i \cdot e^{-\frac{x^2+y^2}{\lambda \cdot b_i \cdot \overline{r^2}}}}{\pi \cdot \overline{r^2} \cdot \lambda \sum_{i=1}^{2} B_i \cdot b_i}$$

where the b_i and B_i, (i = 1, 2) are pre-calculated weighting parameters based on MC calculation in water. $\overline{r^2}$ is the mean square of radial spread.

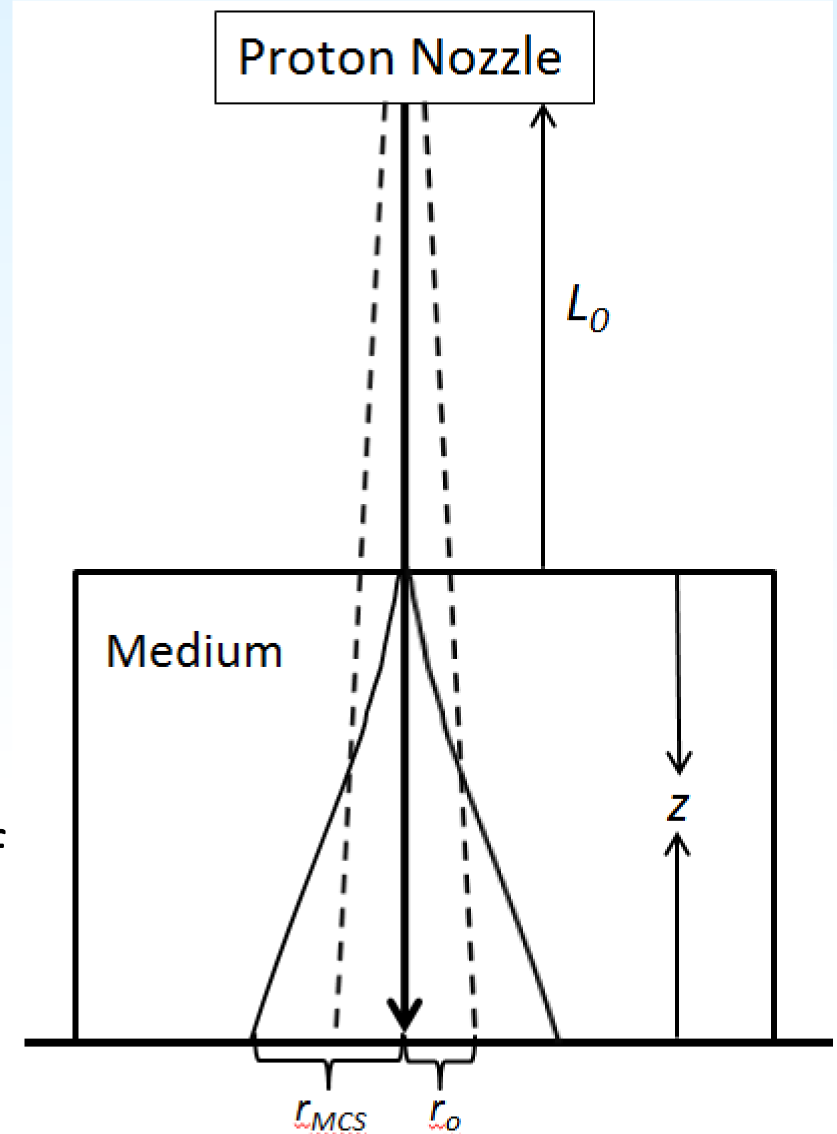
Pencil-beam algorithm – broad beam

- Single gaussian kernel

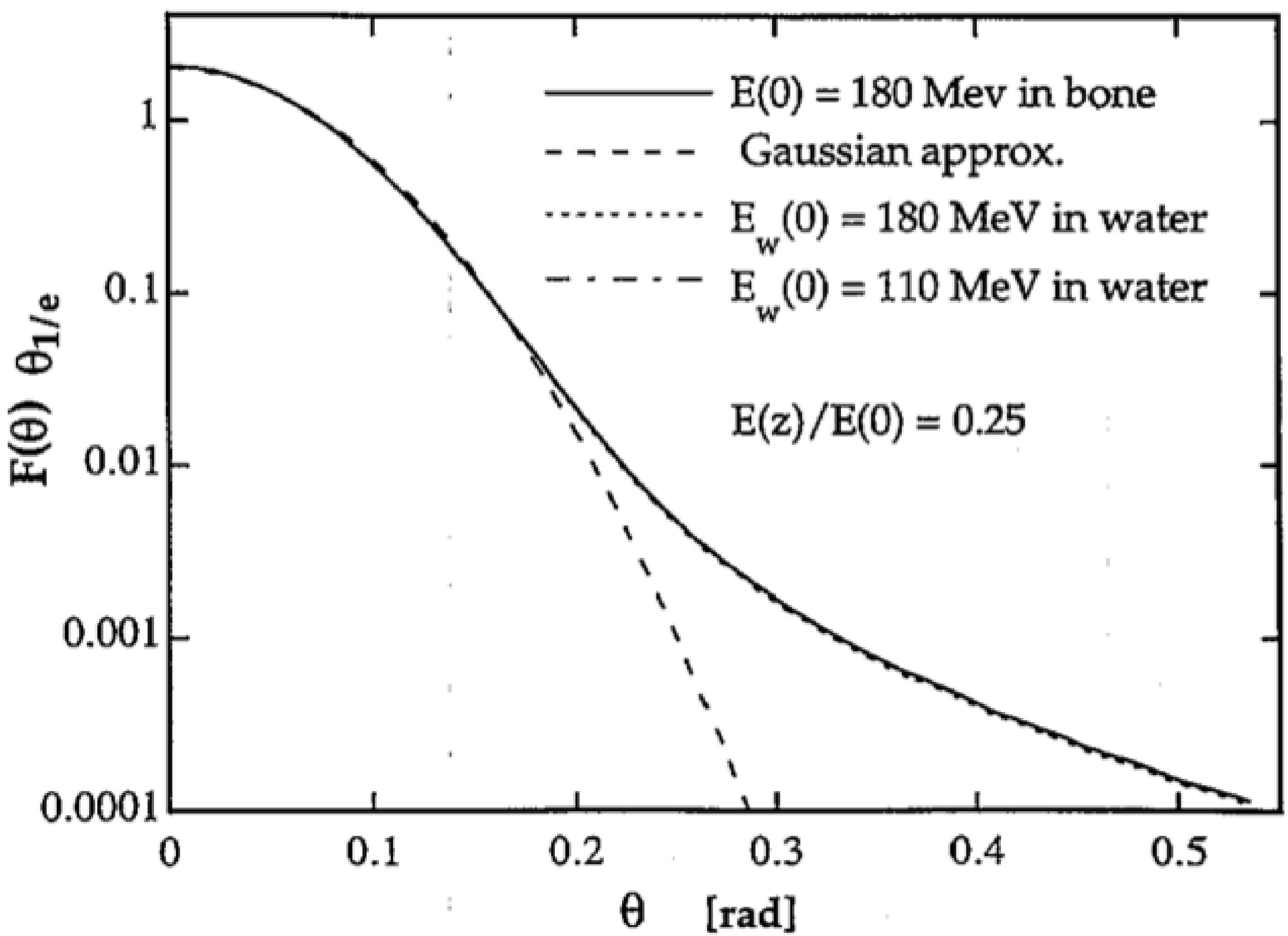
$$f(x,y,z) = \frac{1}{\pi \cdot \overline{r_{MCS}^2}} e^{-\frac{x^2+y^2}{\overline{r_{MCS}^2}}}$$

$$\overline{r_{MCS}^2} = \overline{\theta_0^2}(L_0 + z)^2 + \int_0^z (z-u)^2 T(u)du$$

where *T* is the scattering power, $r_0=\theta_0(L_0+z)$ is the in-air lateral spread at point of interest, L_0 is the distance between bottom of nozzle and patient surface, *z* is the depth in the medium.

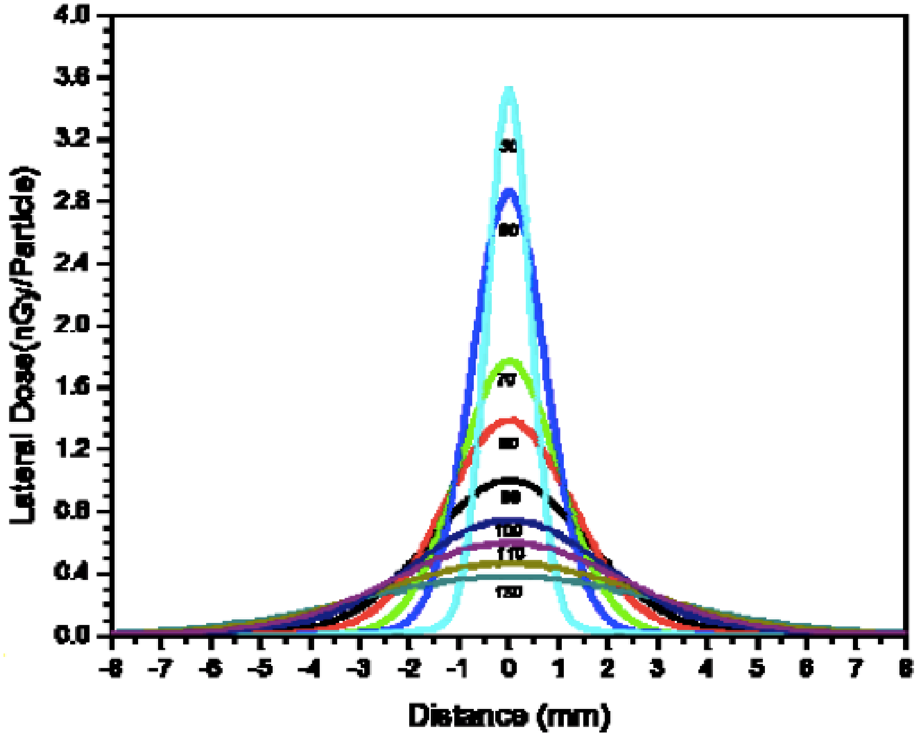


Pencil-beam algorithm – angular spread



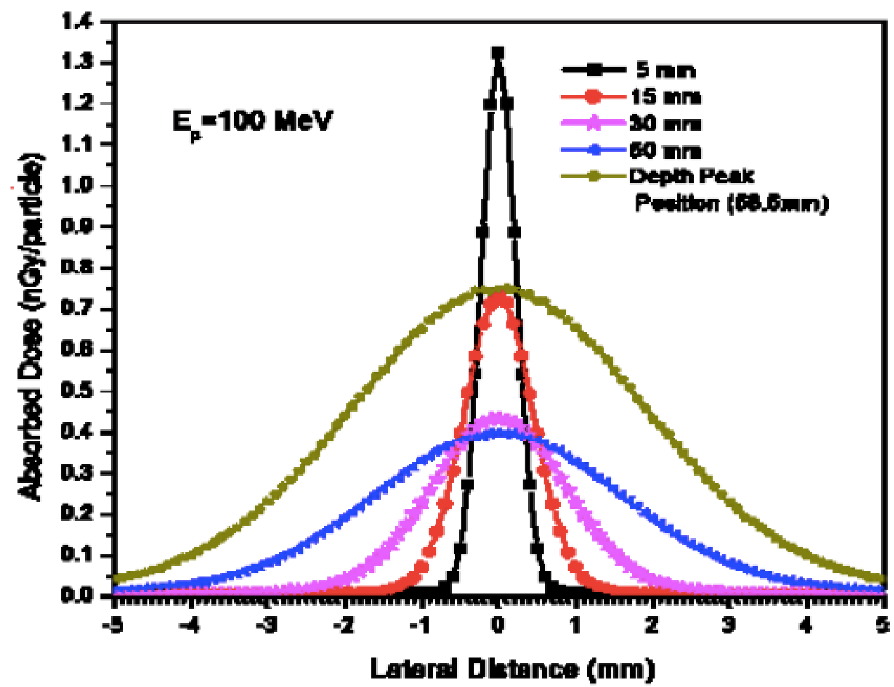
PMB 40 1031-1043 (1995)

Pencil-beam algorithm – angular spread



Lateral dose distribution at Bragg peak positions for 40, 50, 60, 70, 80, 90, 100, 110, 120 MeV.

Lateral dose distribution for 100 MeV for different depths: 5, 15, 30, 50, 68.5 mm.



Pencil-beam algorithm – PBS

- Besides pencil-beam convolution, a ray casting algorithm have been used for PBS dose calculation. The ray casting model of a single spot with nominal energy *E* and a single Gaussian lateral profile:

$$D_{spot}(x, y, z) = MU * IDD(E, WET) \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{(x_0 - x)^2}{2{\sigma_x}^2}\right] \exp\left[-\frac{(y_0 - y)^2}{2{\sigma_y}^2}\right]$$

where *IDD* is the function of spot nominal energy *E* and water equivalent thickness (*WET*) at position (*x, y, z)*. The single Gaussian distribution describes the spot lateral dose distribution in the medium (e.g., water).

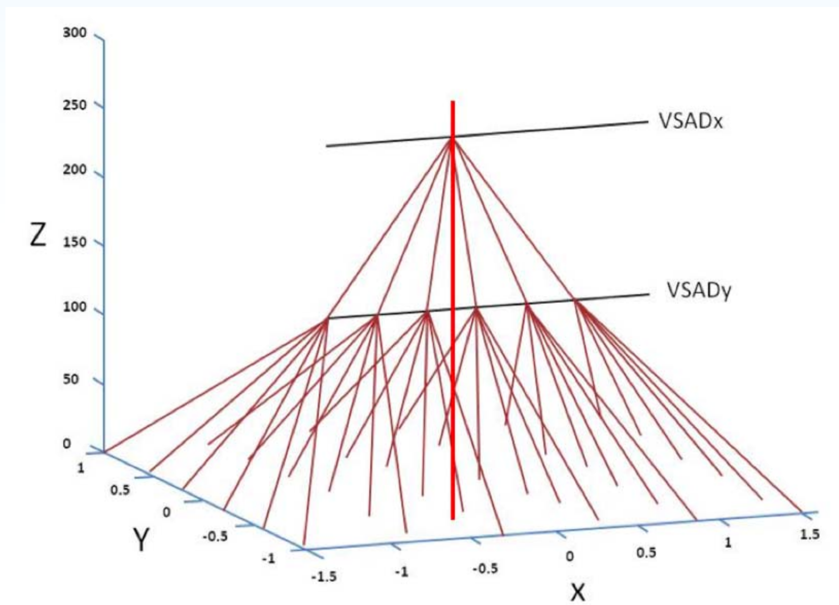
Pencil-beam algorithm – PBS

- When the single spot dose distribution is explicitly established, the total dose is a summation of the contribution from all spots

$$D\ \ (x, y, z) = ISF * \sum_{j=1}^{N} \left[MU_j * IDD_j\left(E_j, WET\right) * \iint LAT_j(x', y', z) dx' d\,y' \right]$$

where

$$ISF = \frac{SAD_{vx} * SAD_{vy}}{\left(SSD_{vx} + z\right) * \left(SSD_{vy} + z\right)}$$



Introduction

Pencil-beam based dose/MU algorithms

Factor-based dose/MU algorithms

Beam data

Dose and MU verification

Fluence-based formalism – broad beam

- Fluence-based formalism separates the measurements into in-air (fluence) and in water (dose) measurements. The basic formalism can be expressed as:

$$D = MU \cdot \frac{D_{ref}}{MU} \cdot PDD(z) \cdot PSF \cdot SF \cdot H_p \cdot ISF \cdot OAR(x,y)$$

where D_{ref}/MU is the dose per MU for each energy option measured under reference. *PSF* is the phantom scatter factor. *SF* is the snout factor. H_p is the proton headscatter factor. P*OAR* is primary off-axis ratio, and *ISF* is the inverse-square factor.

Fluence-based formalism – SOBP

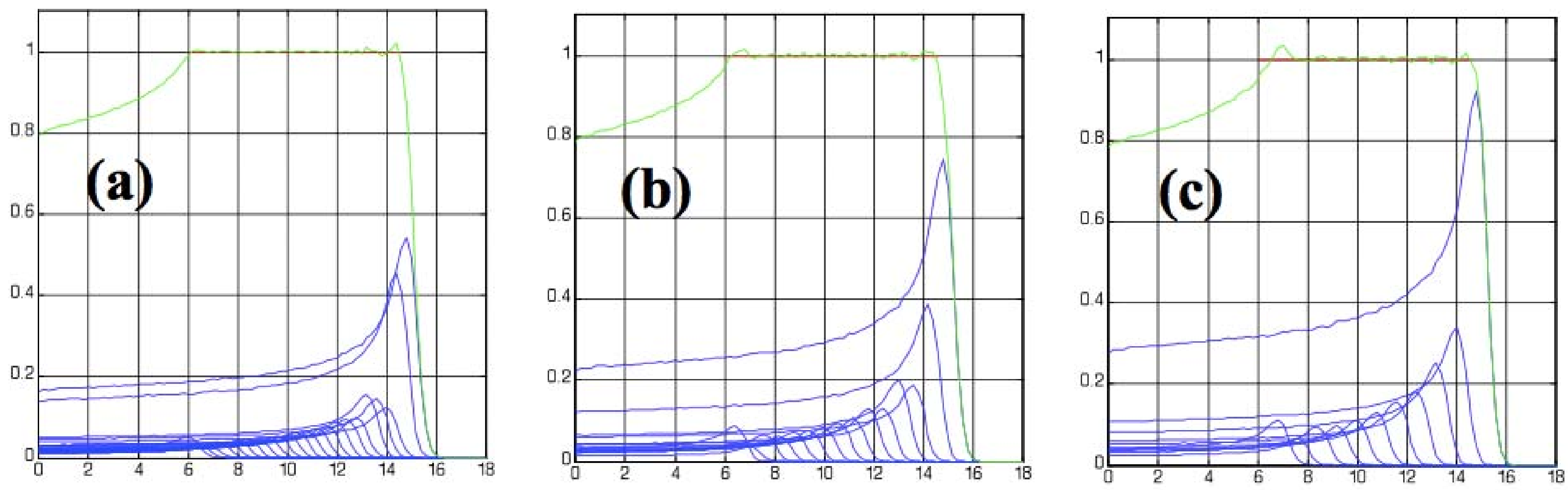
- Most, if not all, broad proton beam are made of SOBP. MU calculation is made in the middle of SOBP, where PDD = 1. Alternatively, PDD can be calculated using the expression:

$$PDD_{SOBP} = \sum_{i}^{n} w(R)_i \, PDD_{pris}(R)_i$$

where $w(R)_i$ is the weight for the pristine PDD with range R.

Fluence-based formalism – SOBP

- Cimmino optimization algorithm to determine SOBP based on R and M.

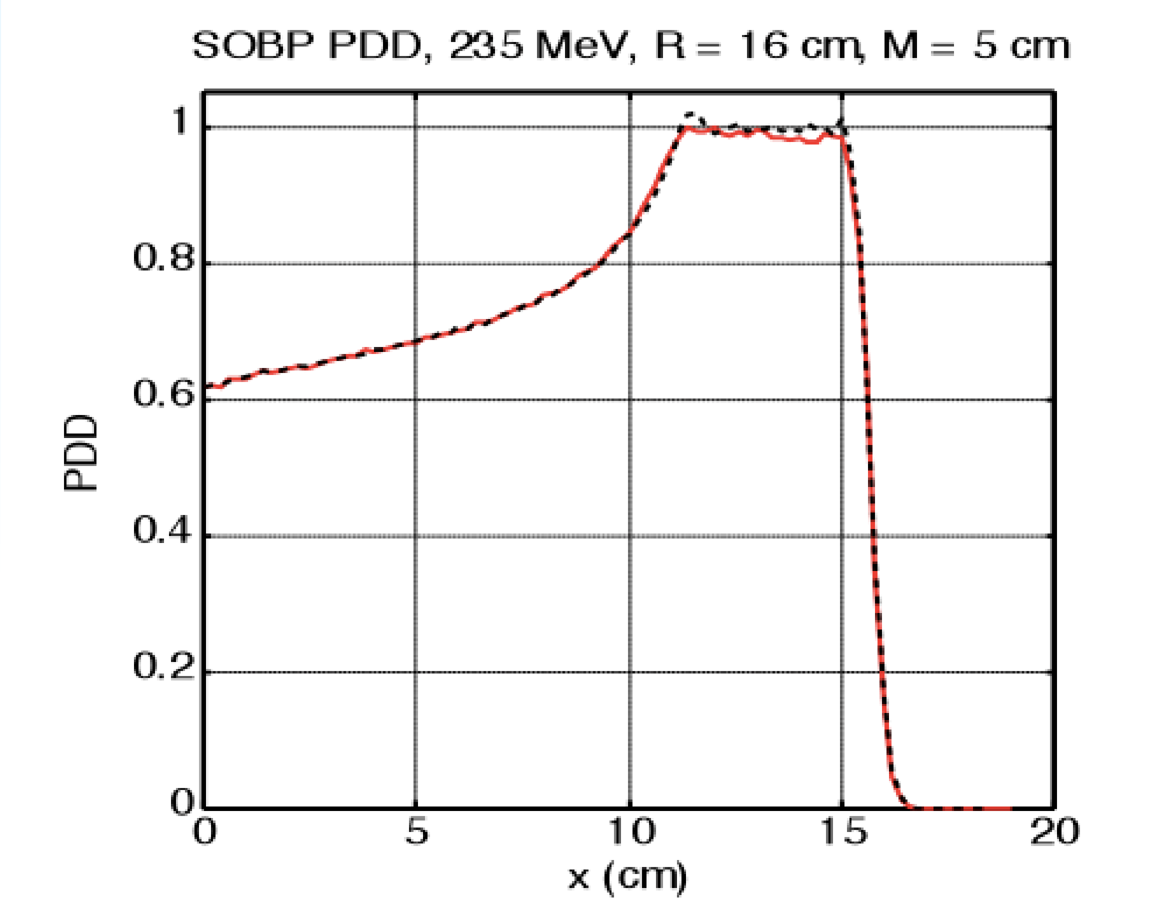


(a) 0.4 cm (b) 0.6 cm (c) 0.8 cm

Med Phys 35, 2766 (2008)

Fluence-based formalism – SOBP

- Comparison between calculation (dashed) and measurement (solid)



Med Phys 35, 2766 (2008)

Fluence-based formalism – ISF

- Inverse square law

$$ISF = \left(\frac{SAD - xv}{SSD + z - xv} \right)^2$$

- SADv= SAD – xv, SSDv=SSD-xv.
- xv represents the virtual source shift due to varied E and nozzle equivalent thickness (NET)
- xv = 0 for SOBP

Fluence-based formalism – Hp

- Output factor for SOBP

$$H_p = \frac{(CF/100)\cdot \psi_C \cdot D_C}{(1+a_1 \cdot r^{a_2})} \cdot (s_0 + s_1(R - R_L)) \cdot H_0(s)$$

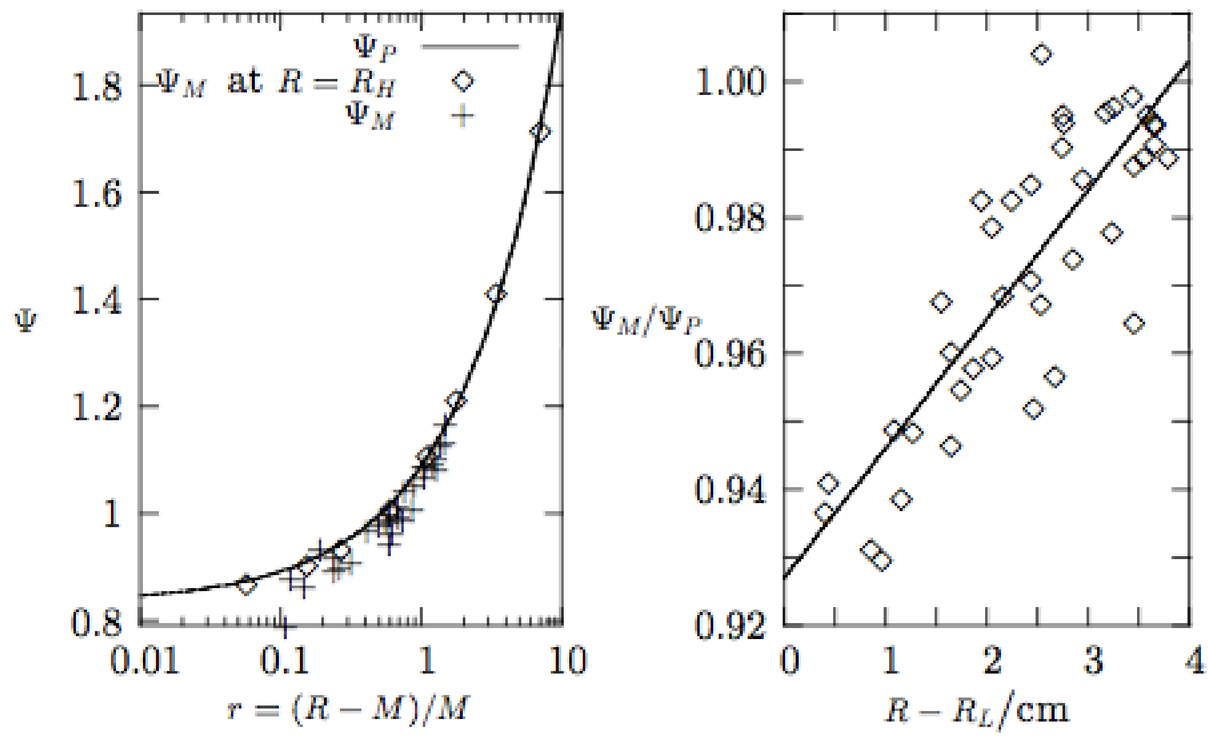
$$r = \frac{(R-M)}{M} = \frac{R_{90} - mM_{90}}{mM_{90}}$$

where *R* is the range, *M* is modulation width, and $m = 0.91$ is a conversion factor, CF is a constant to correct for the output change per option, R_L is the minimal range of the option. The effect of virtual source shift is included as the parameters s_1 and s_2. $H_0(s)$ is field size dependence.

PMB 50: 5847-5856 (2005)

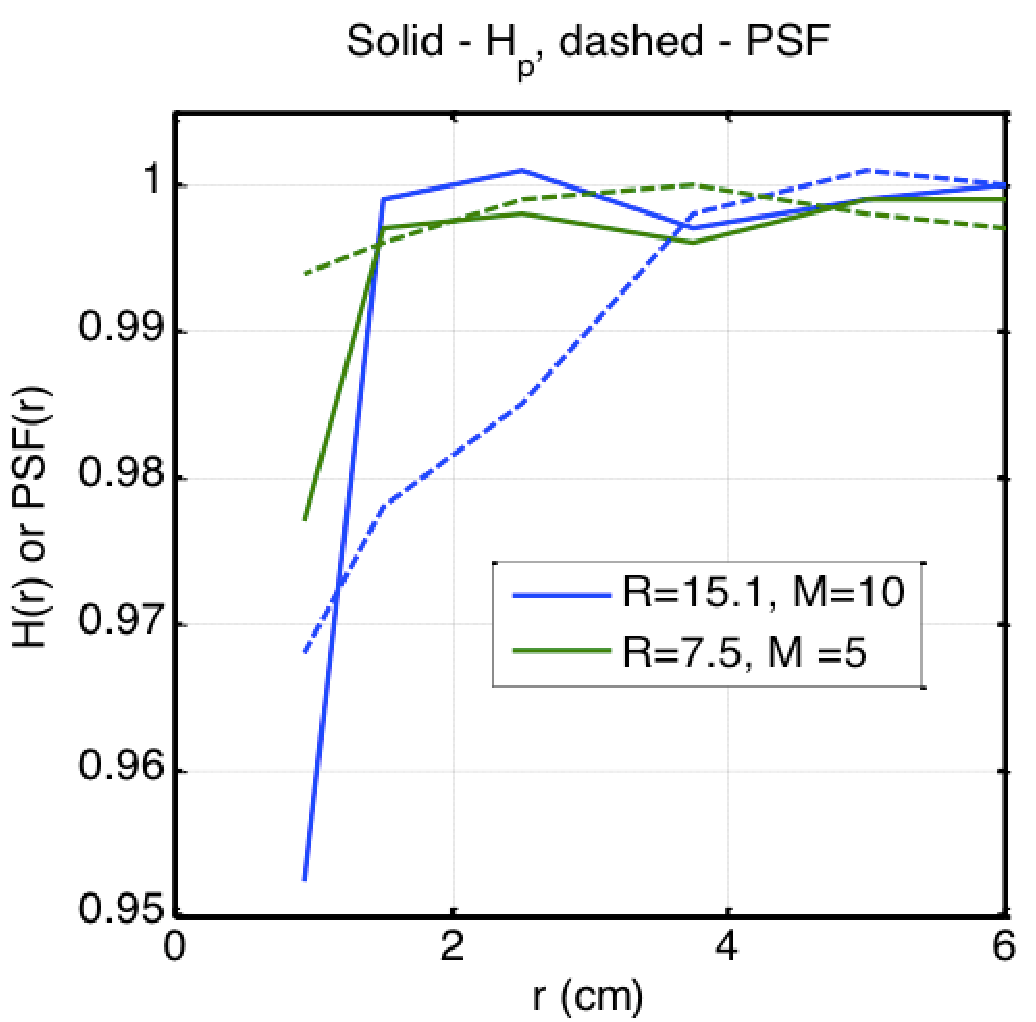
Fluence based formalism – Hp

Option	a_0	a_1	CF	s_0	s_1	R_L	RMS (%)
B3	0.3375	0.7405	0.9970	0.963	0.0196	7.49	2.5
B4	0.3667	0.6963	1.0234	0.946	0.0208	9.55	3.5
B5	0.3552	0.6081	0.9532	0.928	0.0218	11.65	1.4
B6	0.2338	0.8990	1.0549	0.986	0.0070	15.54	0.6
B7	0.1461	0.7843	1.1849	0.952	0.0090	19.83	1.7



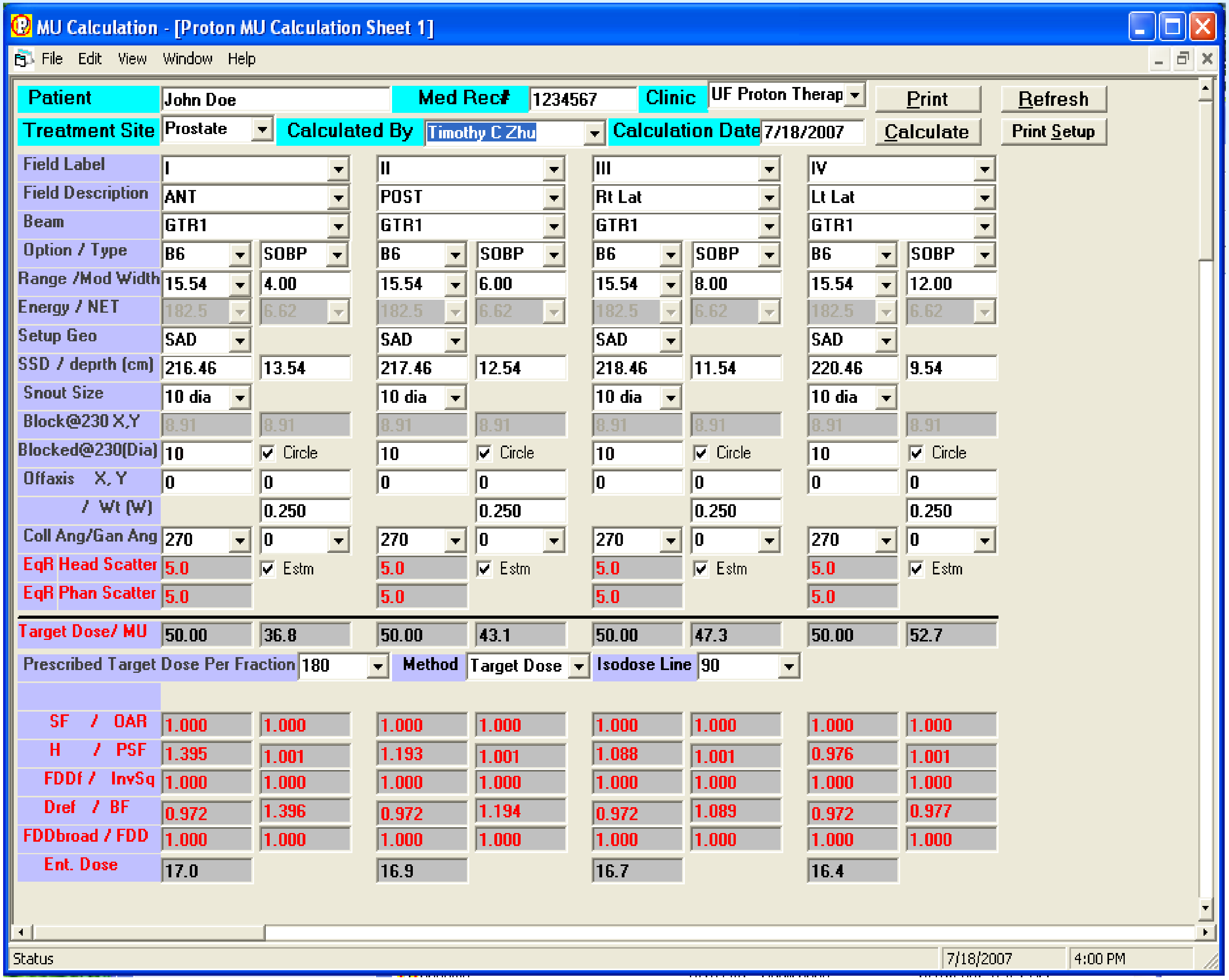
PMB 50: 5847-5856 (2005)

Fluence based formalism – $H_p(s)$ and PSF(s)



Med Phys 34: 2403 (2008)

Fluence-based formalism – broad beam



MU Calculation - [Proton MU Calculation Sheet 1]

File Edit View Window Help

Patient	John Doe	Med Rec#	1234567	Clinic	UF Proton Therap	Print	Refresh
Treatment Site	Prostate	Calculated By	Timothy C Zhu	Calculation Date	7/18/2007	Calculate	Print Setup

Field Label	I		II		III		IV	
Field Description	ANT		POST		Rt Lat		Lt Lat	
Beam	GTR1		GTR1		GTR1		GTR1	
Option / Type	B6	SOBP	B6	SOBP	B6	SOBP	B6	SOBP
Range /Mod Width	15.54	4.00	15.54	6.00	15.54	8.00	15.54	12.00
Energy / NET	182.5	6.62	182.5	6.62	182.5	6.62	182.5	6.62
Setup Geo	SAD		SAD		SAD		SAD	
SSD / deprth (cm)	216.46	13.54	217.46	12.54	218.46	11.54	220.46	9.54
Snout Size	10 dia		10 dia		10 dia		10 dia	
Block@230 X,Y	8.91	8.91	8.91	8.91	8.91	8.91	8.91	8.91
Blocked@230(Dia)	10	☑ Circle	10	☑ Circle	10	☑ Circle	10	☑ Circle
Offaxis X, Y	0	0	0	0	0	0	0	0
/ Wt (W)		0.250		0.250		0.250		0.250
Coll Ang/Gan Ang	270	0	270	0	270	0	270	0
EqR Head Scatter	5.0	☑ Estm	5.0	☑ Estm	5.0	☑ Estm	5.0	☑ Estm
EqR Phan Scatter	5.0		5.0		5.0		5.0	
Target Dose/ MU	50.00	36.8	50.00	43.1	50.00	47.3	50.00	52.7

Prescribed Target Dose Per Fraction 180 | Method Target Dose | Isodose Line 90

SF / OAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
H / PSF	1.395	1.001	1.193	1.001	1.088	1.001	0.976	1.001
FDDf / InvSq	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Dref / BF	0.972	1.396	0.972	1.194	0.972	1.089	0.972	0.977
FDDbroad / FDD	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Ent. Dose	17.0		16.9		16.7		16.4	

Status 7/18/2007 4:00 PM

Fluence-based formalism – broad beam

Option	Range	Mod-Width	SSD	MU	Dose	D/MU (calc.)	D/MU (meas.)	error(%)
B5	15.1	10.4	230.1	114.3	100	0.875	0.857	2.1%
B5	15.1	10.4	225.1	109.6	100	0.912	0.900	1.4%
B5	15.1	10.4	220.1	105	100	0.952	0.952	0.0%
B5	15.1	10.4	215.1	100.5	100	0.995	0.991	0.4%
B5	15.1	10.4	210.1	96.1	100	1.041	1.036	0.4%
B7	21.87	18	227.1	109.3	100	0.915	0.909	0.7%
B7	21.87	18	222.1	104.8	100	0.954	0.948	0.7%
B7	21.87	18	217.1	100.3	100	0.997	0.996	0.1%
B7	21.87	18	212.1	96	100	1.042	1.035	0.6%
B7	21.87	18	207.1	91.8	100	1.089	1.087	0.2%
B4	9.98	7.78	223.9	104.1	100	0.961	0.962	-0.1%
B4	10.4	8.34	223.8	104.6	100	0.956	0.958	-0.2%
B3	9.13	7.97	224.9	112.9	100	0.886	0.888	-0.2%
B3	8.36	6.45	224.9	107.2	100	0.933	0.935	-0.2%
B9	26.54	10.09	208.5	68.3	100	1.464	1.468	-0.3%

Dose-ratio based formalism – broad beam

- Dose ratio can be used for MU calculation from MD Anderson:

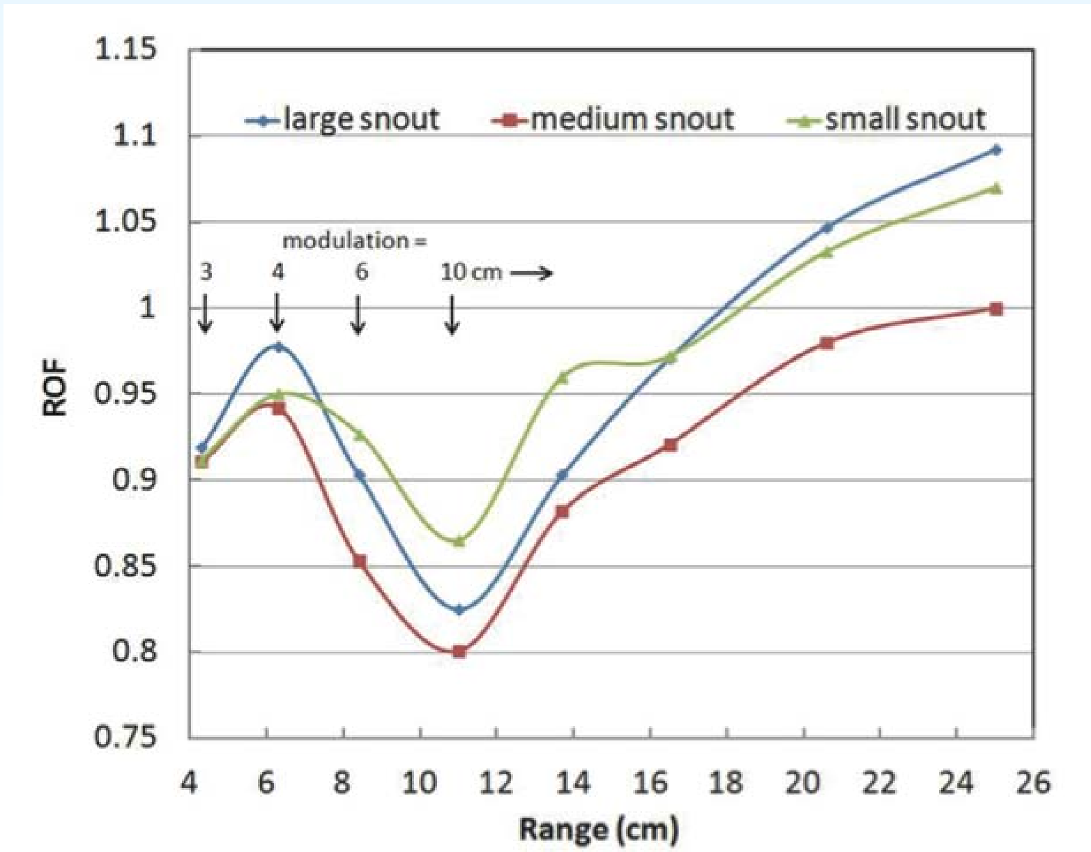
$$\left(\frac{D}{MU}\right)_p = ROF \cdot SOBPF \cdot RSF \cdot SOBPOCF \cdot OCR \cdot FSF \cdot ISF \cdot CPSF$$

where ROF is relative output factor, SOBPF is SOBP factor, RSF is the range shift factor, SOBPOCF is the SOBP off-center factor, OCR is the off-center ratio, FSF is the field size factor, ISF is inverse-square factor, CPSF is compensator and patient scatter factor.

Med Phys 35: 5088-5097 (2008)

Dose-ratio based formalism – broad beam

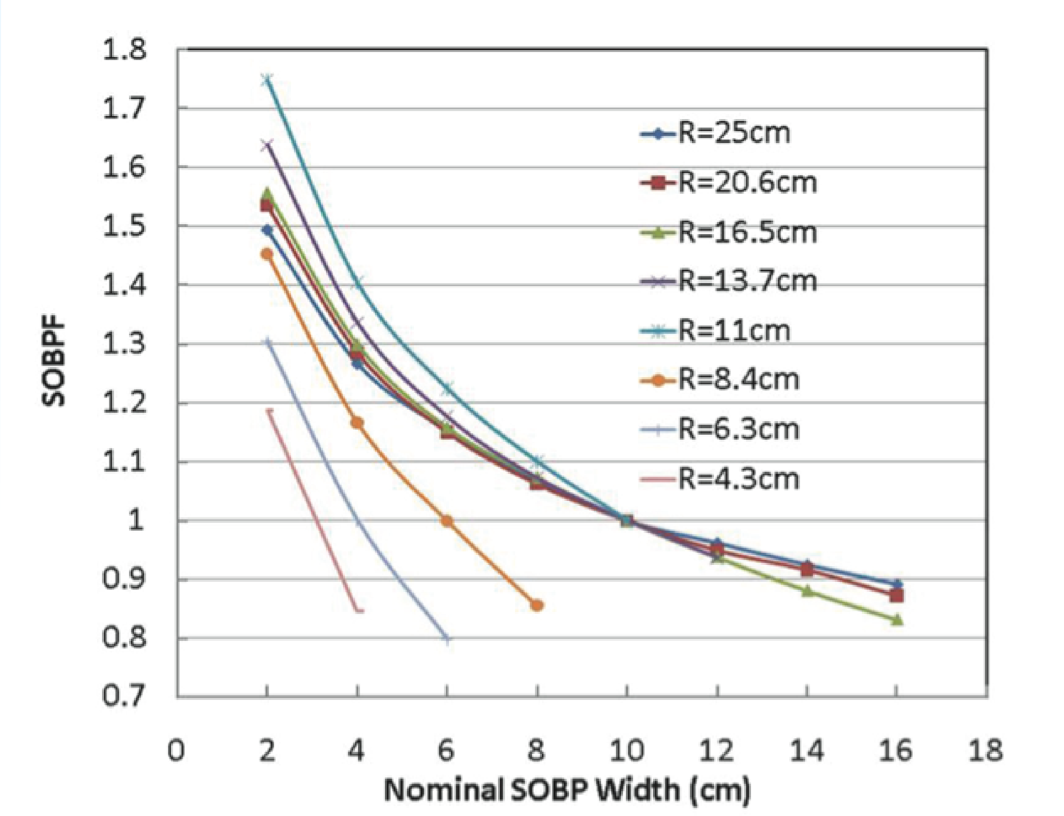
- ROF represents the change in D/MU relative to the reference condition as a function of R and M.



Med Phys 35: 5088-5097 (2008)

Dose-ratio based formalism – broad beam

- SOBPF represents the change in D/MU relative to the reference M used for ROF. SOBPF is a function of M for a fixed R.

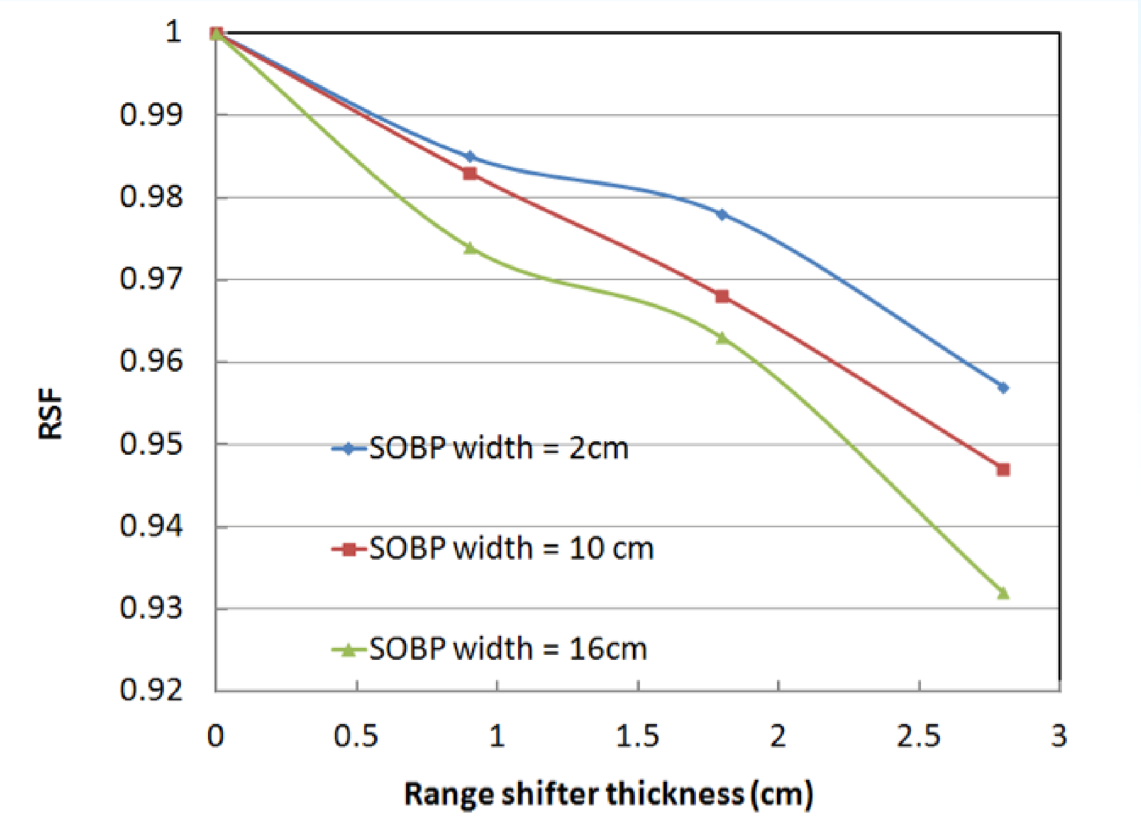


Med Phys 35: 5088-5097 (2008)

Dose-ratio based formalism – broad beam

- RSF accounts the output change due to the presence of the range shifters in the beam. It is a function of range shift thickness.

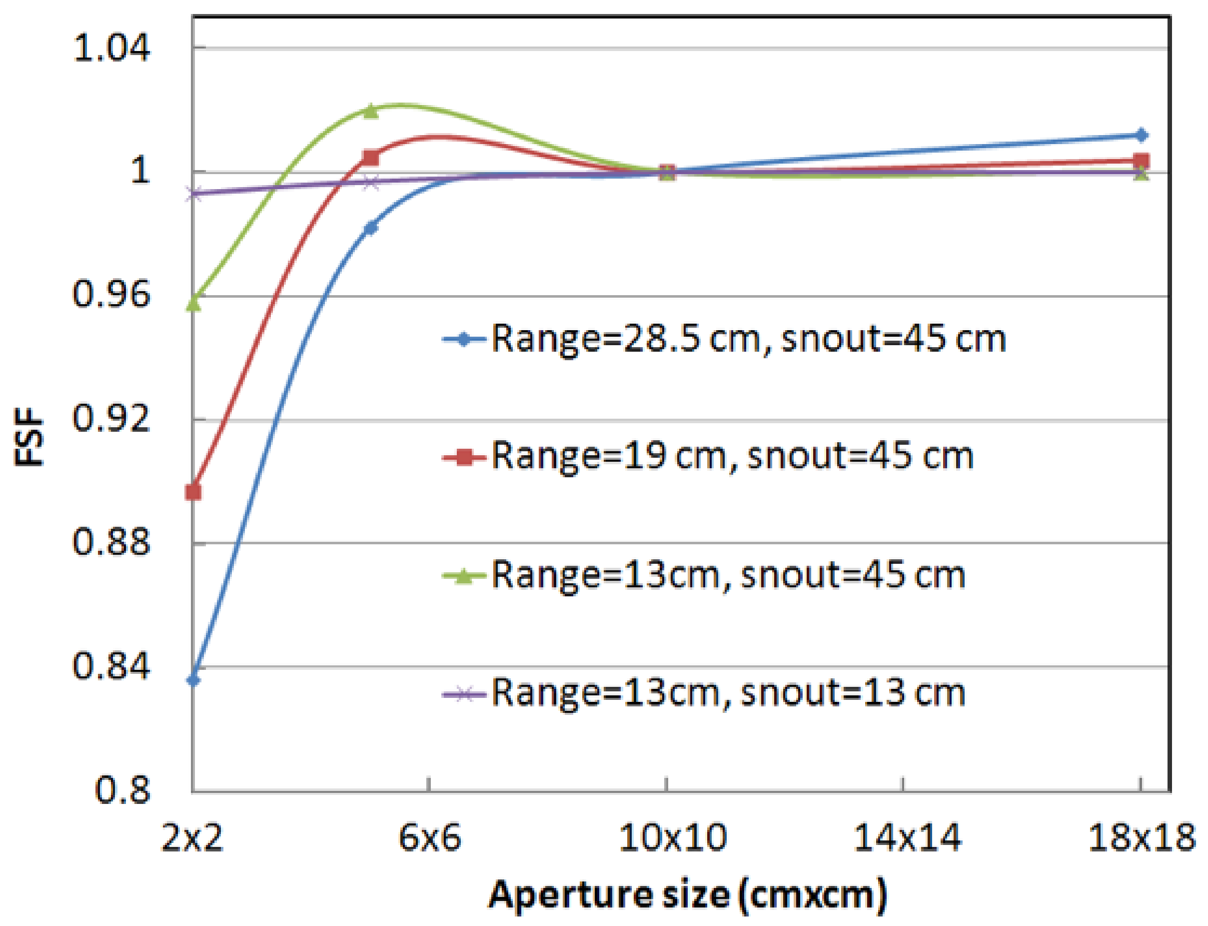
~ 2%



Med Phys 35: 5088-5097 (2008)

Dose-ratio based formalism – broad beam

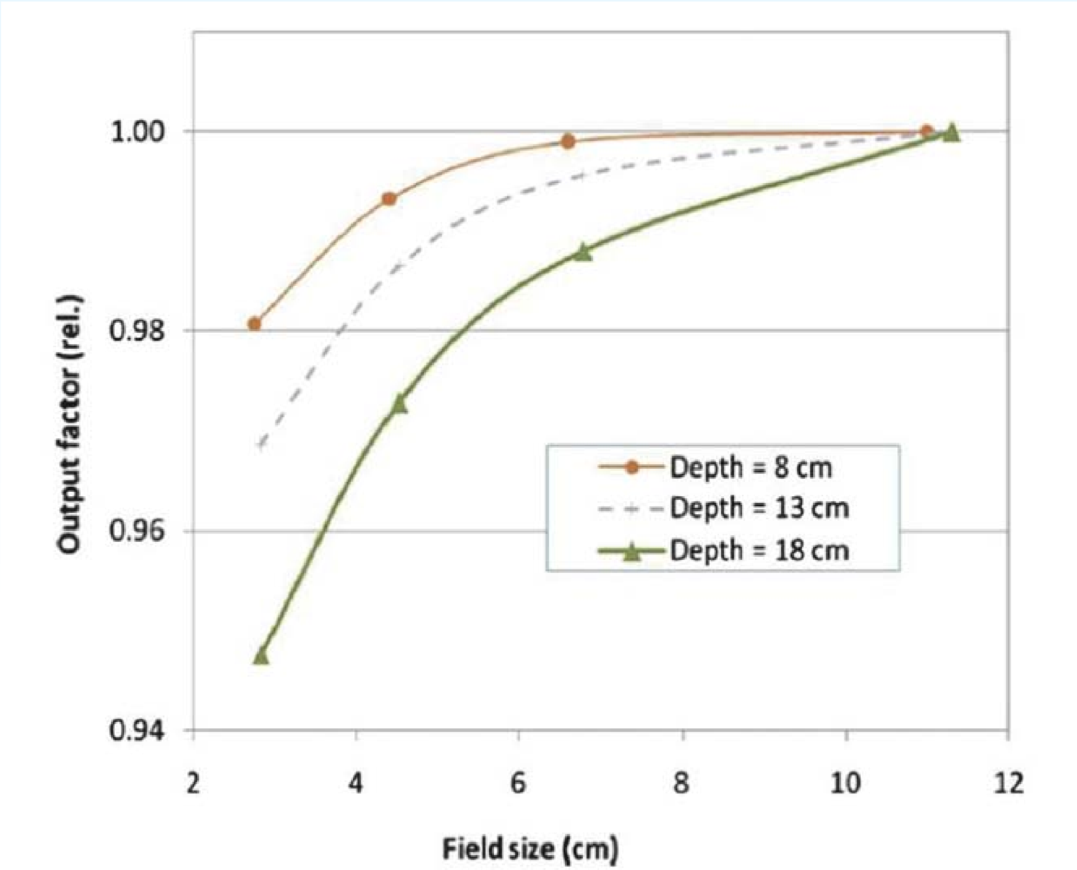
- FSF=PSF*Hp: field size dependence in water. For s > 5 cm, FSF = 1.



Med Phys 35: 5088-5097 (2008)

Dose-ratio based formalism – broad beam

- FSF=PSF*Hp: field size dependence in water for uniform scanning beam (small fields).

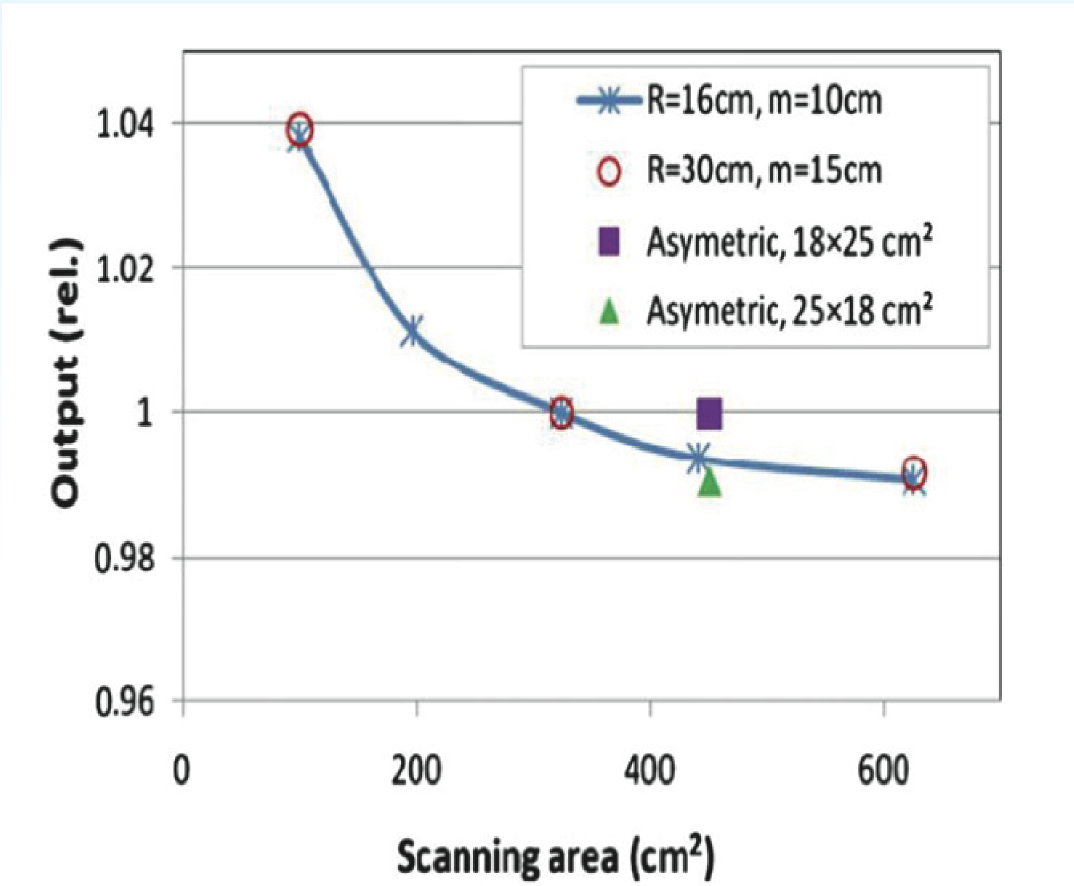


Med Phys 38: 2299-2306 (2011)

Dose-ratio based formalism – broad beam

- FSF=PSF*Hp: field size dependence in water specific to uniform scanning beam.

Snout size = 25 cm



Med Phys 38: 2299-2306 (2011)

Dose-based formalism – broad beam

- SOBPOCF: Relative change when the measurement point is away from the middle of SOBP on central-axis, generally very small.
- OCR: Off-central ratio accounts for proton beam profile, generally equal to 1.
- ISF: inverse square factor.
- CPSF: Compensator and patient scatter factor is the ratio of dose with and without the compensator in a uniform water phantom. This factor is usually directly measured.

Introduction

Pencil-beam based dose/MU algorithms

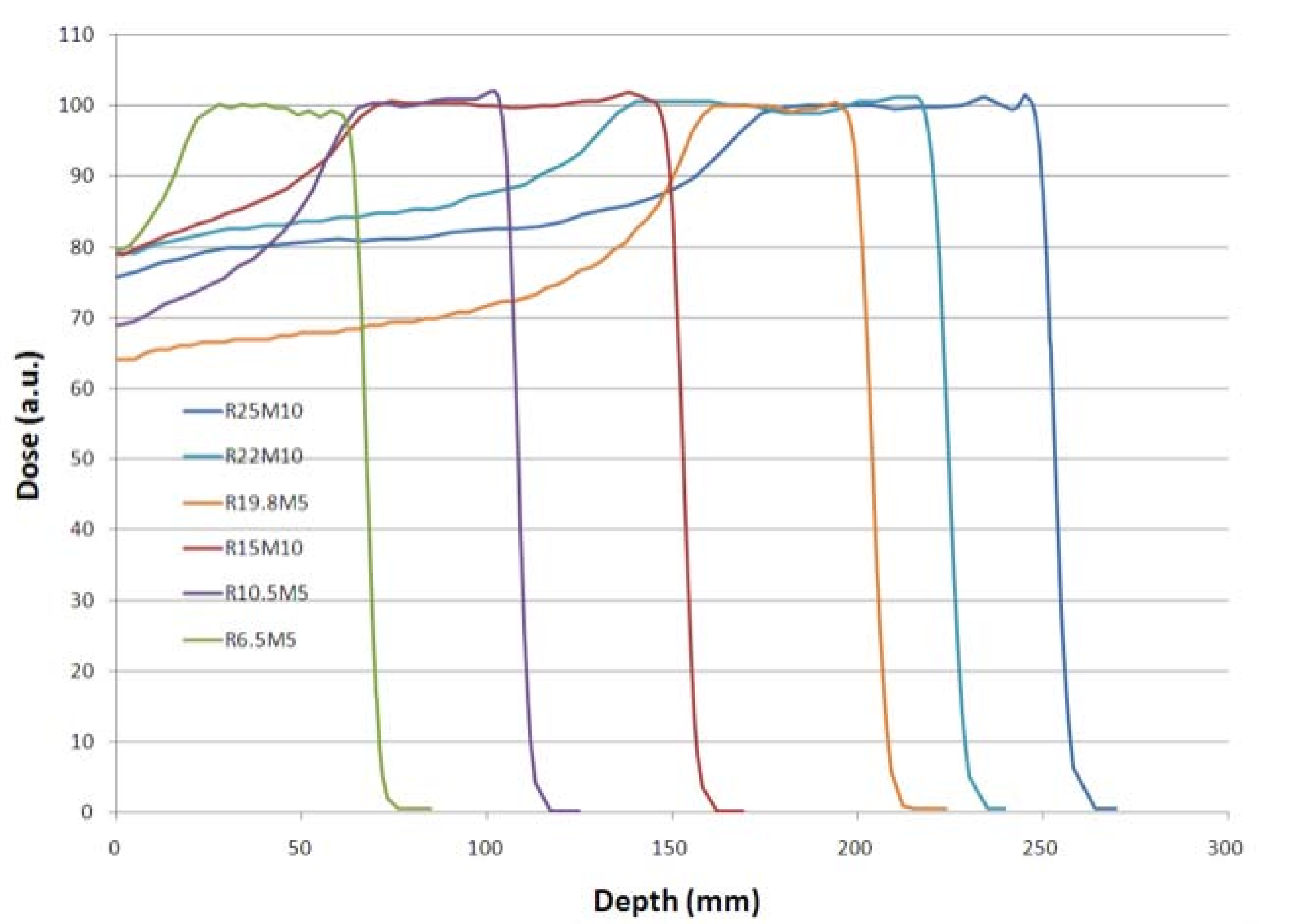
Factor-based dose/MU algorithms

Beam data

Dose and MU verification

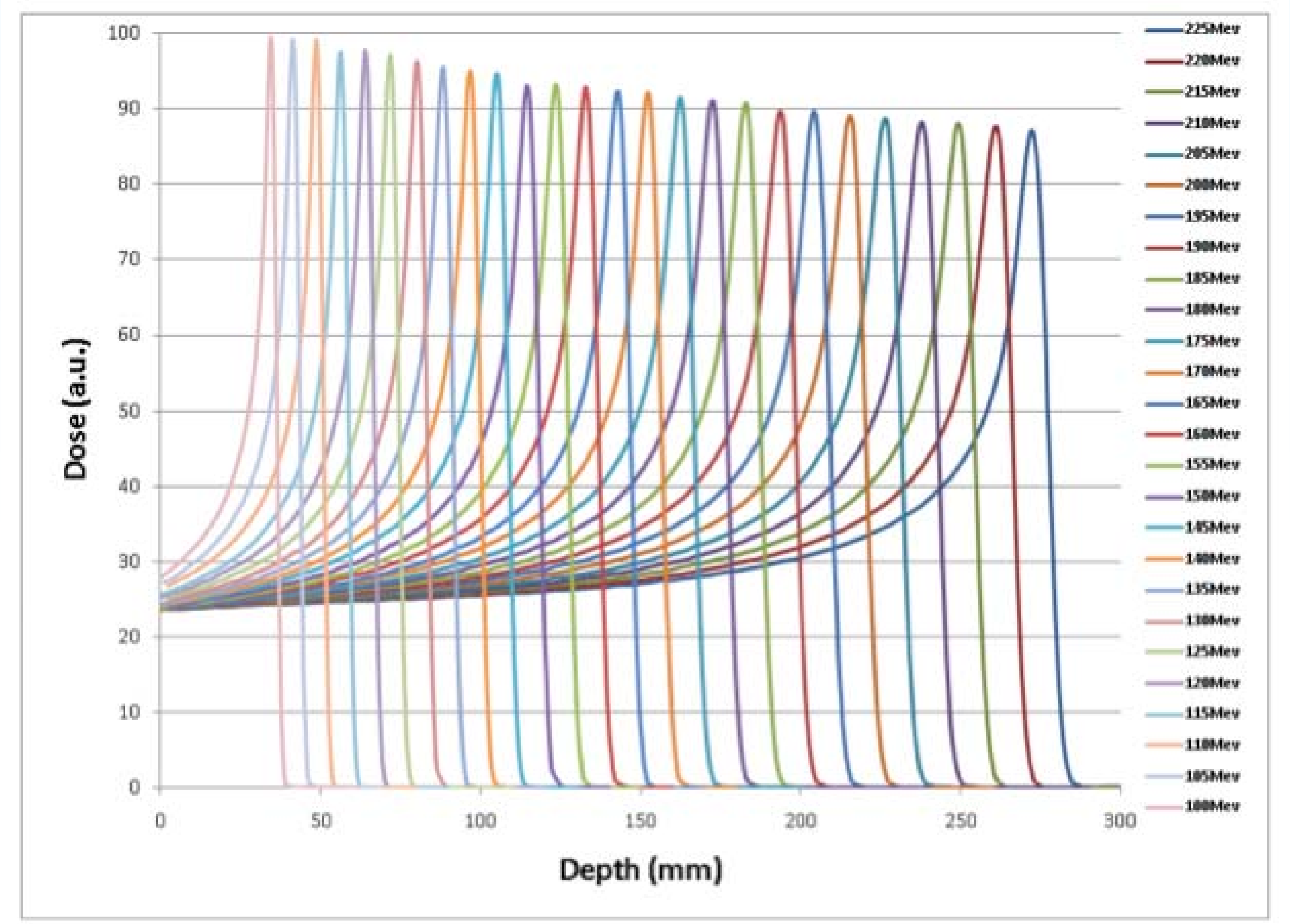
Beam Data – Depth dose for broad beam

- Most broad beam application use SOBP in clinical practice, thus PDD = 1 in middle of SOBP.



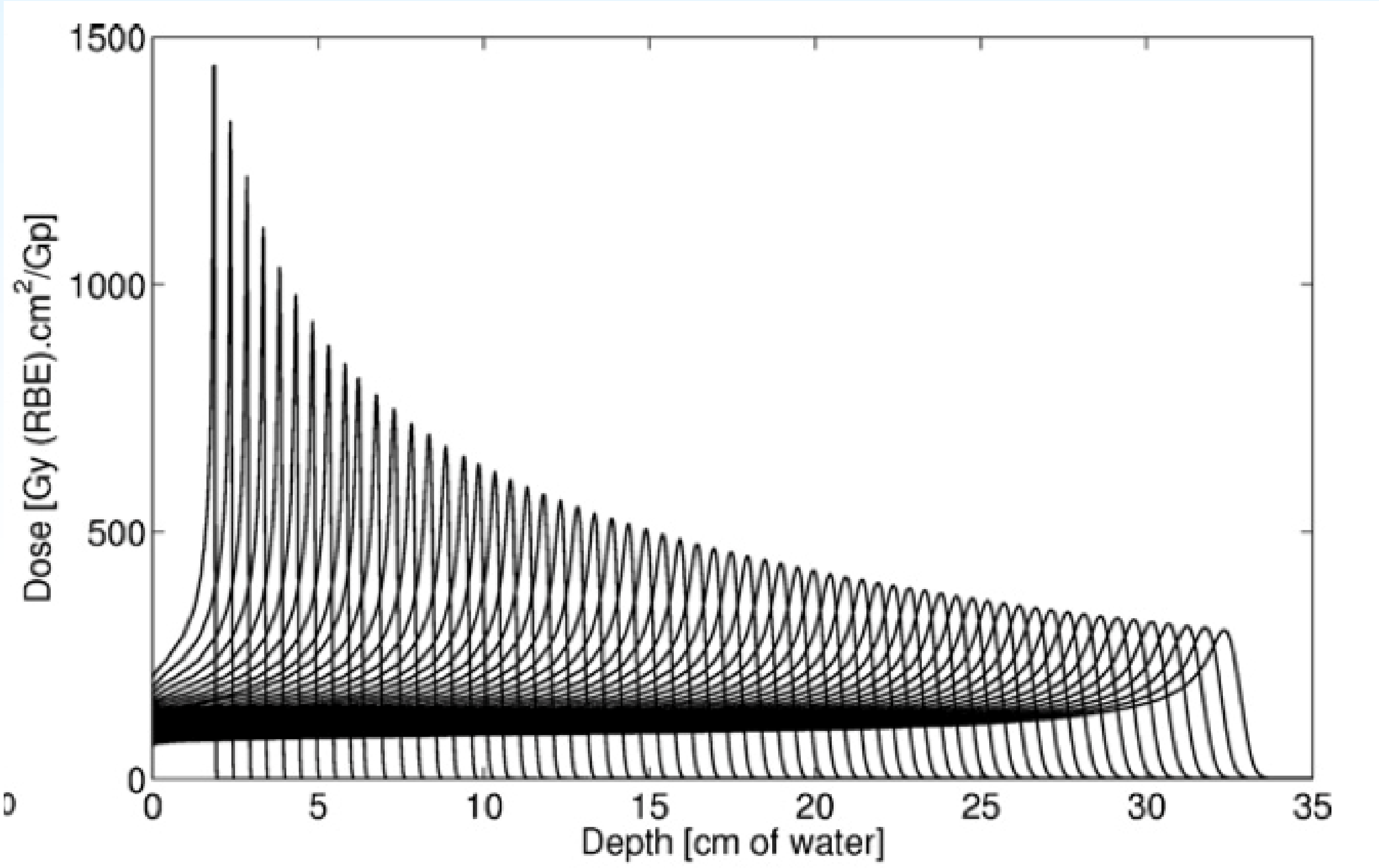
Beam Data – Depth dose for PBS

- For PBS, integrated depth dose (IDD) is need for both fluence and ray-casting models.



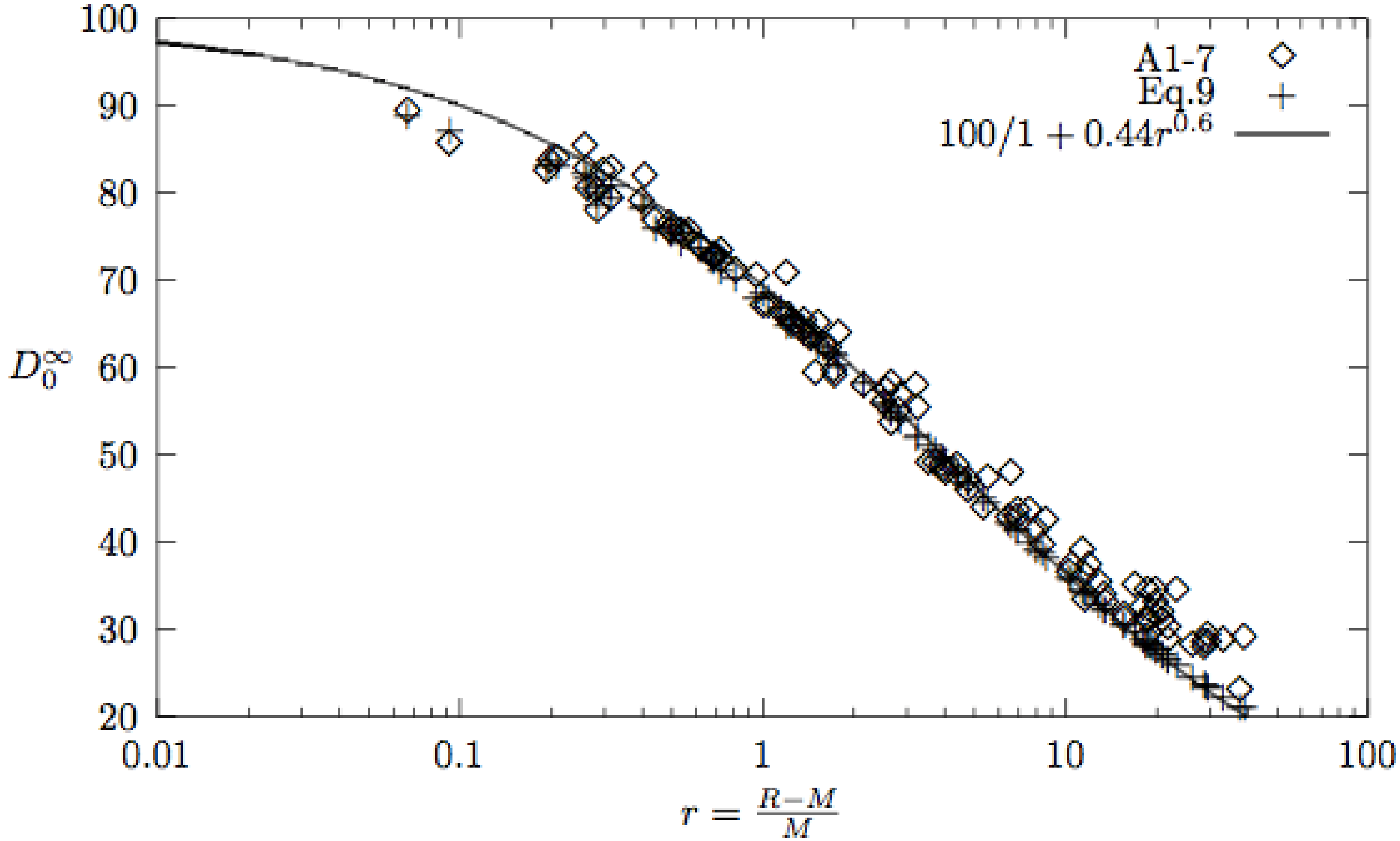
Beam Data – Depth dose for PBS

- MC generated IDD scaled by D/Gp:



PMB 57: 1147 – 1158 (2012).

Beam Data – Output factor in air



PMB, 48, 2797-2808, 2003

Beam Data – Output factor for broad beam

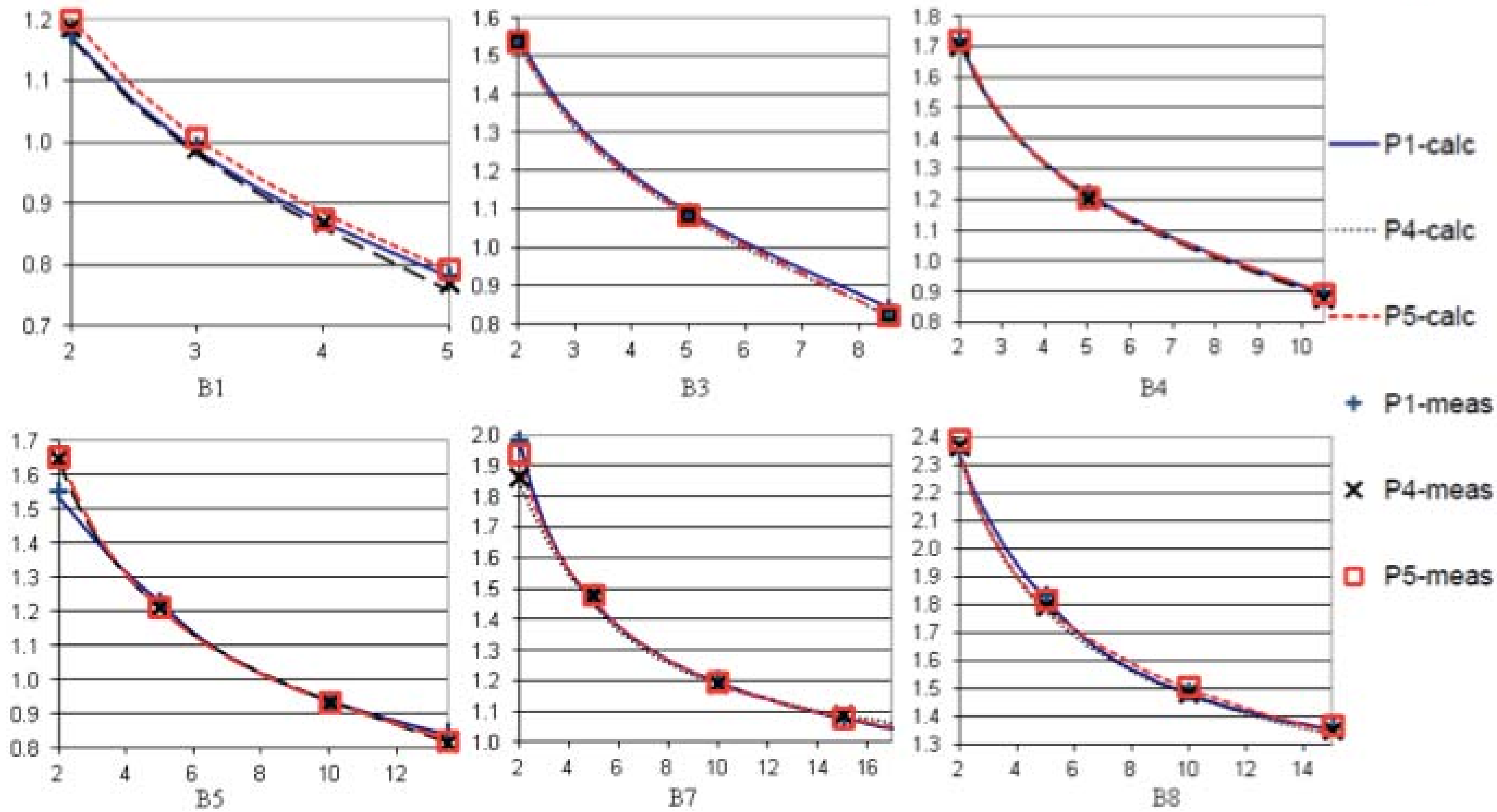
$$H_p = \frac{(CF/100)\cdot \psi_C \cdot D_C}{(1+a_1 \cdot r^{a_2})} \cdot (1+s(R-R_m)), r = b_2 r_n^2 + b_1 r_n + b_0,\ r_n = \frac{R-0.91M}{0.91M}$$

Option	*CF*	a_1	a_2	b_2	b_1	b_0	*s*	R_m *(cm)*
B1	0.677	0.4933	0.69177	0	0.93257	0.07286	0.0276	4.6
B2-1	0.670	0.58345	0.61897	0	0.92578	0.01932	0.0300	5.86
B2-2	0.689	0.57558	0.56474	0	0.97321	-0.00129	0.0000	5.86
B2-3	0.711	0.57693	0.56369	0	0.96567	-0.03908	-0.0200	5.86
B3	0.735	0.50406	0.58053	0.0039	1.0107	0.0033	0.0126	7.49
B4	0.710	0.60838	0.49789	0.04189	0.86883	0.05977	0.0178	9.54
B5 r>2	0.685	0.51036	0.53197	-0.07516	1.25614	-0.14522	0.0173	11.86
B5 r<2	0.685	0.51036	0.53197	0.16743	0.65577	0.09761	0.0173	11.86
B6-1 r>2	0.796	0.35963	0.58597	0.01187	0.99903	-0.00318	0.0000	15.53
B6-1 r<2	0.796	0.35963	0.58597	0.06377	0.89519	0.02232	0.0000	15.53
B6-2 r>2	0.819	0.35324	0.59653	0.00592	1.02821	-0.04525	0.0000	15.53
B6-2 r<2	0.819	0.35324	0.59653	0.06377	0.89519	0.02232	0.0000	15.53
B6-3 r>2	0.858	0.30171	0.62052	0.00215	1.01444	-0.02873	0.0000	15.53
B6-3 r<2	0.858	0.30171	0.62052	0.06377	0.89519	0.02232	0.0000	15.53
B7 r>4	0.813	0.39906	0.52019	0.00356	0.95593	0.07888	0.0038	19.83
B7 r<4	0.813	0.39906	0.52019	-0.0024	0.99999	-0.00468	0.0038	19.83
B8 r>5	1.045	0.30657	0.58088	-0.02376	1.19746	-0.31829	0.0003	22.8
B8 r<5	1.045	0.30657	0.58088	0.05991	0.68095	0.31268	0.0003	22.8

JACMP 15: 297-306 (2011)

Beam data – Output factor for broad beam

Output factor vs. M



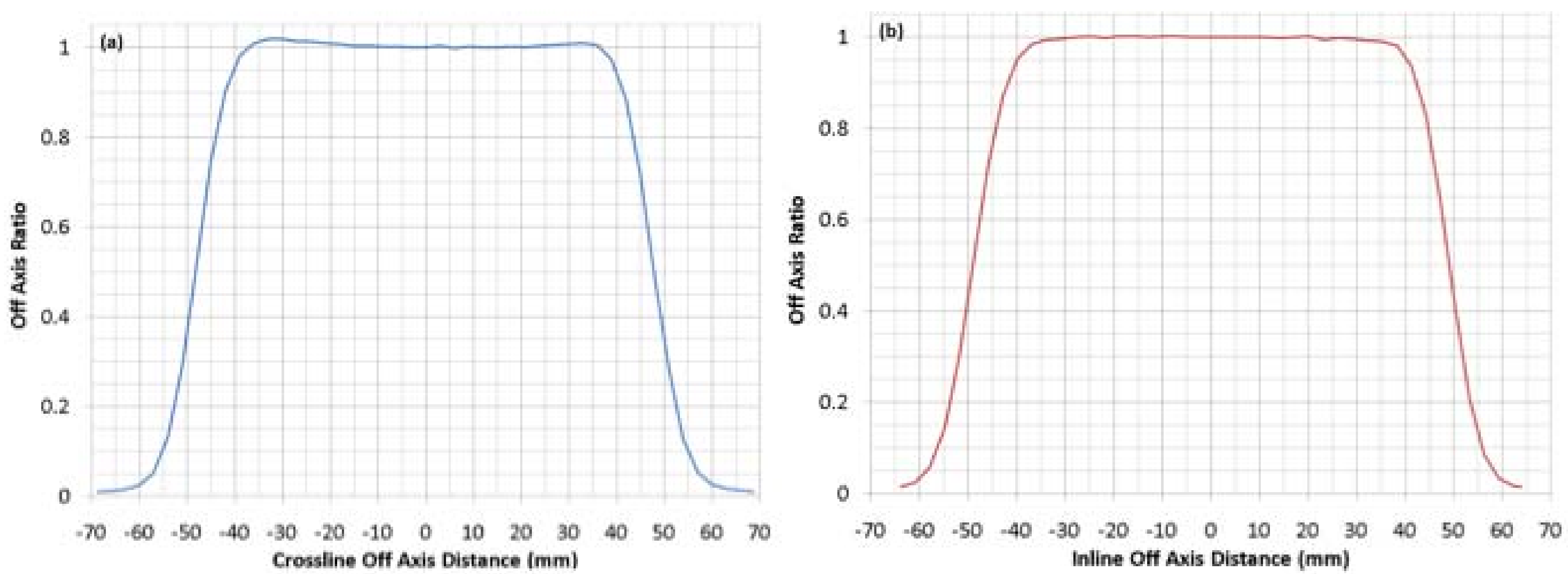
JACMP 15: 297-306 (2011)

Beam data – Output factor for PBS

- Output factor is not directly needed for PBS since pristine peaks is expressed in D/MU. However, small inaccurate modeling of lateral distribution for individual spot will accumulate to significant effect over many spots, the output deviation could be up to 10%. This variation is currently included as field size dependence of output factor.

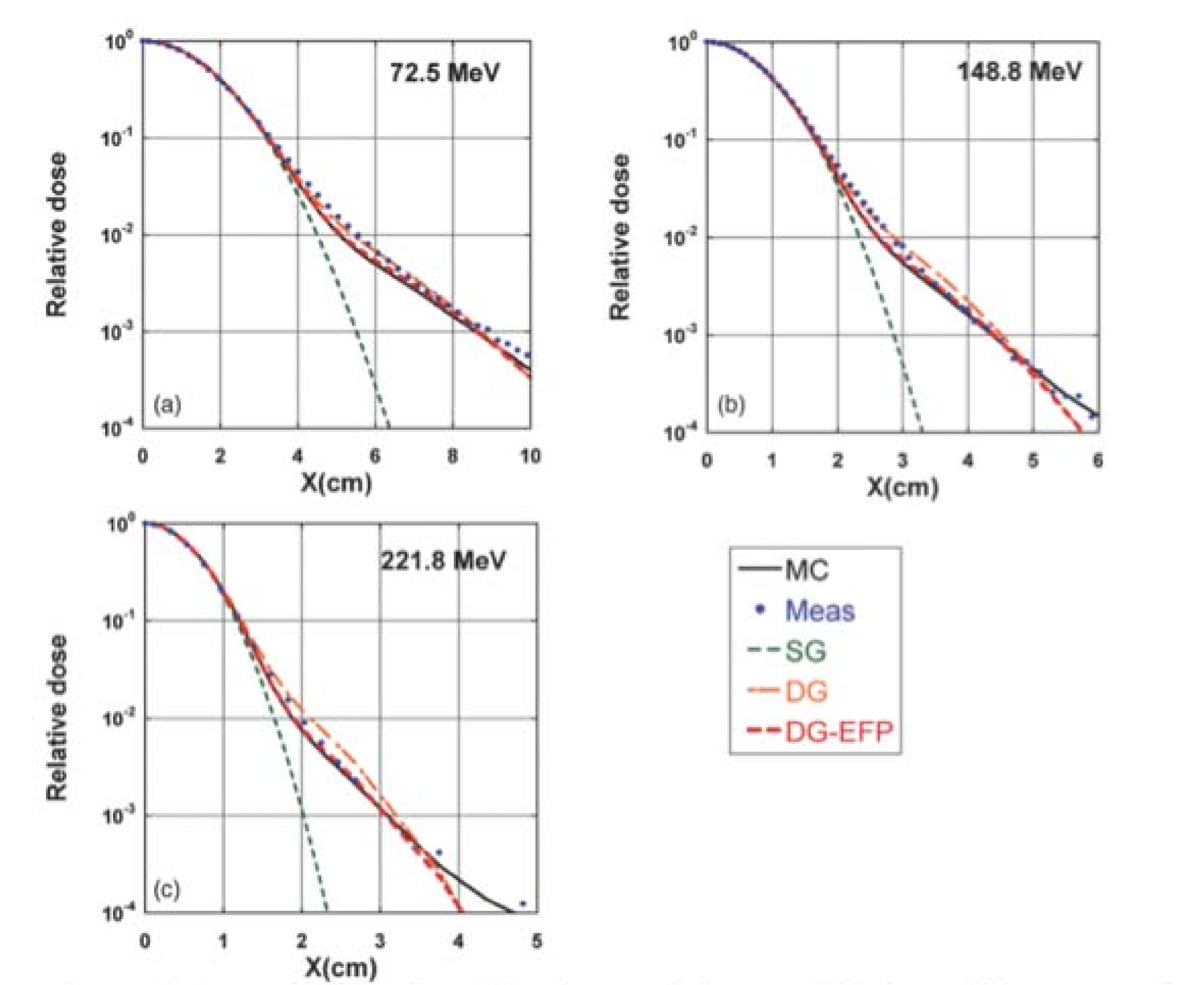
Beam data – OAR for broad beam

- The lateral profile for a broad proton beam is typically very flat, thus OAR = 1



(a) cross line and (b) in-line OAR for10 cm ×10 cm proton beam of SOBP beam of range of 15 cm and modulation of 10 cm.

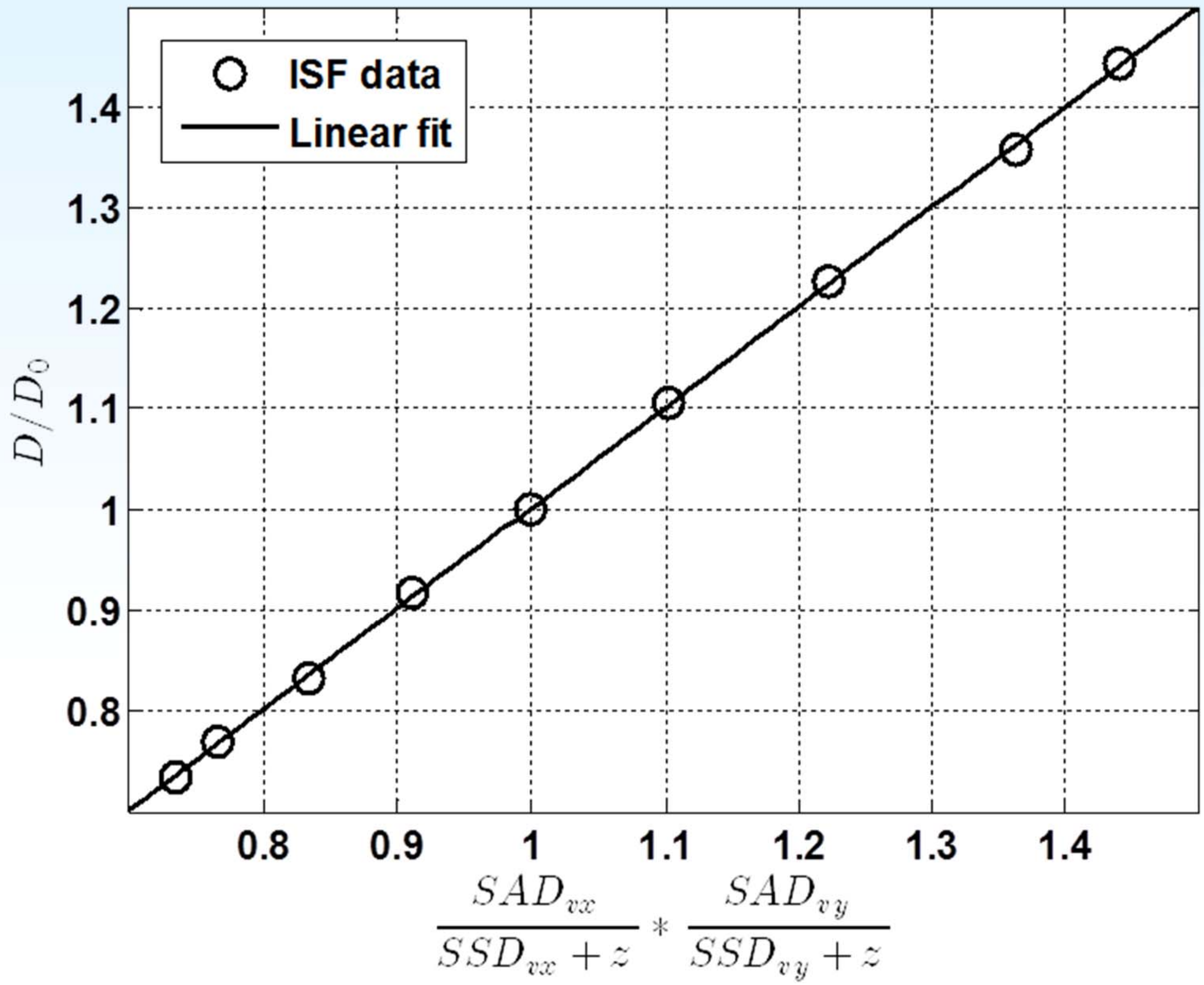
Beam data – OAR for PBS



Med Phys 40: 041723 (2013).

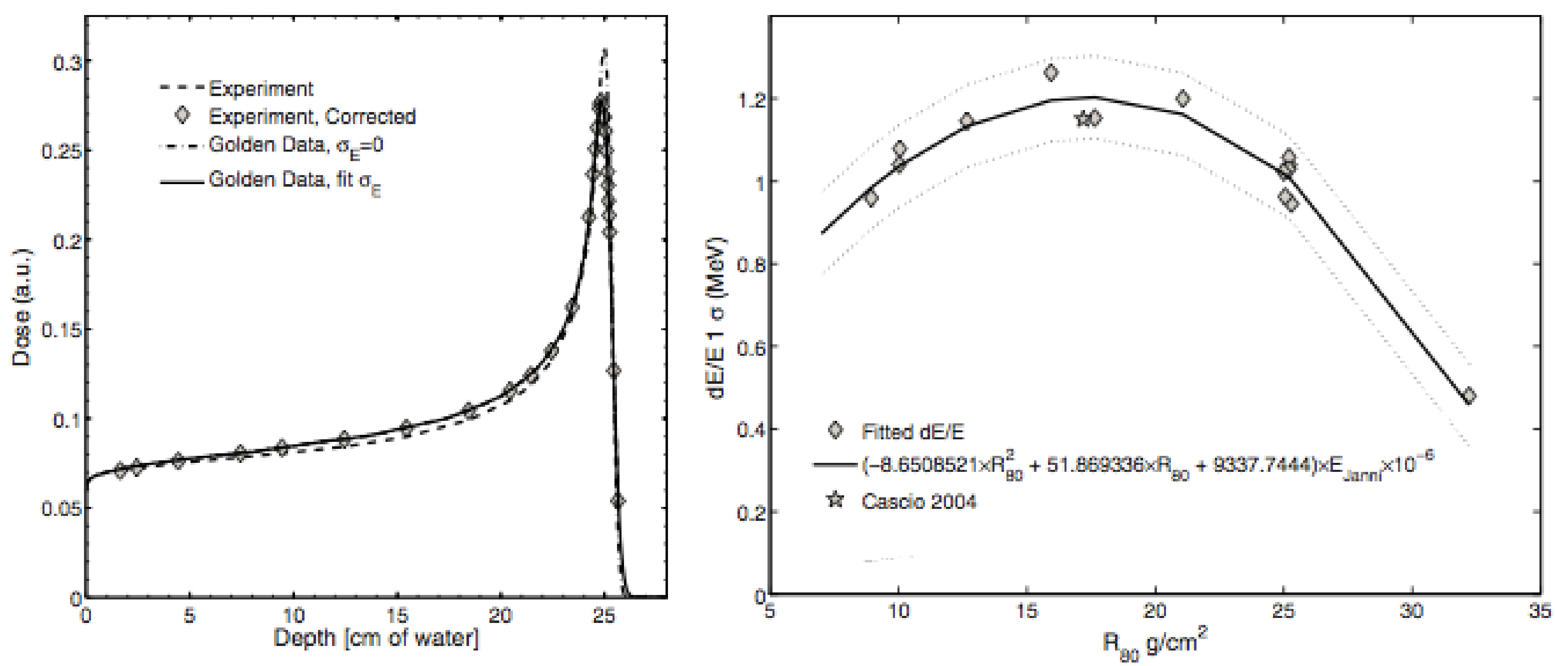
Beam Data – Inverse square law for PBS

$$ISF = \frac{SAD_{vx} * SAD_{vy}}{(SSD_{vx} + z) * (SSD_{vy} + z)}$$



Beam data – Generic

- It is possible to develop generic beam data for PBS, which with appropriate tuning of the proton gaussian energy spectrum can be matched to a specific PT center.



PMB 57: 1147 – 1158 (2012).

Introduction

Pencil-beam based dose/MU algorithms

Factor-based dose/MU algorithms

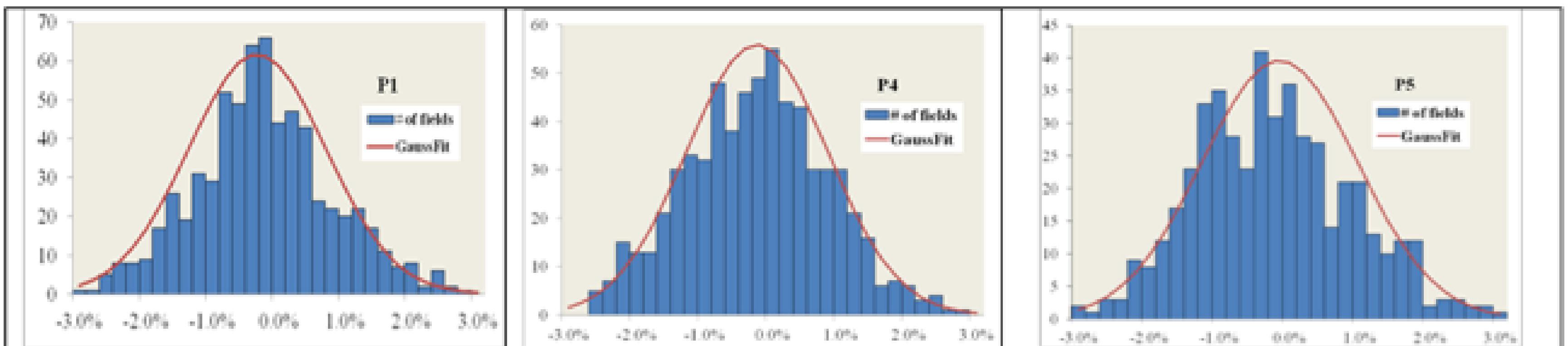
Beam data

Dose and MU verification

Validation Results – Broad beam

Med Phys 35: 5088-5097 (2008)

Treatment site	Number of fields	Mean of difference (%)	Standard deviation (%)
Prostate	228	−0.35	0.83
Thorax	211	0.35	1.21
CNS	167	0.15	1.10
Total (including other treatment fields)	623	0.05	1.09

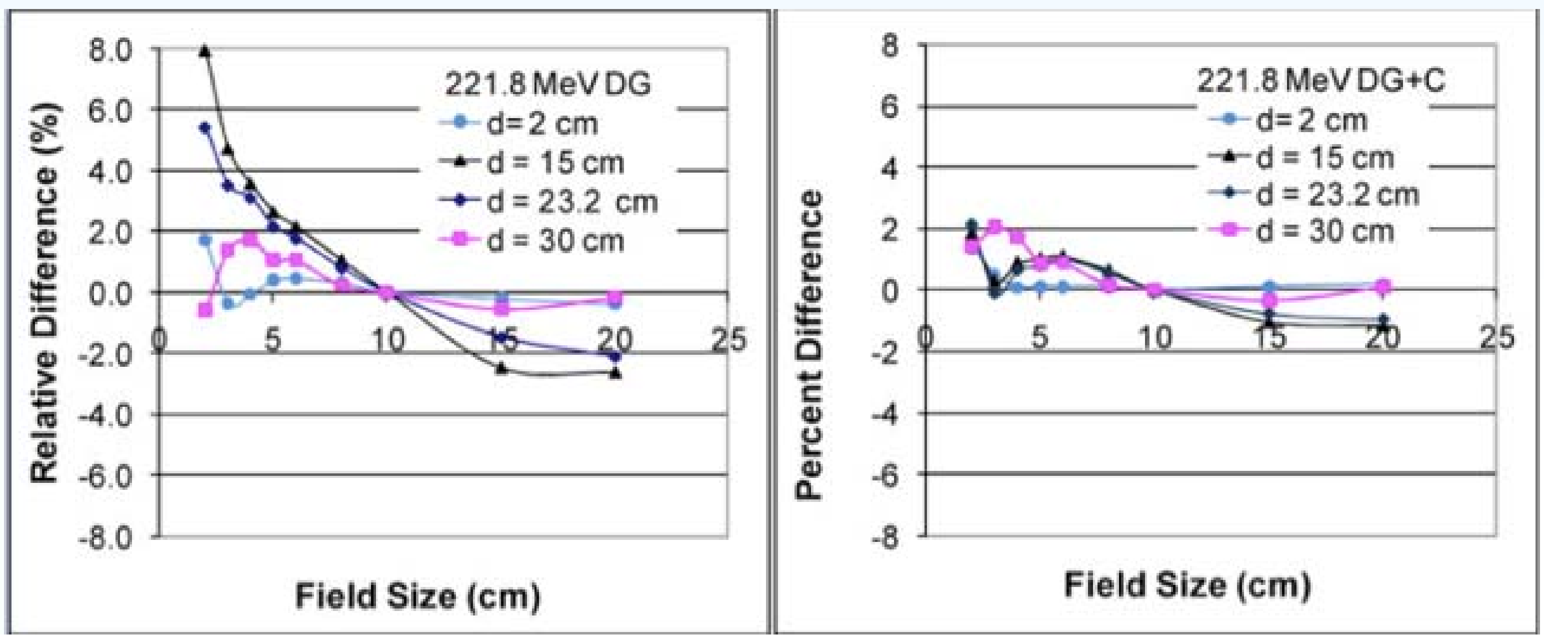


(a) 662 fields with -0.12%±1.0% (b) 647 fields with -0.13%±1.0% (c) 475 fields with -0.14%±1.1%

JACMP 15: 297-306 (2011)

Validation Results – PBS

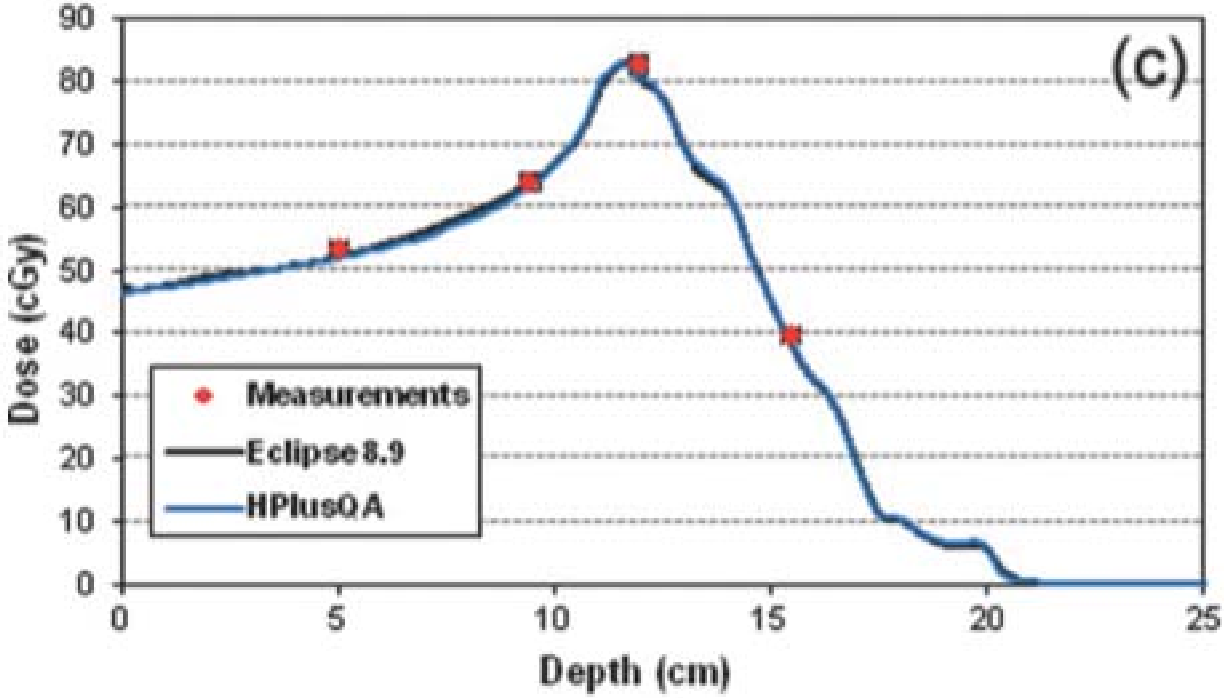
- An analytical model developed at MD Anderson was found to agree with measurement to within 2% at all depths, energies, and field sizes. With a gamma passing rate >99% at 2% and 2 mm.



PMB 57: 983-997 (2012)

Validation Results – PBS

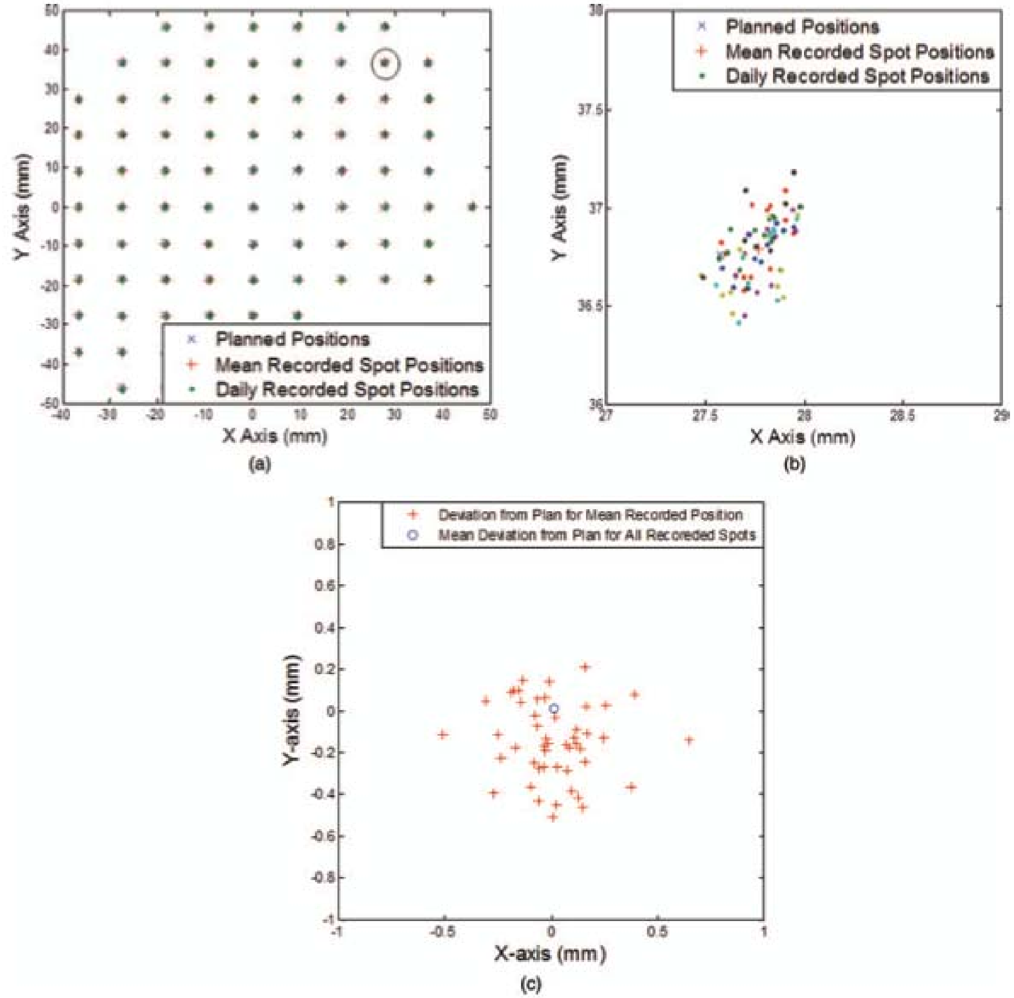
- HPlusQA agrees with measurement for PBS



Med Phys 40: 121708 (2013)

Delivery Validation

- A treatment log file (E,MU,Positions) from PBS (if available) is used for patient-specific quality assurance.



Med Phys 40: 021703 (2013)

Take home messages:

The second MU calculation algorithms are broadly separated into: factor-based and model-based algorithms.

For broad proton beam, MU calculation is almost always taken at middle of SOBP, where PDD = 1. The difference between MU calculation and measurement were generally between 1-3%.

For PBS, Dose distribution validation at several depths becomes necessary. The current practice acceptable agreement were obtained in a uniform phantom, further improvements can be made to improve the PBS lateral profiles.

Novel use of MU calculation include IMPT delivery validation using log file