

Purpose:

To propose an objective function for TCP/NTCP curve fitting.

Method and Materials:

One of the widely used objective functions by physicists, when fitting theoretical models to experimental data, is the χ^2 one, where:

$$\chi^2 = \sum_i ((Y^i_{\text{theoretical}} - Y^i_{\text{experimental}}) / \sigma^i_{\text{experimental}})^2$$

which is the maximum likelihood function in case of Gaussian random variables. In the case of a binary outcome, as is the nature of clinical or experimental animal radiation outcome data, the random variable has a binomial distribution and the maximum likelihood function for ungrouped data becomes:

$$L = \sum_{\text{responders}} \ln(P_{\text{theor}}(\text{par}, \text{DVH})) + \sum_{\text{non-responders}} \ln(1 - (P_{\text{theor}}(\text{par}, \text{DVH}))),$$

where P_{theor} stands for $\text{TCP}_{\text{theor}}$ or $\text{NTCP}_{\text{theor}}$. The maximization of L is often used by different authors for the estimation of the TCP/NTCP model parameters.

Results:

However, sometimes χ^2 is used to fit TCP/NTCP functions for different purposes like theoretical comparison between different models. In this case the application of χ^2 becomes inaccurate and inadequate especially when the function values are close to the ends of the interval in which they are defined, namely 0 or 1. This is why we propose the application of the double logarithmic transformation, presuming that the random variable is not normally but log-log-normally distributed.

Conclusion:

Thus the objective function in case of model comparison would become:

$$\chi^2 = \sum_i ((-\ln(-\ln(P^i_{\text{theoretical}})) + \ln(-\ln(P^i_{\text{experimental}}))) / \sigma^i_{\text{log_experimental}})^2$$

where $\sigma^i_{\text{log_experimental}} = -\sigma^i_{\text{experimental}} / (P^i_{\text{experimental}} \ln(P^i_{\text{experimental}}))$.

Here $P^i_{\text{theoretical}}$ stands for the values of P predicted by one of the models that are being compared and $P^i_{\text{experimental}}$ stands for the values of P predicted by the other model.

Conflict of Interest (only if applicable):