AbstractID: 7169 Title: Energy-range relations and an analytical integration procedure of Bethe-Bloch equation

Purpose: A calculation procedure of energy-range relations of protons, of the residual energy E(z) and its gradient dE(z)/dz according to Bragg-Kleeman rule and its relativistic generalization and to an analytical integration of Bethe-Bloch equation (BBE) is presented. The determination of LET as a function of E(z) only works reliably on the base of BBE.

Method and Materials: The Bragg-Kleeman rule $R_{CSDA} = A\cdot E_0^p$ results from a modified Langevin equation. A relativistic extension reads: $R_{CSDA} = A\cdot(E_0+E_0^2/2Mc^2)^p$, where E_0: initial energy, R_{CSDA}: range according to continuous-slowing-down-approximation, and Mc^2: proton rest energy. A suitable transformation provides a complete integration of BBE. The solutions are compared with GEANT4.

Results: The dimensionless power p of Bragg-Kleeman rule is energy-dependent, if the same accuracy of R_{CSDA}, E(z) and dE(z)/dz as by BBE is required. The exact integration of this theory removes the singularities occurring in numerical Monte-Carlo procedures (cutoff in GEANT4: 1 MeV) mainly by the shell correction, which compensates the logarithm term of BBE. For therapeutic proton energies with $E_0 \leq 300$ MeV simple analytical solutions of $R_{CSDA}(E_0)$, E(z) and dE(z)/dz will be presented in terms of a power expansion and/or of a sum of 5 exponential functions.

Conclusion: The new integration procedure of BBE prevents singularities in the domain of the Bragg peak and at $z = R_{CSDA}$. This provides an accurate determination of the LET (dE/dz as a function of residual energy E). A subjection of the analytical solutions to a convolution (energy-straggling) reduces the LET in the environment of the Bragg peak and at the distal end of the Bragg curves. The calculation of LET according to Bragg-Kleeman rule should also be avoided, since it implies a singularity at $z = R_{CSDA}$.