Dual Energy CT Image Reconstruction
Algorithms and Performance

Joseph A. O’Sullivan
Samuel C. Sachs Professor
Dean, UMSL/WU Joint Undergraduate Engineering Program
Professor of Electrical and Systems Engineering,
Biomedical Engineering, and Radiology

jao@wustl.edu

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J. A. O’Sullivan, AAPM 07/28/09
Collaborators

Faculty

Bruce R. Whiting
David G. Politte
Jeffrey F. Williamson, VCU
Donald L. Snyder

Students

Norbert Agbeko
Liangjun Xie
Daniel Keesing
Josh Evans, VCU
Debashish Pal
Jasenka Benac
Giovanni Lasio, VCU

Chemical Engineers

M. Dudukovic
M. Al-Dahhan
R. Varma

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Dual Energy CT Image Reconstruction

• Data Models ➔ Reconstruction Algorithms
• Image Reconstruction Approaches
  – “Linear” Approaches
  – Statistical Iterative Reconstruction
• Simulation Study of the Dual Energy Alternating Minimization Algorithm
• Performance Quantification: the Cramer-Rao Lower Bound
• Conclusions
Motivation

• Dual Source, Fast kVp Switching, Photon Counting, and Energy Selective Detectors Are Available
  - Dual energy image reconstruction algorithms are needed now
  - Imaging III session yesterday; this session

• System Selection and Algorithm Design
  - Basis for comparing systems based on performance
  - Fundamental approach that extends to new systems (multiple energies, photon counting, etc.)
  - Quantifying the impact of modeling errors
Dual Energy Image Reconstruction

- Multiple inputs with different spectral sensitivities
- Some standard normalizations (e.g., relative to air scans)
- Joint processing combines the data sets
- Post-processing can extract the desired image(s)
  - Components
  - Estimated images
X-Ray Transmission Tomography—Back to Basics

- Source Spectrum, Energy $E$
- Beer’s Law and Attenuation $S(E)e^{-\int \mu(x,E)dx}$
- Mean Photons/Detector $I_0$ $I_0S(E)e^{-\int \mu(x,E)dx}$
- Beam-hardening
- Detector Sensitivity Spectrum
- Mean (Photon) Flux $D(E)$, $\Phi(E) = S(E)D(E)$ $I_0\Phi(E)e^{-\int \mu(x,E)dx}$ $\int_0^{kVp} I_0\Phi(E)e^{-\int \mu(x,E)dx} dE$


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“Linear” Image Reconstruction

- Normalize relative to an air scan
- Beam-hardening correction to target energy
- Negative log to estimate line integrals at target energy
- Linear inversion, normalize (e.g. water-equivalent)

\[
\int_{l(y)} \mu(x, \bar{E}) dx \approx -\log \left[ BH^{-1} \left( \frac{\int_{0}^{kVp} I_0 \Phi(E) e^{-\int_{l(y)} \mu(x, E) dx}}{\int_{0}^{kVp} I_0 \Phi(E) dE} \right) \right]
\]

\[
\hat{\mu}(x, \bar{E}) = FBP \left( \int_{l(y)} \mu(x, \bar{E}) dx \right), \quad \hat{c}(x) = \frac{\hat{\mu}(x, \bar{E})}{\mu_{water}(\bar{E})}
\]
X-Ray Transmission Tomography—Back to Basics

• Detector Sensitivity $D(E)$
  - Probability that a photon of energy $E$ is detected
  - Mean response to a photon of energy $E$:
    • photon counting $\Rightarrow$ response $= 1$
    • energy integrating $\Rightarrow$ response $= E$
    • other

• Detector Statistics
  - Photon counting, Beer’s Law as survival probability
    $\Rightarrow$ Poisson distribution
  - Energy integrating or other $\Rightarrow$ compound Poisson

Poisson mean $\lambda$:
  $$P(N = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k \geq 0$$

Log-likelihood function:
  $$l(\lambda) = k \ln \lambda - \lambda$$
X-Ray Transmission Tomography—Reconstruction Principles

- Deterministic Model
  - Data equal a function of the desired image
  - Approximately invert that function to reconstruct image (minimize a measure of error between the data and the model)
    \[ \min_{\mu} \|d - g(\cdot; \mu)\|^2 \]

- Random Model
  - Find the log-likelihood function for the data
  - Maximize the (possibly penalized) log-likelihood function over possible images
    \[ \max_{\mu} l(d \mid g(\cdot; \mu)) \]
Statistical Image Reconstruction

• Source-detector pairs indexed by $y$; voxels indexed by $x$

• Data $d_j(y)$ Poisson, means $g_j(y; \mu)$, log-likelihood function

$$l(d_j | g_j(\cdot; \mu)) = \sum_{y \in Y} d_j(y) \ln g_j(y; \mu) - g_j(y; \mu)$$

$$g_j(y; \mu) = \sum_{E} I_j \Phi_j(y, E) \exp \left(-\sum_{x \in X} h(y, x) \mu(x, E)\right) + \beta_j(y)$$

• Mean unattenuated counts $I_j$, mean background $\beta_j$

• Attenuation function $\mu(x, E)$, $E$ energies

$$\mu(x, E) = \sum_{i=1}^{I} c_i(x) \mu_i(E)$$

• Maximize over $\mu$ or $c_i$

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Attenuation Function Approximation

• Voxels as function approximation
  - Constant attenuation over a small volume, or
  - Linear combination of basis functions

• Energy dependence
  - Water equivalent
  - Linear combination of basis functions

• Basis functions
  - Physics (photoelectric and Compton scatter)
  - Physiological (e.g., fat and bone)
  - Signal processing (e.g., SVD)
  - Hand selected (e.g., CaCl and styrene)

• Constrained system
  - Dual Energy, 3 images

\[ \mu(x, E) = \sum_{i=1}^{I} c_i(x) \mu_i(E) \]
\[ c_i(x) \geq 0, \quad c_1(x) + c_2(x) + c_3(x) = 1 \]
Statistical Image Reconstruction—AM Algorithm

- Alternating Minimization Algorithm
  - Several papers from our group (O’S & Benac, TMI 2007)
  - Monotonically increasing log-likelihood

\[ g_j(y: \mu) = \sum_E I_j \Phi_j(y, E) \exp \left( -\sum_{x \in X} h(y, x) \sum_{i=1}^I \mu_i(E)c_i(x) \right) + \beta_j(y), \]

\[
\max \left\{ c_i(x) \right\} \sum_{j=1}^2 \ln \left( \frac{d_j(y \mid g_j(\cdot: \mu))}{\sum_{j=1}^2 \sum_{y \in y} d_j(y) \ln g_j(y: \mu) - g_j(y: \mu)} \right) = 0
\]

- Compare using backprojection of data, estimated means

\[
\hat{c}_i^{(k+1)} = \hat{c}_i^{(k+1)}(x) - \frac{1}{Z_i(x)} \ln \left( \frac{\sum_j \tilde{b}_{i,j}^{(k)}(x)}{\sum_j \tilde{b}_{i,j}^{(k)}(x)} \right)
\]

\[
\tilde{b}_{i,j}^{(k)}(x) = \sum_{y,E} h(y,x) \mu_i(E) q_j^{(k)}(y,E)
\]

\[
\tilde{b}_{i,j}^{(k)}(x) = \sum_{y,E} h(y,x) \mu_i(E) \frac{d_j(y) q_j^{(k)}(y,E)}{\sum_{E'} q_j^{(k)}(y,E')}
\]

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Dual Energy CT Image Reconstruction

• Data Models → Reconstruction Algorithms

• Image Reconstruction Approaches
  – “Linear” Approaches
  – Statistical Iterative Reconstruction

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• Performance Quantification: the Cramer-Rao Lower Bound

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- Multiple inputs with different spectral sensitivities
- Some standard normalizations (e.g., relative to air scans)
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Dual Energy Image Reconstruction

- Multiple inputs with different spectral sensitivities

Selected Technologies
- Multiple sources
- Fast kVp switching
- Multiple detectors
- Energy selective photon counting

Selected Issues
- Data quality versus dose
- Clinical issues, including motion
- Image use (application)

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Dual Energy Image Reconstruction

- Joint processing combines the data sets
  - Linear combination of attenuations, then
    reconstruct individual images independently
Dual Energy Image Reconstruction

- Joint processing combines the data sets
  - Nonlinear inversion of raw data, then
  reconstruct individually

\[ g_1(L_1, L_2) = \hat{a}_E \ I_1 F_1(E) \exp\left(-L_1m_1(E) - L_2m_2(E)\right) \]
\[ g_2(L_1, L_2) = \hat{a}_E \ I_2 F_2(E) \exp\left(-L_1m_1(E) - L_2m_2(E)\right) \]

Dual Energy Image Reconstruction

- Joint processing combines the data sets
  - Reconstruct individual images from each data set, then (linearly) combine the resulting attenuation images to estimate desired outputs

• Joint processing combines the data sets
  – Use knowledge of source spectrum and detector sensitivity spectrum
  – Use a model of joint data dependence on unknown underlying attenuation maps
  – Jointly estimate component images based on that model (e.g., statistical iterative reconstruction)

See also: Sukovic and Clinthorne, TMI 2000.
Dual Energy CT Image Reconstruction

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Simulations: Post-reconstruction vs. Joint Statistical Image Reconstruction

- **Large Phantom**: 20cm in diameter; thin outer lucite shell; water; four rods in inner 60mm lucite cylinder.
  - Calibration rods: calcium chloride, ethanol, teflon and polystyrene in the 12, 3, 6, and 10 o'clock positions, resp.
  - Test phantom rods: muscle, ethanol, teflon and substance X (a bonelike material) in the 12, 3, 6, and 10 o'clock positions, resp.

- **Small Phantom**: 60mm diameter lucite cylinder with rods

- **FBP-BVM** (Basis Vector Method; JF Williamson, et al. 2006)
  - Water-equivalent beam hardening correction
  - Requires calibration data to estimate linear transformation that generates the component images
  - FBP uses a ramp filter

- **AM-DE** (AM Dual Energy Algorithm)
  - NO pre-correction of data
  - NO calibration data
  - NO regularization
## Materials Used - Fractions

<table>
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<tr>
<th>Substance</th>
<th>Styrene Fraction</th>
<th>Ca. Chloride Fraction</th>
</tr>
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<tr>
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<tr>
<td>Substance X</td>
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<td>2.8613</td>
</tr>
</tbody>
</table>

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Large Phantom ($I_0=10^5$) – Component Image

Styrene

Calcium Chloride

Truth  FBP-BVM  AM-DE
Large Phantom – Synthesized Images

20 keV

Truth

FBP-BVM

AM-DE

65 keV

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Large Phantom Profiles

Profiles through column 256 of 20keV image(left) and 65keV image(right)

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Mini CT – Small Phantom

- 360 Source positions
- 92 detectors
- Based on Siemens Somatom Plus 4 scanner
- 64 by 64 image
Small Phantom – Component Images

Styrene

Calcium Chloride

Truth FBP-BVM AM-DE

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Small Phantom – Synthesized Images

20 keV

65 keV

Truth

FBP-BVM

AM-DE

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Small Phantom – 20keV profiles

Profile through row 25 of 20keV image

Profile through row 32 of 20keV image

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Small Phantom – 20keV ratio images and profiles

FBP-BVM

AM-DE

Profiles through row 25 of 20keV ratio images

Profiles through row 32 of 20keV ratio images

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Small Phantom: Relative Errors

$I_0 = 10,000$

- Lucite – Center: A region of interest in the center of the phantom.
- Lucite – Edge: A region of interest near the edge of the phantom, between Substance X and Teflon.
- Relative error equals absolute difference divided by truth

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Small Phantom: Relative Errors

$I_0 = 100,000$

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Small Phantom: Relative Errors
$I_0 = 1,000,000$

- AM-DE relative errors decrease consistently as the number of counts increases
- AM-DE performs at least an order of magnitude better

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Small Phantom – Ensemble mean and variance (from 15 samples) $I_0 = 100,000$  *Styrene Component*

**AM-DE**

**FPB-BVM**

**Truth**

**Mean**

**Variance**

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Small Phantom – Ensemble mean and variance (from 15 samples) $I_0 = 100,000$ Ca. Chloride Component

Variance is inversely proportional to $I_0$
Predicting Performance: Fisher Information and the Cramer-Rao Bound

- The variance of an unbiased estimate is greater than or equal to the Cramer-Rao lower bound (CRLB)
- CRLB is conditioned on a model
- CRLB is independent of the algorithm
- AM-DE is biased (in part due to nonnegativity constraint)
- CRLB is derived from the inverse of Fisher information
- Fisher information measures the joint dependence of values within a component image and across images
- Fisher information is proportional to $I_0$

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**Fisher Information: Within a Component Image**

Calcium chloride  
**Maximum 1.2E5**

Styrene  
**Maximum 1.6E4**
Fisher Information: Across Images

Maximum 4.2E4

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CRLB Images: Diagonals of Inverse of Fisher Information

Ratio of variance images to CRLB. Different display windows.

Styrene

Calcium Chloride

CRLB

FBP-BVM

AM-DE
CRLB Images: Diagonals of Inverse of Fisher Information

Ratio of variance images to CRLB. SAME display windows.

Styrene

Calcium Chloride

CRLB

FBP-BVM

AM-DE

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Conclusions

• Reviewed Models for CT Image Reconstruction
• Outlined Various Approaches for Dual Energy CT Image Reconstruction
• Summarized a Simulation Study Showing Performance of a Dual Energy Alternating Minimization Algorithm
• Outlined and Applied a Method for Quantifying Achievable Performance Based on Fisher Information
• Quantitative Analysis Shows Benefit of AM-DE algorithm
• AM-DE Algorithm is Computationally Demanding
• Other Work: Quantify Impact of Modeling Errors
References


Small Phantom – Ensemble mean and variance (from 15 samples) $I_0 = 100,000$
Small Phantom – Ensemble mean and variance (from 15 samples) \( I_0 = 100,000 \)

65keV Image

AM-DE

FBP-BVM

Truth

Mean

Variance

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CRLB images for 20keV (above) and 65keV (below) images
Model-Based Imaging: 

*Principled Algorithm Development*

- Model as much of the underlying physics, biology, chemistry as possible
- Derive an objective function based on physical model, problem definition, and implementation constraints (complexity, robustness) → loglikelihood function
- Derive algorithms to optimize objective function → maximum likelihood; mathematical considerations
- Predict and evaluate performance using simulations and phantom experiments
- Revisit physical models, algorithms
- Transition to clinical settings → improve algorithms, implementations, connect to imaging sensors or scanners, identify key applications