AbstractID: 12592 Title: Proton Computed Tomography Reconstruction using a Bayesian Inference based Proton Path Probability Map

Introduction: Proton computed tomography (pCT) is a potential low dose imaging modality. Since protons undergo multiple Coulomb scattering (MCS) inside the imaging object, pCT image reconstruction requires the estimation of proton path based on the position and direction of proton when entering and leaving the imaging object. Recently Schulte et al[1] have proposed a maximum likelihood (ML) formalism for estimating the most likely path (MLP) for pCT reconstruction. It has been further proposed that a likelihood envelope around the MLP could be employed for pCT reconstruction. In this work, we propose an alternative method for proton path estimation that can be employed to generate a proton path probability map, from which the most likely path and a likelihood envelope around the most likely path can be extracted.

Mathematical Derivations: Referring to **Figure 1**, by construction we impose for the incoming condition $A(z_A, x_A, \boldsymbol{\theta}_A)=(0, 0, 0)$, i.e. we assume that we have a proton incident along *z*-axis. Moreover, we assume that we have the means of measuring the exit condition D_θ, which is given by (z_D, x_D, θ_D). The posterior probability of proton passing through the intermediate condition C_θ can be expressed in a Bayesian inference framework as follows:

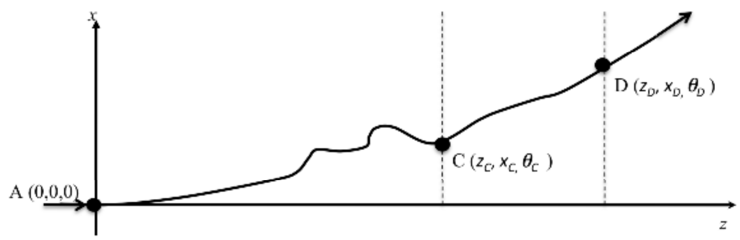

Figure 1 Illustration of how to the calculate the conditional path probability at Point C

$$P(A \rightarrow C_\theta \mid D_\theta) = \frac{P(A \rightarrow C_\theta) \cdot P(C_\theta \rightarrow D_\theta)}{P(A \rightarrow D_\theta)} \tag{1}$$

Note that the subscript θ indicates the conditions *C* and *D* are for a *specific direction* of the proton. For path estimation however, we are interested in the probability, $P(A \rightarrow C \mid D_\theta)$, that a proton moves through (z_C, x_C) *regardless of the direction at C* given the specific exit condition D_θ:

$$P(A \rightarrow C \mid D_\theta) = \frac{\int_{\theta_C} P(A \rightarrow C_\theta) \cdot P(C_\theta \rightarrow D_\theta)}{P(A \rightarrow D_\theta)} \tag{2}$$

No specific conditions on *P*(•) need to be imposed in this framework. In this study a form of the distribution based on Fermi's MCS theory[2] has been employed. In particular, a semi-analytical formula $F_u(z, x, \theta)$ is used. For a proton entering at (0,0) along *z*-axis with certain energy, $F_u(z, x, \theta)$ is the probability of it scattered to position (*z*,*x*) with direction θ, given by

$$F_u(z,x,\theta) = \frac{\sqrt{3}}{2\pi}\frac{u(z)^2}{z^2}\exp\left[-u(z)^2\left(\frac{\theta^2}{z} + \frac{3x\theta}{z^2} + \frac{3x^2}{z^3}\right)\right] \tag{3}$$

Integrating $F_u(z, x, \theta)$ for all θ at a given depth *z* yields the distribution of proton's vertical deflection *x* irrespective of its direction:

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$$H_u(z,x) = \frac{\sqrt{3}}{2\sqrt{\pi}} \frac{u(z)}{z^{3/2}} \exp\left(-\frac{3}{4}\frac{u(z)^2 x^2}{z^3}\right) \tag{4}$$

This is a Gaussian with variance

$$\sigma_z^{\;2} = \frac{2}{3}\frac{z^3}{u(z)^2} \tag{5}$$

This variance can be determined using Geant4 Monte Carlo simulation, and the parameter $u(z)$ can be obtained from it through a least-square fit, yielding a specific form for $F_u(z, x, \theta)$.

Estimated Proton Paths: **Figure 2** shows an example of path estimation.
a) The proton path as recorded during Monte Carlo simulation in 1 mm resolution; **b)** Cubic Spline Path (CSP) estimate with bilinear interpolation; **c)** path probability density map generated using the Bayesian inference based method, with MLP highlighted; **d)** MLP90 envelope, i.e., pixels with 90% or higher probability of being passed by the proton, relative to the probability of MLP.

Reconstructed Images: Monte Carlo simulation of proton projections on a Shepp-Logan phantom was performed using Geant4 simulation toolkit. **Figure 3** shows the reconstruction of pCT image using different proton path estimations. **a)** Reconstructed using the CSP method with bilinear interpolation; **b)** Reconstructed using the full BI probability density map; **c)** Reconstructed using MLP extracted from the BI probability density map; **d)** Reconstructed using MLP90 envelope.

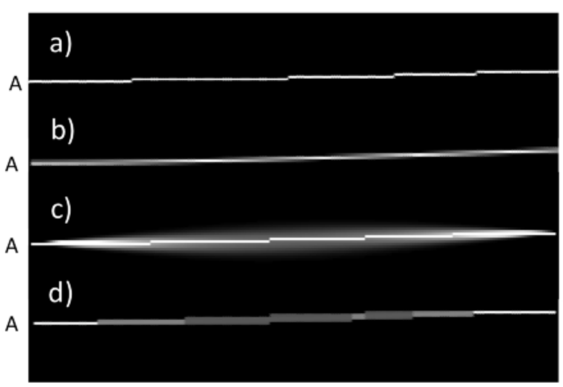

Figure 3 Estimated proton paths using different methods

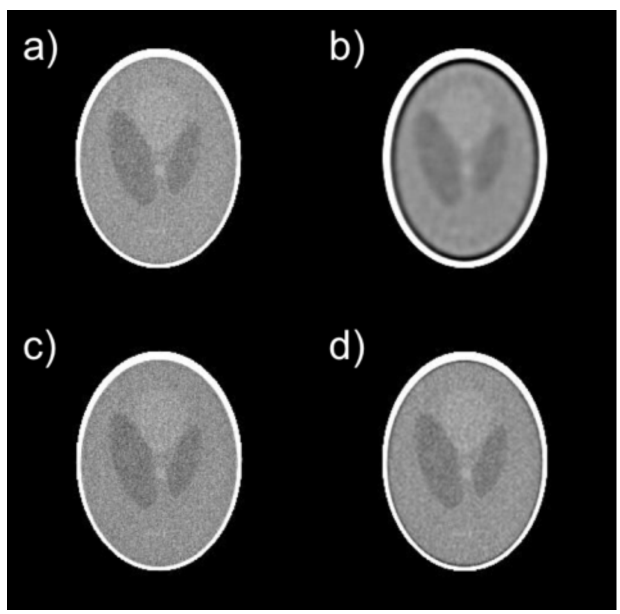

Figure 3 Reconstructed Images using different path estimation methods

References:

1. R.W. Schulte, S.N. Penfold, J.T. Tafas and K.E. Schubert, "A maximum likelihood proton path formalism for application in proton computed tomography," Med. Phys. **35**, 4849-4856 (2008).

2. B. Rossi and K. Greisen, "Cosmic-ray theory," Rev. Mod. Phys. **13**, 240-309 (1941).