Statistical phantoms

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Introduction

- Statistical phantoms:
  - Where did they come from?
    - Some background on backgrounds
  - What does it mean?
    - An attempt at a definition
  - How to generate?
    - A recipe
  - Have we actually learned anything?!?
    - New insights that we have gained thanks to these phantoms

Why phantoms?

- Phantoms are useful for
  - quality control (not discussed here)
  - investigation of the imaging chain, system optimization
  - assessment of image quality

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What is image quality?

How to define image quality?

A useful definition of image quality in medical imaging is

the ability of an observer to perform a well-defined task based on a set of images

(task-performance)

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Image quality is a statistical concept

Factors that affect image quality

- The statistics and physics governing the image formation and the statistics characterizing the object being imaged all contribute to the ability, or inability, of an observer to perform tasks and, hence, image quality.

JOSA A 24(12) B1 2007, Introduction to special issue on image quality

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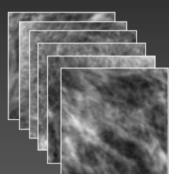
➢ object characteristics affect image quality

JOSA A 24(12) B1 2007, Introduction to special issue on image quality

Determining image quality: 1. images



Determining image quality: 2. task

- Task: detection of an exactly known signal
- Possible hypotheses:

  H_0: g = n (signal absent)

  H_1: g = n + s (signal present)

- Given an image g, observer decides whether signal is present/absent

Determining image quality: 3. observer performance

- Performance of the ideal observer in the task of detecting an exactly-known signal s:

$$d'^2 = s^t K^{-1} s$$

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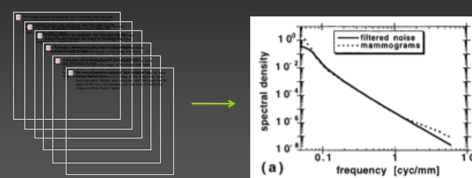
$$d'^2 = s^t K^{-1} s$$

for a given signal *s*, covariance *K* will determine *d'*

if the image statistics are (cyclo-)stationary, K is diagonalized by Fourier transform, with diagonal elements P(f), the power spectrum

Image statistics for a set of mammographic ROIs: Power spectrum

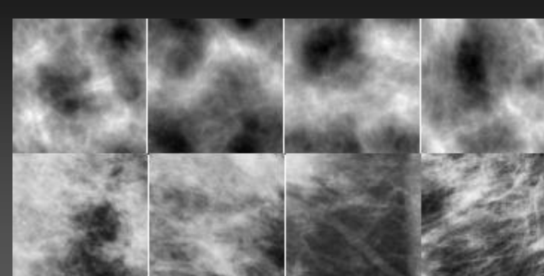


Burgess AE, Jacobson FL, Judy PF. Human observer detection experiments with mammograms and power-law noise. Medical Physics 2001;28(4):419.

Statistically defined backgrounds

- Background with well-defined 2nd order statistics (covariance matrix K)
- Pattern is due to randomness, rather than anatomical structure
- Typically stationary (K ⇔ Power spectrum)
- Examples:
  - lumpy backgrounds (generated in spatial domain)
  - power-law backgrounds (freq. domain)

Lumpy backgrounds

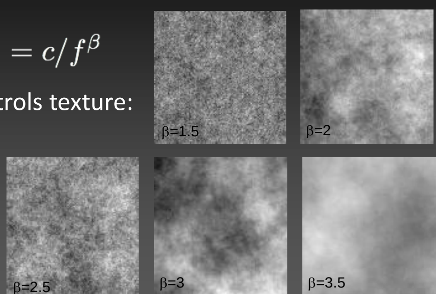


Bochud F, Abbey C, Eckstein M. Statistical texture synthesis of mammographic images with super-blob lumpy backgrounds. Optics express 1999 Jan;4(1):33-42.

Power-law backgrounds

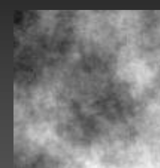
$$P(f) = c/f^{\beta}$$

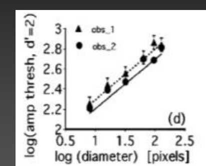
β controls texture:



Burgess AE, Judy PF. Signal detection in power-law noise: effect of spectrum exponents. JOSA A, 2007 Dec;24(12):B52-60.

Lesion detection in statistically defined backgrounds (β=3)





Mass detection in mammography is limited by the normal anatomy of the breast, rather than quantum noise (Burgess et al., 2001, 2007)

Statistical phantoms

- Inspired by statistically defined backgrounds
  - Pattern is due to randomness, rather than anatomical structure
- Modifications to account for object properties:
  - gray-scale to binary conversion to mimic tissue characteristics in a breast (adipose/glandular)
  - gray-scale conversion: step function, smooth transition

Structured background simulation

1. Generate power-law filtered noise volume
   - initialize in spatial frequency domain:

$$V(f_r) = \frac{c}{f_r^{\beta/2}} e^{i\tilde{\phi}} \qquad \tilde{\phi} \in [-\pi, \pi)$$

   - inverse Fourier transform to get filtered noise volume:

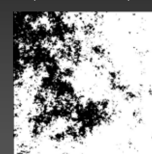
$$v(x, y, z) = F^{-1}\{V(f_r)\}$$

Structured background simulation, cont'd

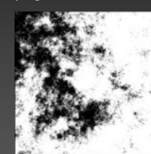
2. Apply gray-value threshold to obtain binary volume
   – structure: (% dense, β)
   – tissue properties (μ_{ad}, μ_{gland})

Statistical phantom: binarized power-law noise volume

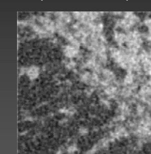
simulated volume slice (0.08 mm thick)



simulated volume slice (avg. across 10 slices)



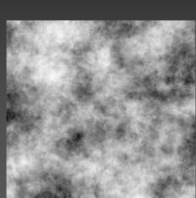
breast CT slice (0.2 mm thick)



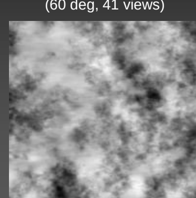
courtesy of J. Boone, UC Davis

Examples: projection view and tomosynthesis slice

cone-beam projection of simulated volume

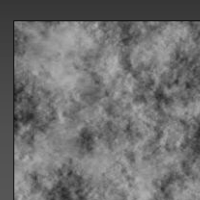


tomosynthesis reconstructed slice (60 deg, 41 views)

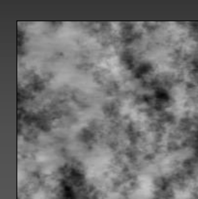


Comparison: tomosynthesis slice, 15 vs. 60 degree scan angle

tomosynthesis slice, 15 degree scan



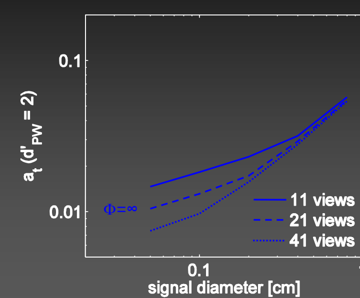
tomosynthesis slice, 60 degree scan



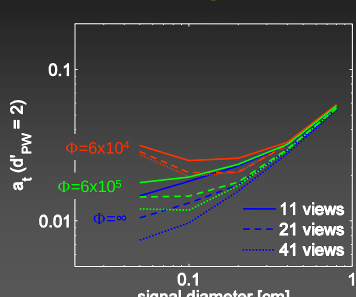
Phantom study: lesion detectability in tomosynthesis

- effect of # of views
- effect of quantum noise
- effect of scan angle

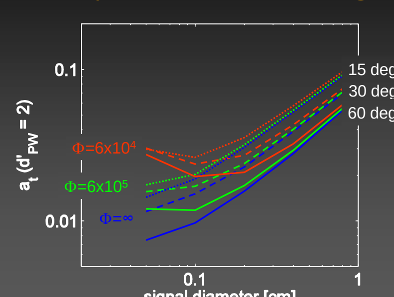
effect of # of views (α = 60 deg)



effect of quantum noise (α = 60 deg)



effect of scan angle α (angular step size $\Delta\alpha$ = 1.5 deg)

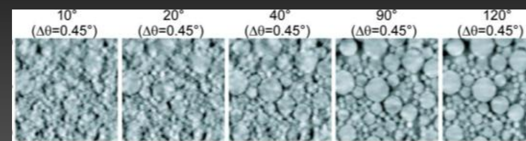


“clutter phantom” (random spheres)

- Inspired by lumpy backgrounds (?)
- Physical phantom:
  - mix of solid spheres of different radii (materials)
  - adjust size mix to achieve a given power-spectrum

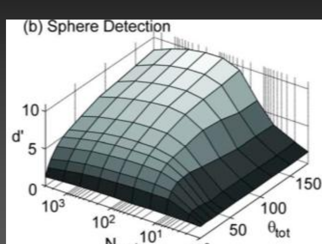
Gang GJ, Tward DJ, Lee J, Siewerdsen JH. Anatomical background and generalized detectability in tomosynthesis and cone-beam CT. Medical Physics 2010;37(5):1948

“clutter phantom” image examples



- β=3 is necessary but not sufficient condition for similarity with anatomic structure

detection of a 3mm sphere in “clutter phantom”



Gang GJ, Tward DJ, Lee J, Siewerdsen JH. Anatomical background and generalized detectability in tomosynthesis and cone-beam CT. Medical Physics 2010;37(5):1948.

Studies based on phantoms with power-law spectrum

Lesion detectability in tomographic imaging

- “clutter phantom” (random spheres)
  - acquisition parameters in tomosynthesis, cone-beam CT (Gang, Siewerdsen et al. 2010)
- power-law filtered noise phantom
  - detectablity in mammography, tomosynthesis and CT (Gong, Glick et al.; 2006)
  - tomosynthesis acquisition parameters, quantum noise (Reiser, Nishikawa; 2010)

Effect of scan angle, view number on signal detectability in tomographic imaging

- Conclusions:
  - imaging of tumor-sized objects benefit from large scan angle
  - imaging of small structures are affected by view sampling and quantum noise
- Same overall conclusion reached with different phantom, as long as β~3 for phantom volume

Central slice theorem

implies relationships between image volume, volume slice, projection:

- volume:

$$P_{vol}(f_r) \propto f_r^{\beta}$$

- volume slice:

$$P_{sl}(f_r) \propto / f_r^{\beta-1}$$

- projection:

$$P_{proj}(f_r) \propto / f_r^{\beta}$$

Metheany KG et al., Characterizing anatomical variability in breast CT images. Medical Physics 2008;35(10):468

Study: Compressive sensing in CT

- Compressive sensing: Relationship between image sparsity (Δ) and necessary number of samples
- If a sparse representation of image can be found, compressive sensing predicts that exact reconstruction is possible from "fewer" data samples
- Need to know ratio of necessary number of samples to image sparsity, N_s/Δ

CT iterative image reconstruction

- Linear algebra:
  - Reconstruction of NxN CT image
  - need at least NxN equations to solve for NxN unknowns
  - in reality: need 2N views, 2N detector bins (stability)*

*J. H. Joergensen et al.: Analysis of discrete-to-discrete imaging models for tomographic iterative image reconstruction and compressive sensing. To be submitted to Inverse Problems.

Compressive sensing: How many samples are needed?

- How many samples is enough?
  - Theoretical formulas for ratio of necessary number of samples to data sparsity exist only for special random matrices*
- Image sparsity in CT: gradient magnitude
- No theoretical N_s/Δ for CT
  - Phantom studies

* Candes, Wakin, IEEE Signal Processing Magazine 2008

Phantom study (head phantom): image sparsity

f | grad(f) |

N = 256x256 Δ = 2500

Compressive sensing-based image reconstruction for breast CT

	head phantom	
Image sparsity Δ	2,500	
Required N_s for exact reconstruction from discrete image	12 views 512 detector bins	

J. H. Joergensen et al.: Toward optimal X-ray flux utilization in breast CT. 11th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine 2011

Phantom study (breast phantom): image sparsity

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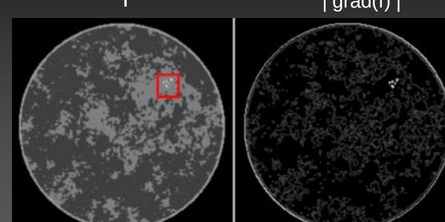

N = 256x256 Δ = 10,000

Compressive sensing-based image reconstruction for breast CT

	head phantom	**breast phantom**
Image sparsity Δ	2,500	10,000
Required N_s for exact reconstruction from discrete image	12 views 512 detector bins	50 views 512 detector bins

Required sampling density derived from head phantom does not result in exact image reconstruction for breast CT

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Image sparsity Δ	2,500	10,000
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N_s/Δ	~2-2.5	2-2.5

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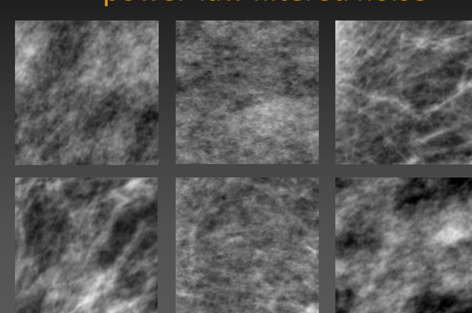
J. H. Joergensen et al.: Toward optimal X-ray flux utilization in breast CT. 11th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine 2011

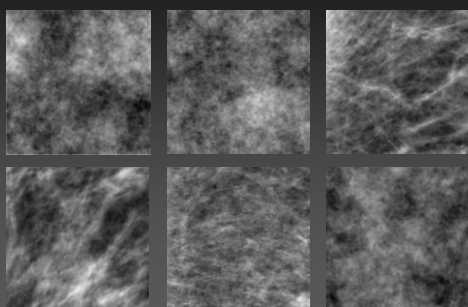
Towards a better phantom?

- Radiologists: Structure in power-law backgrounds “too undefined”
  - add directionality

Mammographic ROIs vs. directional power-law filtered noise

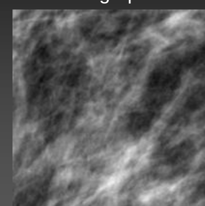


Mammographic ROIs vs. power-law filtered noise



Periodogram when structure has directionality

mammographic ROI



periodogram

directionality rotates power spectrum:

$$P(f) = \frac{c'}{(f^t R^t Q^{-1} R f)^{\beta/2}}$$

R: rotation
Q: scaling

I. Reiser, S. Lee et al.: On the orientation of mammographic structure. Accepted as Medical Physics Letter.

Summary

- Quantify image quality through task-based performance
  - Image quality is a statistical concept
  - Statistical properties of phantom should be similar to those of object of interest
- Statistical phantoms
  - "definition": Structure is due to randomness
  - provide reasonable representation of breast structure at small scale
  - provide insight into imaging fundamentals
    - mass detection in mammography
    - system parameters in tomosynthesis
    - iterative image reconstruction for breast CT

Conclusions

- Statistical phantoms have advantages
  - easy to generate noise realizations
  - "easy" to generate (random number generator)

… and limitations:
  - Representation of large-scale anatomic features (breast anatomy such as Cooper's ligaments, ducts; glandular tissue distribution) is limited
- An inappropriate phantom can give results that do not hold for real data
- Put phantoms to use!

Future work – (open questions)

- Effect of projector
  - physical phantom: continuous-to-discrete projection
  - most software phantoms: Numeric array
  - -> discrete-to-discrete projector is an approximation to continuous-to-discrete projector
  - -> how to minimize artifacts
- Statistical phantoms seem to work reasonably for breast imaging, but what about imaging of other organs?

Closing remarks:

- Phantoms should be available to other researchers!
  - the purpose of phantoms is to advance systems design, image reconstruction
  - phantoms should be easy to use
    - provide algorithm in detail, code, or realizations
    - provide good documentation
    - example: NCAT phantom (Paul Segars)
- Image reconstruction/system design studies can help phantom development

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Make everything as simple as possible, but not simpler.
Albert Einstein