

Photoacoustic Scanning Tomography (PHAST) with Coded Optical Excitation: Theory and Experiment

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Photoacoustic Tomography: Objectives and Methods

Imaging Objectives:

1. Positions and dimensions of photoacoustic (PA) sources.
2. Characteristics of PA sources: absorption coefficient, chemical composition, blood flow rate etc.

Standard PA Methods:

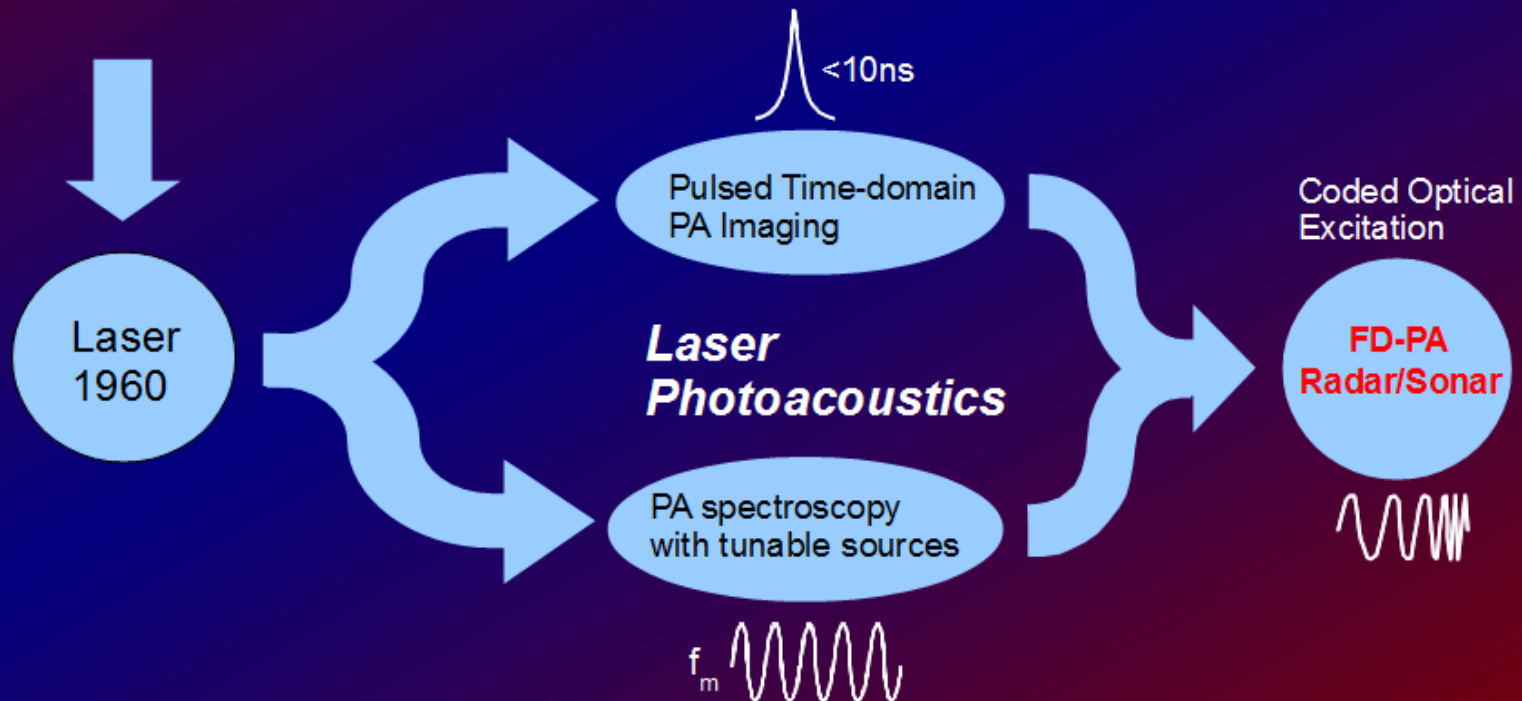
1. Short (nanosecond) laser irradiation and broadband detection.
2. Photoacoustic microscopy with high frequency (> 30 MHz) sources.
3. Photoacoustic spectroscopy with narrow band tunable sources.

Photoacoustics: Historical Developments

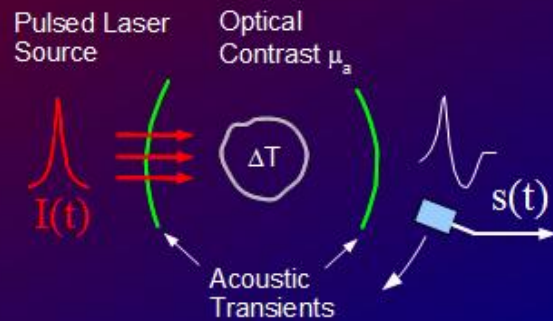
Original Patent of "Photophone"

My invention consists in a method of utilizing radiant energy and of applying it by suitable apparatus to produce audible signals and to produce electric signals.

Alexander G. Bell



Pulsed Laser Photoacoustics



$$\nabla^2 p(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 p(\vec{r}, t)}{\partial t^2} = \frac{-\beta}{C_p} \frac{\partial q(\vec{r}, t)}{\partial t}$$

Confinement conditions:

$$\sqrt{D_T t_L} < \mu_a^{-1}; t_L < \frac{1}{c \mu_a}$$

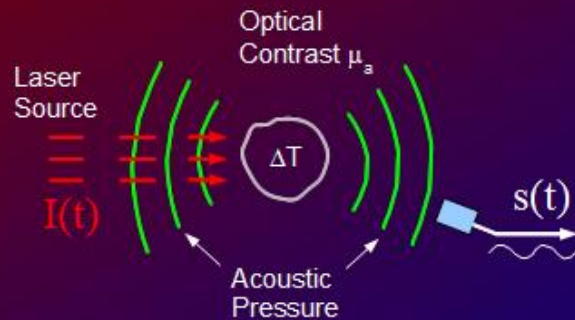
$$q(\vec{r}, t) \approx \mu_a \Phi_0(\vec{r}) \delta(t)$$

$$p_0(z) = \Gamma \mu_a \Phi_0 \exp(-\mu_a z)$$

Key Characteristics

- High magnitude acoustic transients
- Signal profile carries information on light absorption and chromophore dimensions
- Depth information obtained from “time-of-flight” measurements
- Relatively expensive laser source, especially with tunable wavelength and high repetition rate options
- Broadband ultrasound detection – excessive level of noise and interference

Photoacoustic Imaging with Intensity Modulated CW Laser Source (Frequency Domain PA)



$$\nabla^2 \tilde{p}(\vec{r}, \omega) + k^2 \tilde{p}(\vec{r}, \omega) = \frac{-i\omega\beta}{C_p} \tilde{q}(\vec{r}, \omega)$$

$$\tilde{q}(\vec{r}, \omega) = \mu_a(\vec{r}) I(\vec{r}) \tilde{F}(\omega)$$

Confinement conditions in FD:

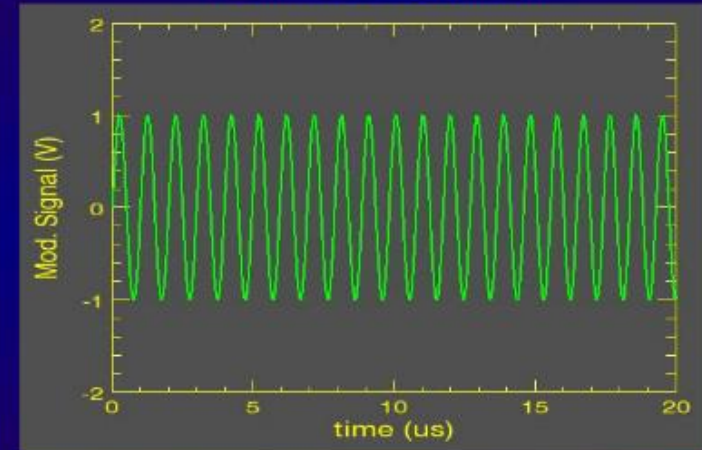
$$L_T = \left(\frac{D_T}{\omega} \right)^{1/2} \ll \mu_a^{-1}$$

$$\omega < \omega_a = \mu_a c_a$$

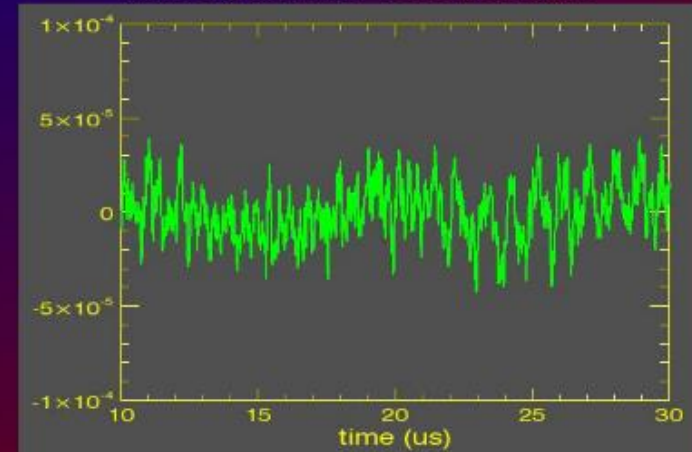
Difficulties of FD photoacoustics:

1. Low optical power (0.1 – 1 W) → Low SNR
2. Long pulse duration (> 1 ms) → Poor spatial resolution

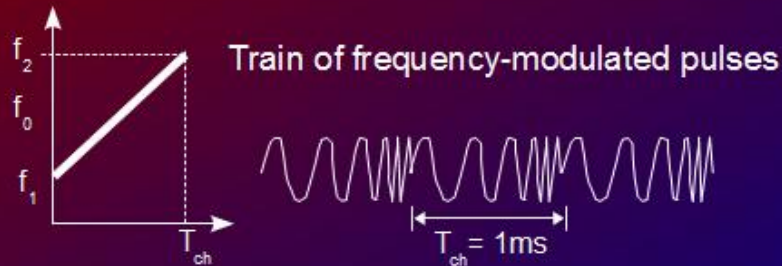
Modulation waveform



Raw signal after 1000 averages

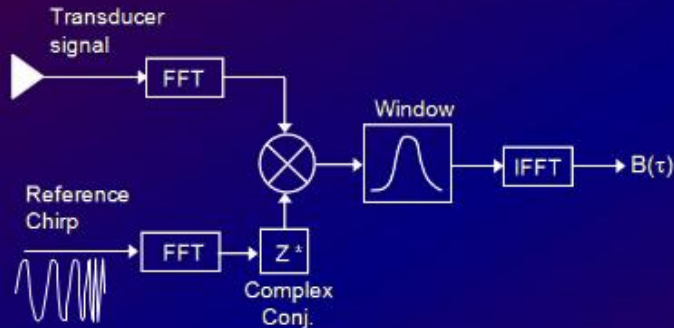


Spatially-Resolved PA Imaging with Chirped Waveforms – Photoacoustic Radar



Correlation Processor

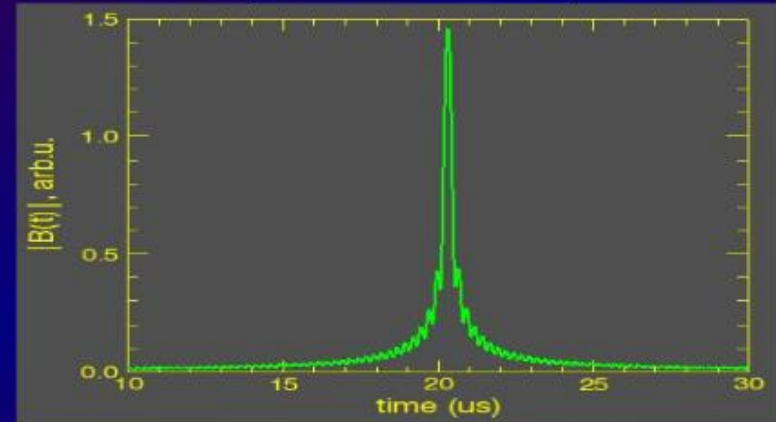
$$B(\tau) = \int_{-\infty}^{\infty} r(t+\tau) s(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{R}^*(\omega) \tilde{S}(\omega) e^{j\omega\tau} d\omega$$



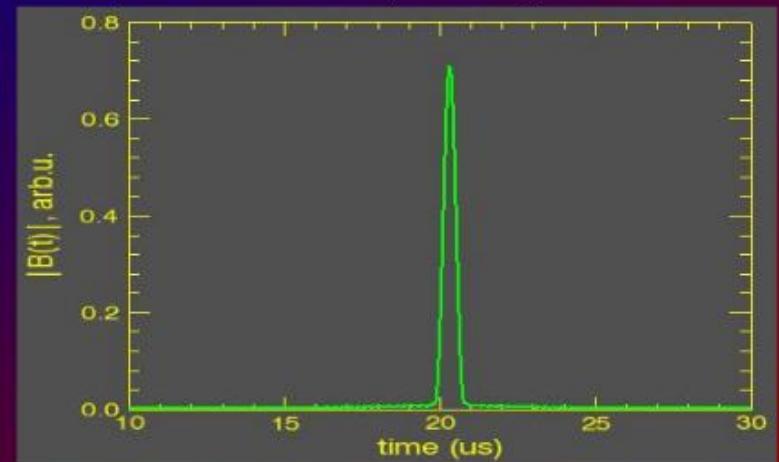
$$B(\tau) = \underbrace{\frac{A^2 T_{ch}}{2}}_{\text{Peak Amplitude}} \underbrace{\frac{\sin\left[\frac{\pi m \tau}{T_{ch}} \left(1 - \frac{\tau}{T_{ch}}\right)\right]}{\pi m \tau / T_{ch}}}_{\text{Side lobes}} \underbrace{\cos(\omega_0 \tau)}_{\text{Harmonic carrier}}$$

$$m = T_{ch} \Delta f \text{ - Time-Bandwidth product}$$

Correlation amplitude of a linear chirp 1 – 5 MHz

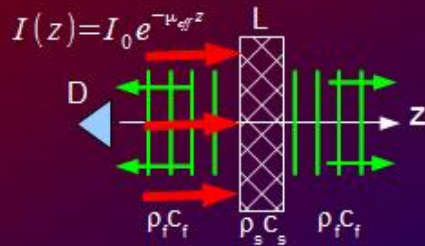


Spectral windowed (Hamming) correlation



Analytical Model of PA Generation

1-D Model with Acoustic Impedance Discontinuity



$$\tilde{p}_{zz}(z, \omega) + k^2 \tilde{p}(z, \omega) = \frac{-i \omega \beta}{C_p} \tilde{q}(z, \omega)$$

$$\tilde{q}(z, \omega) = \mu_a I_0 e^{-\mu_a z} \tilde{F}(\omega), \quad \tilde{F}(\omega) \text{ - Spectrum of the laser modulation waveform}$$

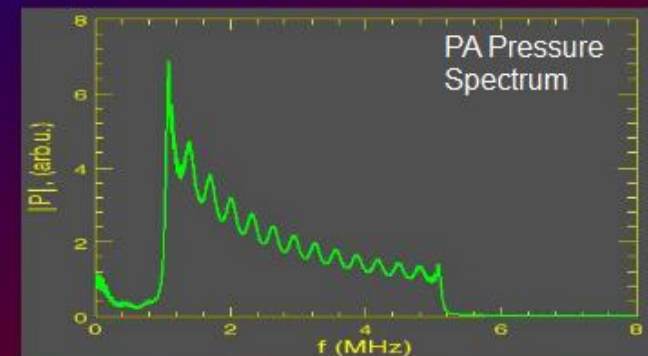
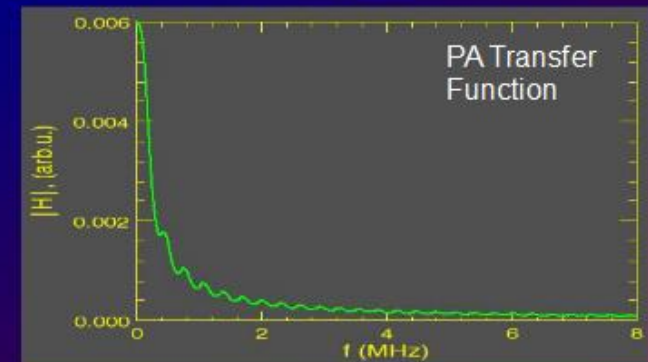
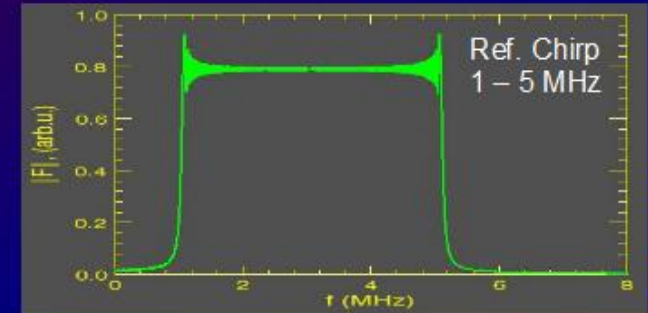
Method of transfer functions:

$$\tilde{p}(\tilde{r}, \omega) = \tilde{H}_{PA}(\omega) \cdot I_0 \cdot \tilde{F}(\omega) \cdot \Phi(\tilde{r})$$

1-D solution for exponential source:

$$H_{PA}(\omega) = \frac{-i \beta \mu_a c_s}{C_p (\mu_a^2 c_s^2 + \omega^2)} \cdot \frac{(\zeta k_f + i \mu_a) \cos(k_s L) + (i k_s - \zeta \mu_a c_s / c_f) \sin(k_s L) - (\zeta k_f + i \mu_a) e^{-\mu_a L}}{i (1/c_s^2 + \zeta^2 / c_f^2) \sin(k_s L) + (2 \zeta / c_s c_f) \cos(k_s L)}$$

$$\tilde{p}(z, \omega) = \tilde{H}_{PA}(\omega) \cdot I_0 \cdot \tilde{F}(\omega) \cdot e^{-i k_f z}, \quad k_f = \omega / c_f$$



Correlation Processing of Chirped PA Signals

Simulation Results for a 1-D layer:

Layer thickness: 5 mm

$\rho_s c_s = 1.54$ MRyals

$\rho_f c_f = 1.48$ MRyals

Absorption: 4 cm^{-1}

Optical Modulation:

Sine chirp: 1 – 5 MHz

Chirp duration: 1 ms

Zero-mean Gaussian noise:

Input SNR = -40 dB

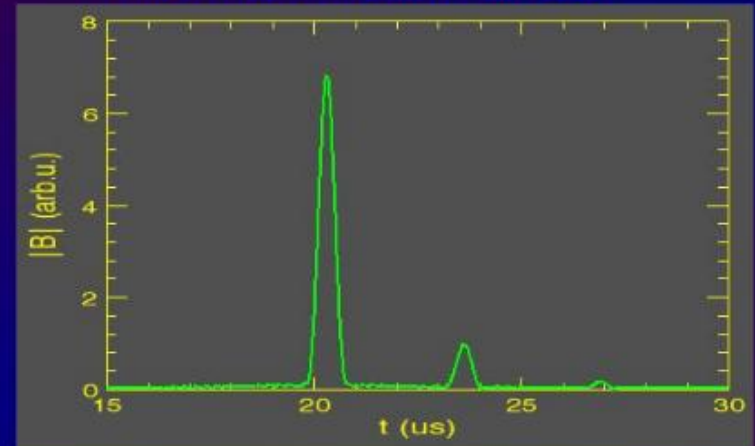
Coherent averaging of 1000 chirps

Axial Resolution: $c_a / \Delta f < 1 \text{ mm}$

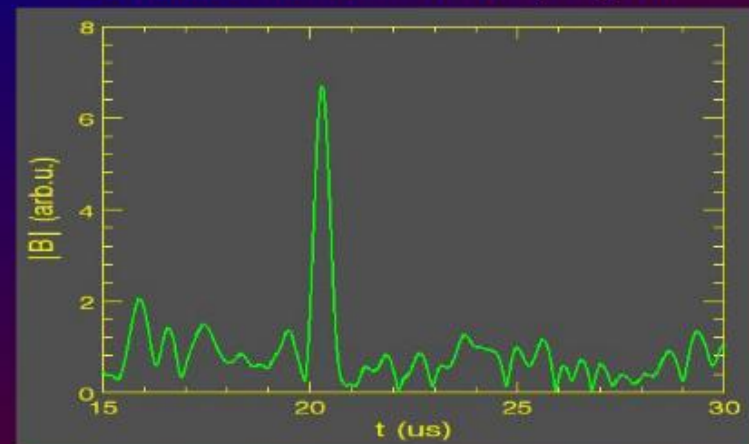
SNR Improvement ~ 56 dB

Estimated SNR of the pulsed PA under the same conditions exceeds that of CW by ~ 10 dB

Noise-free correlation function



Correlation function of noisy signal



Correlation Processing for a Distributed Source

PA Source:

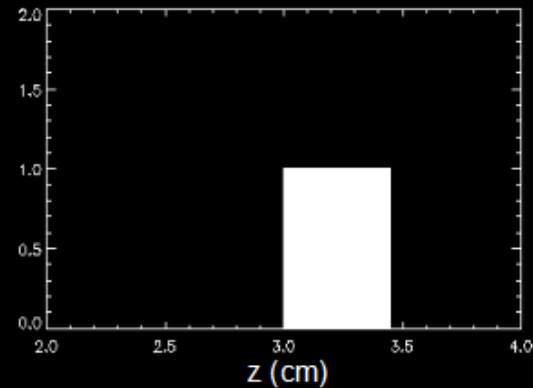
$$q(z, t) = \mu_a I_0 e^{-\mu_a(z-z_0)} f(t)$$

$f(t)$ – linear chirp

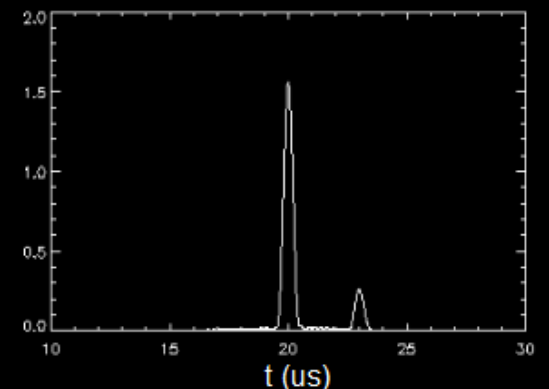
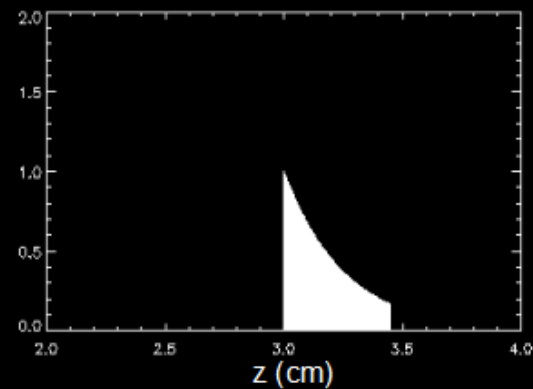
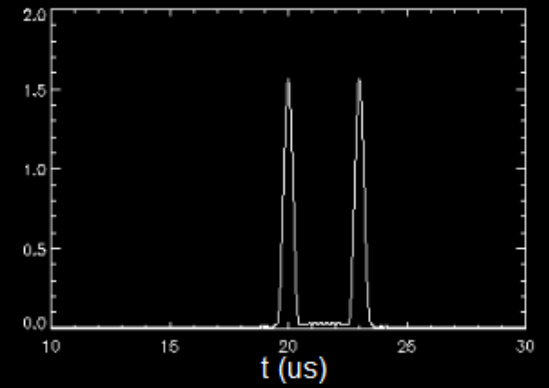
1. Uniform distribution
(weak absorption)

2. Exponential distribution
 $\mu_a = 4 \text{ cm}^{-1}$

Spatially distributed source



Matched filter output



Discontinuities of source function produce correlation peaks

Signal-to-Noise of Frequency Domain PA Measurements

Matched Filter (Correlation):

$$B(\tau) = \frac{1}{2\pi} \int \tilde{R}(\omega) \tilde{S}(\omega) e^{i\omega\tau} d\omega$$

$$|B(\tau)| = \sqrt{\Re^2 B(\tau) + \Im^2 B(\tau)}$$

Gaussian noise PSD:

$$N_0 = \frac{\langle P_N \rangle}{f_s/2} = \frac{2\sigma^2}{f_s}$$

Noise of matched filter
(Rayleigh distribution):

$$PDF = \frac{A}{\sigma_A^2} \exp(-A^2/2\sigma_A^2)$$

$$E[A] = \sqrt{\frac{\pi}{2}} \sigma_A, \sigma_A^2 = \frac{E_s \sigma^2}{f_s}$$

$$Var = 0.43 \sigma_A^2$$

SNR of Matched Filter (Single Chirp):

$$SNR_{MF} = \frac{B^2(0)}{\langle P_{NB} \rangle} = \frac{E_s f_s}{0.43 \sigma^2}$$

Multiple Chirps: Coherent vs
Incoherent Averaging of N_p Chirps

1) Coherent Averaging (Phase retained):

$$\langle P_N \rangle = \frac{\sigma^2}{N_p}; \quad E[B_N] = \sqrt{\frac{\pi E_s}{2 f_s N_p}} \sigma \quad \text{- Noise Background}$$

$$Var[B_N] = 0.43 \frac{E_s \sigma^2}{f_s N_p} \quad \text{- Noise Variance}$$

$$SNR = \frac{(E_s - E[B_N])^2 f_s N_p}{0.43 \sigma^2 E_s}$$

2) Incoherent Averaging (Post processing):

$$B_{av}(\tau) = B(\tau) + \frac{1}{N_p} \sum_{i=1}^{N_p} B_N(\tau) = B(\tau) + n_B$$

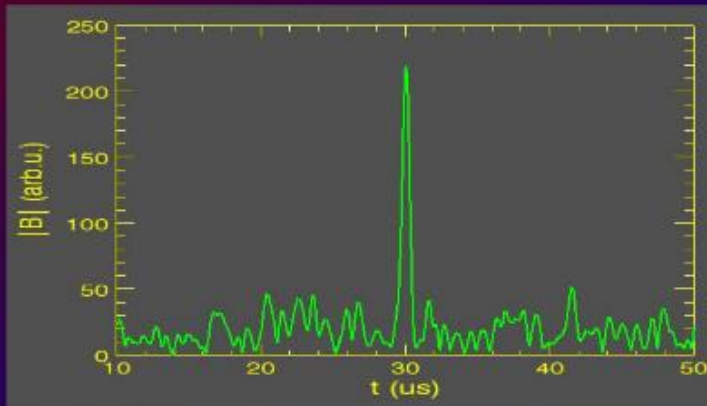
$$E[n_B] = \sqrt{\frac{\pi E_s}{2 f_s}} \sigma; \quad \text{- Independent on } N_p$$

$$SNR = \frac{\left(E_s - \sqrt{\frac{\pi E_s}{2 f_s}} \sigma \right)^2}{0.43 \sigma^2 E_s} f_s N_p$$

SNR of Coherent vs Incoherent Averaging

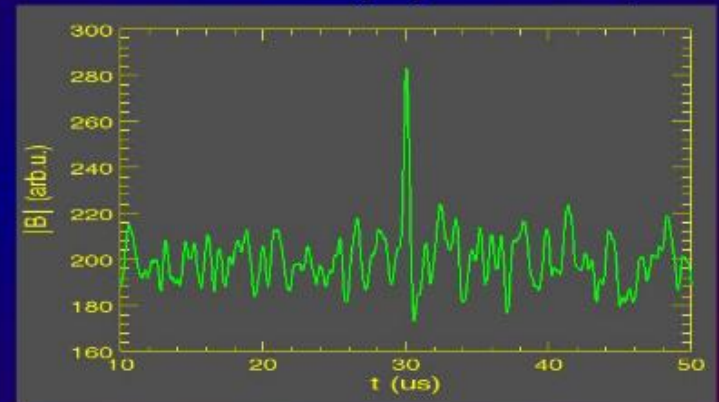
*Two methods of signal detection with different level of the input noise
(zero-mean Gaussian noise with std deviation σ)*

Coherent Averaging of 100 chirps

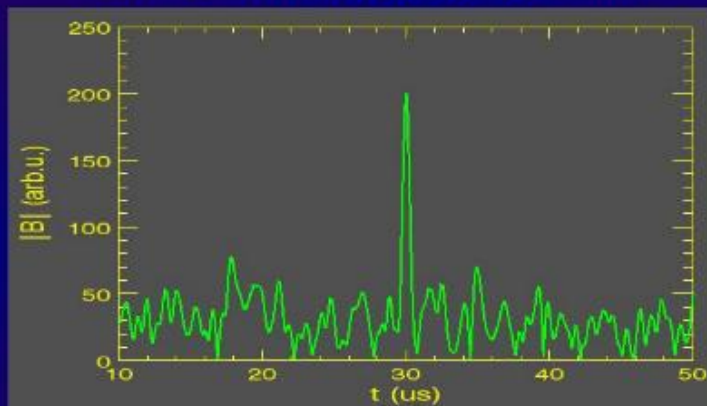


$\sigma = 100$
SNR = -23 dB

Incoherent Averaging of 100 chirps

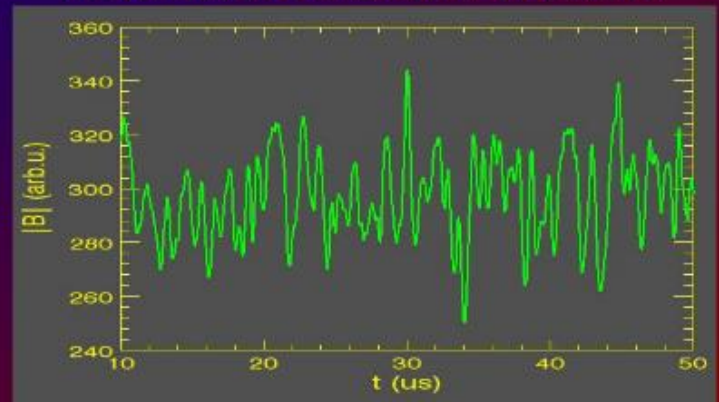


Coherent Averaging of 100 chirps



$\sigma = 150$
SNR = -25 dB

Incoherent Averaging of 100 chirps



SNR and Laser Safety Limit

Maximum Permissible Exposure (1064nm, 10⁻⁷ – 10s):

$$E_{MPE} = 5.5 \cdot T^{1/4} [J/cm^2]$$

SNR of Matched Filter Processing:

$$SNR_{MF} \sim E_s \sim A_s^2 T_{ch}$$

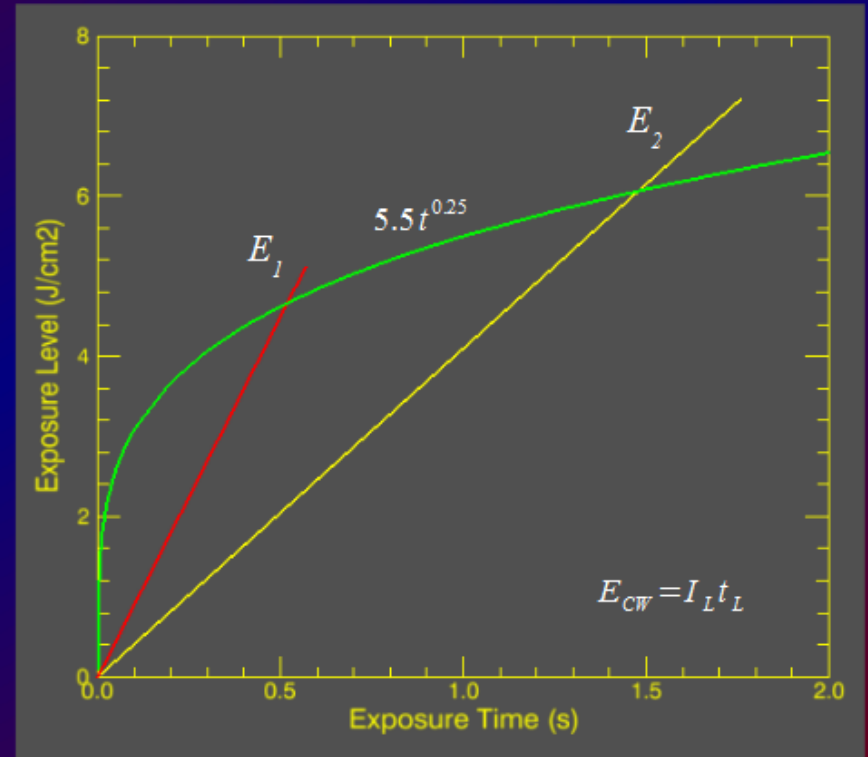
Signal Amplitude:

$$A_s \sim I_L \quad \text{- Laser Irradiance [W/cm}^2\text{]}$$

Assuming: $I_L = I_{MPE} = 5.5 \cdot T_{ch}^{-3/4}$

Then: $SNR \sim I_{MPE}^2 \cdot T_{ch} \sim T_{ch}^{-1/2}$

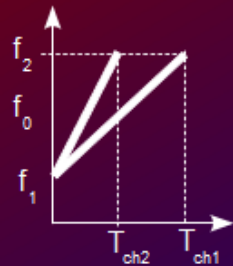
For $I = I_{MPE}$ shorter chirp duration is expected to give higher SNR



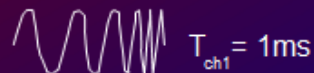
— P = 1.76 W; — P = 0.8 W (diam = 5 mm)

— Laser safety curve

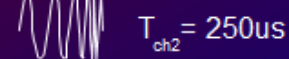
Effect of the Chirp Duration



Chirp: 1 – 5 MHz



$T_{ch1} = 1\text{ms}$



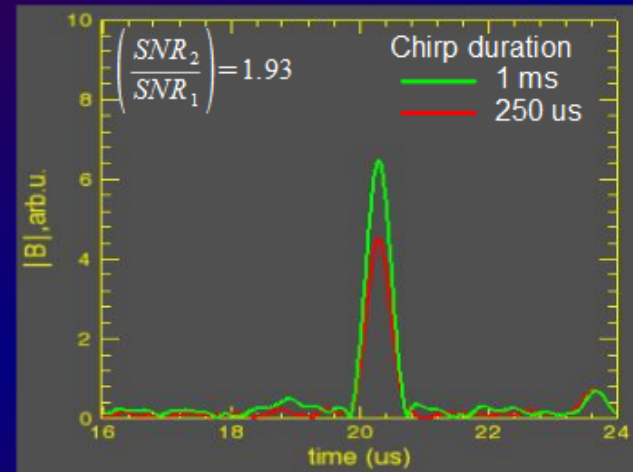
$T_{ch2} = 250\text{us}$

+ Gaussian Noise



Theoretical
SNR increase: $\left(\frac{SNR_2}{SNR_1}\right) = \left(\frac{T_{ch1}}{T_{ch2}}\right)^{1/2} = 2$

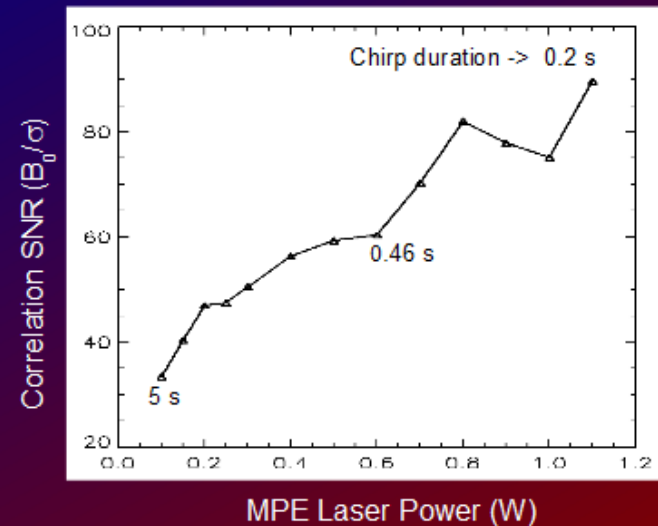
Numerical Simulation using 1D Model



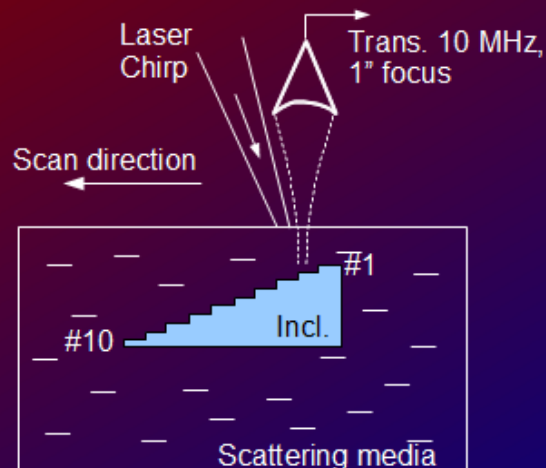
Predicted increase of SNR was
observed in experiments with
chromophores in liquid phantoms



Optical inclusion in scattering media



Axial Resolution Measurements of Correlation PA

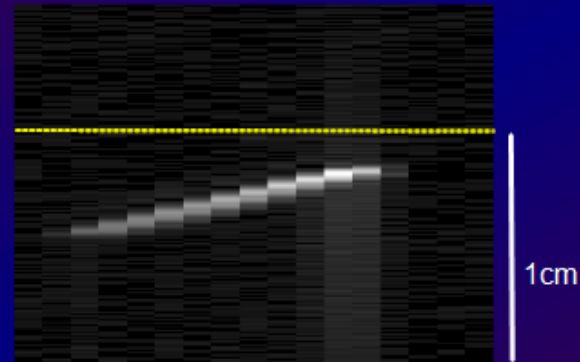


Plastic sample with 10 steps
~ 250 μm height inside of
a scattering substrate

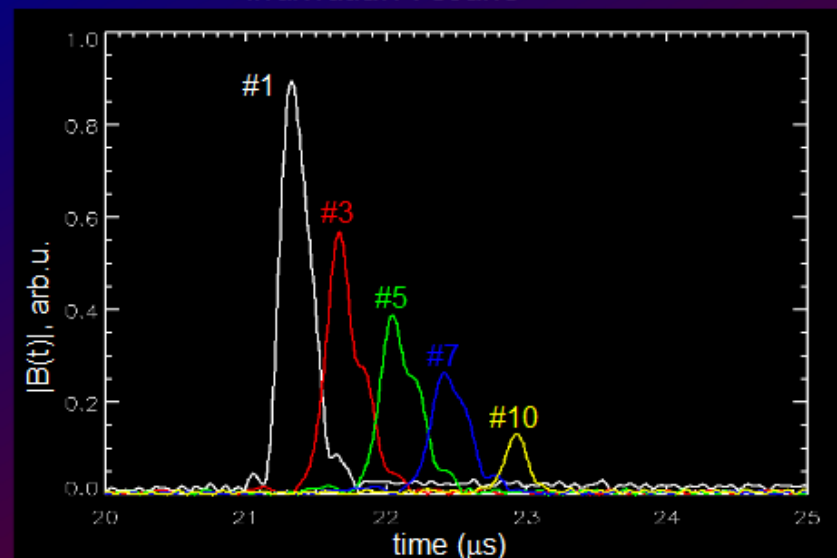
Laser chirp: 1 – 12 MHz, 1 ms

Correlation peak FWHM: ~250 ns
translates into axial resolution ~ 375 μm

2D image composed of 17 A-scans



Individual A-scans



Measurements of Maximum Imaging Depth

1. Modulation chirp: 1 – 5 MHz

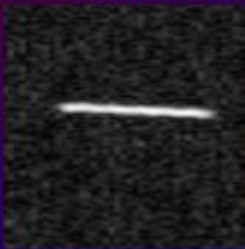
Chromophore: 12x8mm (cross section), $\mu_a = 4\text{ cm}^{-1}$

Transducer: 3.5 MHz, $F = 2.54\text{ cm}$, Laser spot diam = 1.5 mm

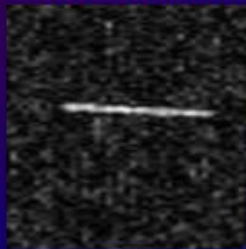
Intralipid solution (0.47%): $\mu_{\text{eff}} \approx 1.3\text{ cm}^{-1}$

Depth, Power

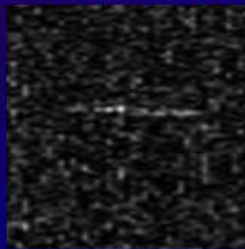
3mm, 190 mW



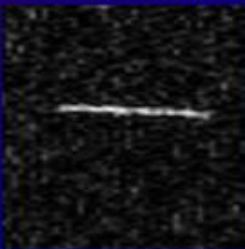
6mm, 190 mW



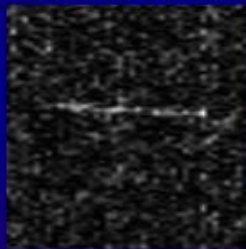
8mm, 190 mW



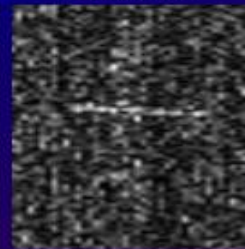
8mm, 540 mW



10mm, 540 mW



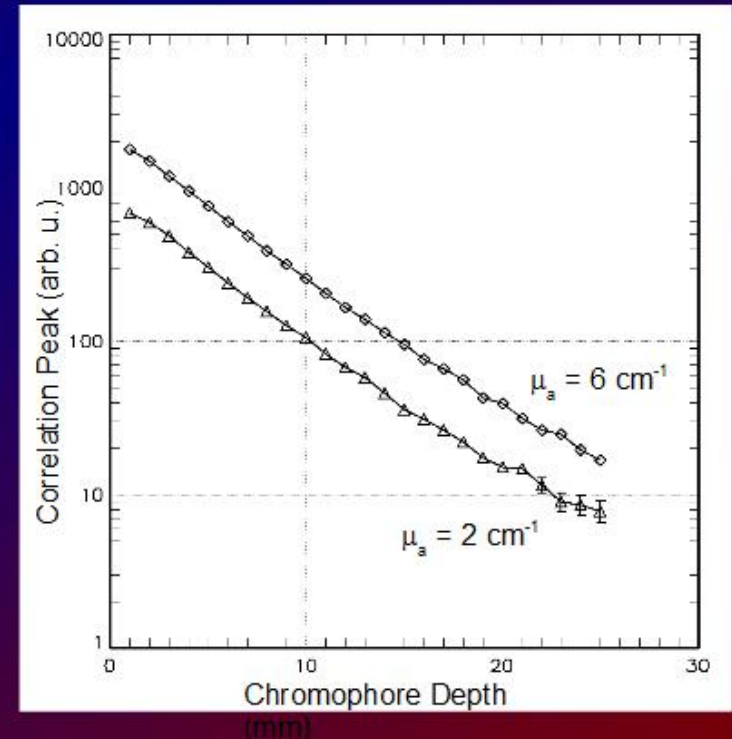
12mm, 1W



2 cm

2. Modulation chirp: 200 – 800 kHz

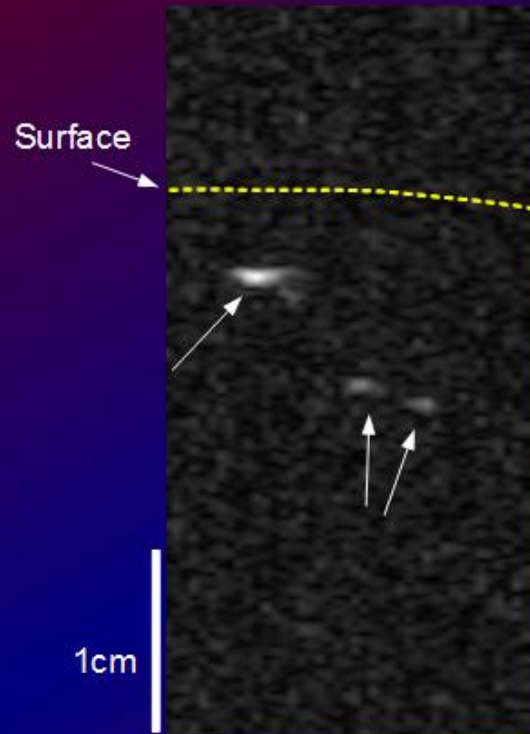
Transducer: 500 kHz, $F = 5\text{ cm}$



Scanning Chromophores in Tissue

Ex-vivo muscle tissue
of small animal (rat)

(Three wires inserted in muscle tissue)



Imaging of blood vessels
in human wrist *in-vivo*

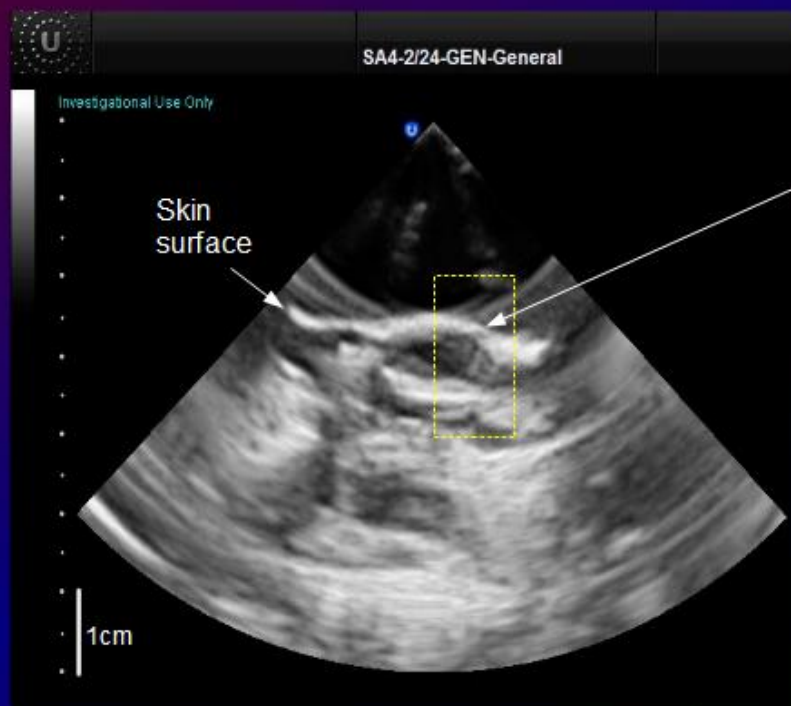
3.5 MHz focused transducer



Imaging of Subcutaneous Cancer in-vivo

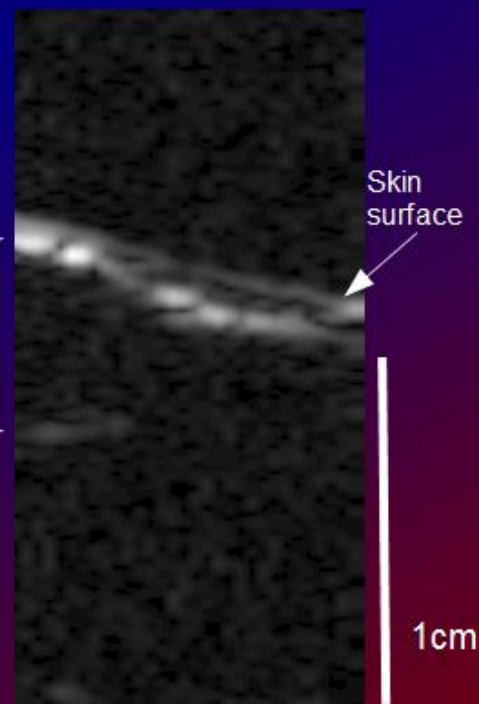
Tumour implanted subcutaneously in a thigh of immunodeficient rat

Conventional Ultrasound Image



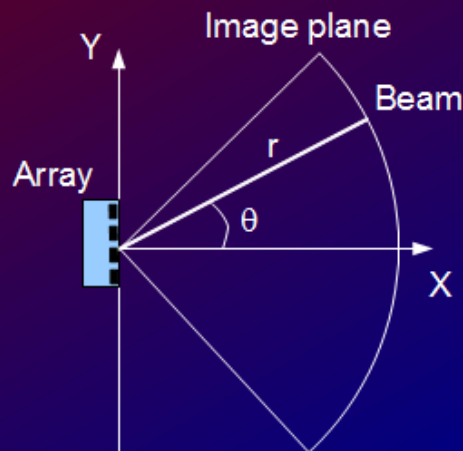
PA Correlation Image
(Chirp: 1 – 5 MHz)

Suspected
Tumour



Phased Array Correlation Imaging

Correlation Phased Array – multichannel matched filter processing and beamforming in frequency domain



1) Array acquisition and FFT of signal matrix

$$\tilde{B}_i(\omega) = \tilde{W}(\omega) \cdot \tilde{R}^*(\omega) \cdot \tilde{S}_i(\omega) \quad - \text{matrix } N_e \times N_t, N_t = 100k$$

2) Digital beamforming, i.e. spatial filtering by creating directional beams and beam steering

$$\tilde{U}(\omega, \theta) = \sum_{n=1}^{N_t} w_n \cdot \tilde{B}_n(\omega) \cdot \exp(-i\omega t_n(\theta)) \quad t_n(\theta) = \frac{y_n}{c_a} \sin(\theta) + t_f$$

$$k = \frac{\omega}{c_a} = k_x^2 + k_y^2 \quad \Rightarrow \quad \tilde{U}(k, \theta) \quad - \text{Spatial spectrum}$$

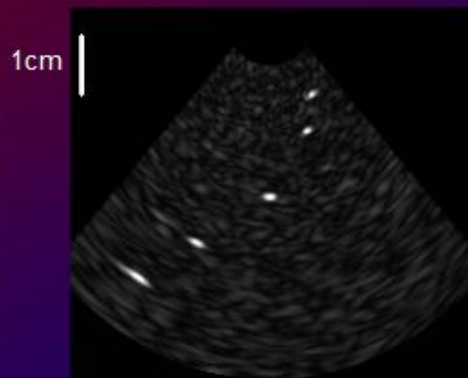
3) Backprojection:

$$u(x, y) = FFT^{-1}[\tilde{U}]$$

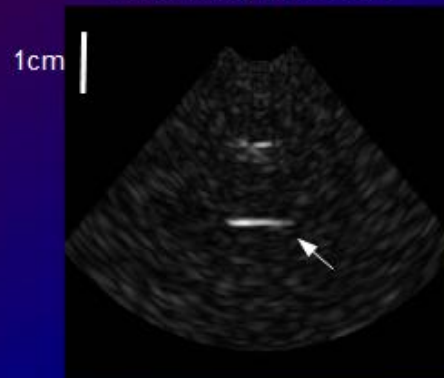
Phased array PA Correlation Imaging

Transducer phased array: 64 elements, 3.5 MHz central frequency
Laser mean power: 1W, spot diam 2.8 mm, chirp $f = 1 - 5$ MHz, 1ms

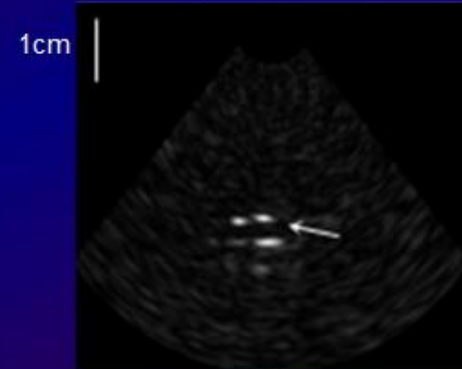
System PSF, SNR = -34dB



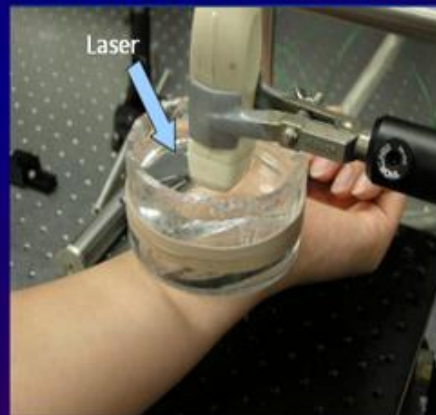
Optical inclusion in scattering phantom



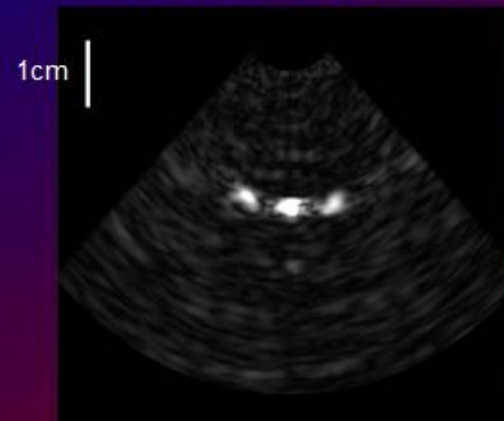
Discrete chromophores in Intralipid



Phased array probe for PA imaging



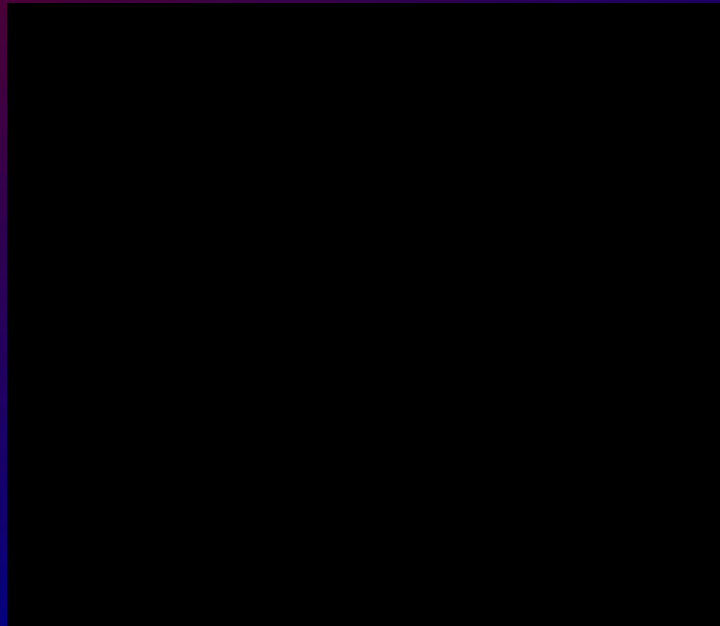
PA correlation image of wrist



Phased Array Correlation PA Imaging in-vivo

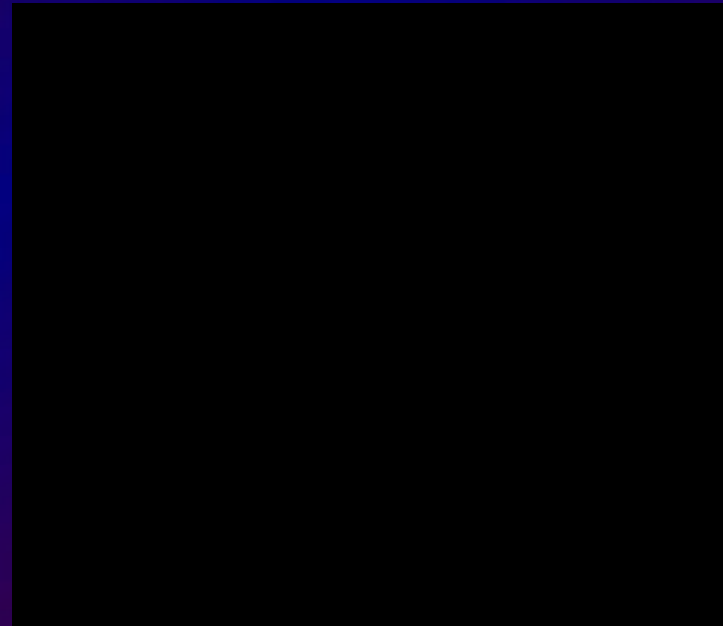
Immunodefficient rats with FaDu (carcinoma cells) injected intramuscularly

(video clip)



Day 4

(video clip)



Day 11

Laser beam (1 W, 1.5 mm in diameter) is scanned along the array axis

Conclusions

Depth-resolved PA imaging with CW laser sources is feasible using coded optical excitation (linear chirps) and correlation signal processing.

Correlation processing of coded PA response provides significant increase of SNR (> 50 dB).

Axial resolution determined by the correlation peak width can be < 300 μm for the chirp bandwidth > 10 MHz.

Imaging of tissue chromophores within 2 cm depth range is feasible with inexpensive laser sources and coded modulation waveforms.

To achieve maximum SNR performance multiple chirps must be averaged coherently in pre-processing and chirp duration should be set according to MPE.

Phased array PA correlation imaging was demonstrated using conventional ultrasound array and a frequency-domain reconstruction algorithm.

Other forms of coded optical excitation including phase-coded waveforms are possible.

Acknowledgements

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